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KART STEERING SYSTEM DESIGN

Luiz Carlos do Vale Filho

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Abstract: This study was conducted with the aim of designing the steering system for the prototype kart being developed by the Optimus Kart team. The ideal is to keep the 4-bar system as close as possible to the system proposed by Ackerman. The first step was to measure the bars of other karts and, based on this, make changes according to the errors found. A comparison was made between the error and the radius of the curve, and a decrease in error was observed with an increase in radius. The importance of the study lies in presenting the optimized arrangements and dimensions of the steering bars.
Keywords: steering, bar, Ackerman, angle.

INTRODUCTION

This paper will show the procedure taken to design the steering system for the kart being developed by the Optimus Kart team, under the guidance of Professor Ademyr Oliveira. Initially, some measurements were taken on karts available in workshops or stores for automotive parts. After measuring the steering system components, adjustments and dimensioning were made. The final step consisted of analyzing the error in the designed system, since the four-bar mechanism adopted deviates from the ideal angular relationship given in Ackerman's theorem.

As stated in the material "Steering Theory," the Ackerman geometry (ideal geometry) for a 4-wheel vehicle without trailers or semi-trailers and with a rear axle without differential is shown in Figure 1.0 below:

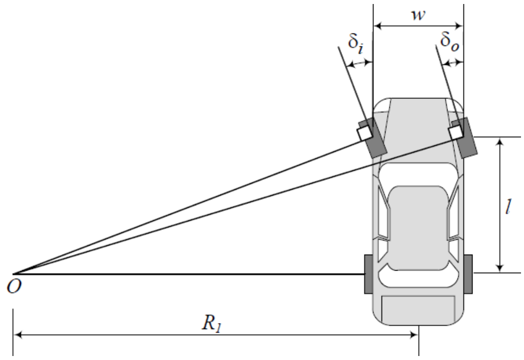


Figure 1.0 - Steering geometry in a curve.

Source: *Steering Theory.*

Figure 1 shows the simplest mechanisms that make it possible for a vehicle such as a kart to turn, and through these mechanisms, Ackerman's condition for turning was developed, shown in equation 1.0 below:

$$cot\delta_0 - cot\delta_i = \frac{\omega}{l} \qquad (1.0)$$

Where δ_0 and δ_i are the angles that the outer and inner wheels make with the car's axis of symmetry during a turn, and ω is the ratio between the width and length of the vehicle, l respectively.

Ackerman's conditions are ideal, but almost impractical. Therefore, in mechanical engineering, more specifically in machine dynamics, four-bar mechanisms are used, which are machines formed by three movable bars and a fourth fixed bar. The law that describes this behavior is Grashof's Law, which is simply a formula used to analyze the type of movement that the mechanism will perform. For continuous movement, the sum of the length of the shortest bar and the longest bar cannot be greater than the sum of the lengths of the other bars.

Figure 2.0 shows an application of the four-bar linkage mechanism in a go-kart.

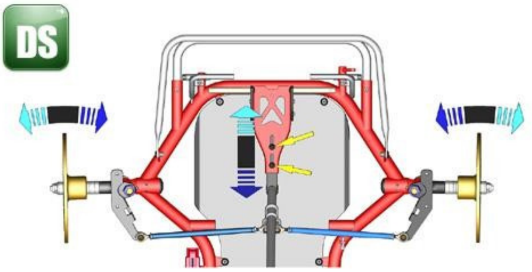


Figure 2.0—Application of the four-bar mechanism in a go-kart.

Source: https://sites.google.com/site/zagarcorse/prodotti

OBJECTIVES

Analyze the 4-bar steering system of a go-kart. Observe the responses of this system, given the movements of the steering wheel, obtain errors from the comparison of this real system with the system proposed by Ackerman, and analyze how these errors will influence performance in a race.

METHODOLOGY

VECTOR EQUATION OF THE FOUR-BAR MECHANISM

The system below (Figure 3.0), developed in GeoGebra, illustrates the vector treatment that will be given to the four-bar mechanism, considering a curve made to the right (the principle adopted is the same as for a curve to the left).

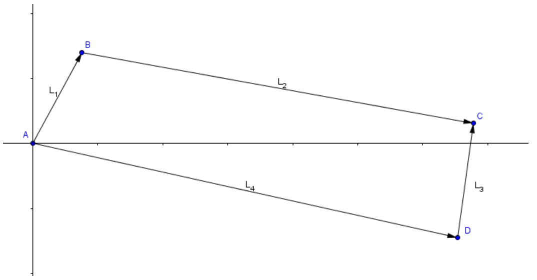


Figure 3.0 - Four-bar vector mechanism.

Source: Authors themselves.

Using vector properties, we obtain:

$$\vec{L}_1+\vec{L}_2=\vec{L}_3+\vec{L}_4 \quad (2.0)$$

Using Euler's relation, which describes complex vectors, each bar can be expressed as follows:

$$\vec{L} = Le^{i\theta} = L(\cos\theta + i\sin\theta) \quad (3.0)$$

Substituting the terms of equation 3.0 into 2.0, we have:

$$L_1(\cos\theta_1+i\sin\theta_1) + L_2(\cos\theta_2 - i\sin\theta_2) = L_3(\cos\theta_3 + i\sin\theta_3) + L_4(\cos\theta_4 - i\sin\theta_4) \quad (4.0)$$

Separating the real and imaginary parts, we obtain expressions 5.0 and 6.0 below:

$$L_1\cos\theta_1+L_2\cos\theta_2 = L_3\cos\theta_3+L_4\cos\theta_4 \quad (5.0)$$

$$L_1\sin\theta_1 - L_2\sin\theta_2 = L_3\sin\theta_3 - L_4\sin\theta_4 \quad (6.0)$$

L_1, L_2, L_3 and L_4 are the lengths of the bars, therefore the modulus of the complex vectors shown in the image. θ_1, θ_2, θ_3 and θ_4 are the angles that the bars make with the x-axis.

From the above equations, the term $\cos\theta_{(4)}$ is isolated in the first equation and the term $\sin\theta_4$ is isolated in the second equation, knowing that the angle θ_4 will not influence the speed transmission mechanism of the bars, since the bar L_4 is static. The expressions obtained are represented by equations 7.0 and 8.0 below.

$$\cos\theta_4 = (L_3\cos\theta_3 + L_2\cos\theta_2 - L_1\cos\theta_1)/L_4 \quad (7.0)$$

$$-\sin\theta_4 = (L_3\sin\theta_3 - L_2\sin\theta_2 - L_1\sin\theta_1)/L_4 \quad (8.0)$$

The two expressions can be combined into one using equation 9.0 below, known as the fundamental trigonometric relation:

$$\sin^2\gamma + \cos^2\gamma = 1 \quad (9.0)$$

γ is the angle considered.

Using equation 9.0 as proposed, we obtain:

$(L_{3\cos}\theta_3 + L_{2\cos}\theta_2 - L_{1\cos}\theta_1)^{(2)} + (L_{3\sin}\theta_3 - L_{2\sin}\theta_2 - L_{1\sin}\theta_1)^{2} = L^2$

RESULTS

ANGLES DURING EXECUTION OF A RIGHT TURN.

After visiting parts stores and kart workshops and taking some measurements, the following was found:

- $\vec{L_1} = 80.0(\cos 61°\,\hat{i} + \sin 61°\,\hat{j})$;
- $\vec{L_2} = 320.0(\cos 13°\,\hat{i} - \sin 13°\,\hat{j})$;
- $\vec{L_3} = 125.0(\cos 85°\,\hat{i} + \sin 85°\,\hat{j})$;
- $\vec{L_4} = 362.8(\cos 21°\,\hat{i} - \sin 21°\,\hat{j})$;

Therefore, the lengths of bars 1 to 4 are: 80mm, 320mm, 125mm, and 362.8mm, respectively. To find θ_1 as a function of θ_3 and the lengths of the bars and angles, the following program, written in Matlab, was used:

```
function wheel_angles i = [];
o= [];
for a= 1:105; b = 61+a;
e= 61-a;
c= 80*sind(b) - 71.98;
d= 80*cosd(b)+ 311.8;
C= 80*sin(e) - 71.98;
D= 80*cosd(e)+311.8; syms x
outerwheel= solve(c*sqrt(1-x^2)+ d*x ==(-116013.4 +c^2 +d^2)/250,x); syms x
innerwheel = solve( C*sqrt(1-x^2)+ D*x ==(-116013.4 +C^2 +D^2)/250,x); tetai= double(innerwheel);
thetao= double(outerwheel); i(a) = 85 - acosd(thetao);
o(a) = acosd(tetao) - 85; end
vector_i = i' vector_o= o' end
```

This program simulates all angles that the wheels will make with the normal, for steering wheel rotations from 0° to -105°. That is, for variations in the bars of up to 105° clockwise, all angles δ_i and δ_0 of the four-bar mechanism were obtained for a right turn. This resulted in Table 1.0 on the next page:

MECHANISM ERRORS

Using the angles of the inner wheel to the curve with the normal referring to the 4-bar system, found in the program made in Matlab, the external angles corresponding to these were calculated according to Ackerman's formula. Then, an error was found that re-related the two sets of external angles, Ackerman's and the 4-bar system. The data are presented in Table 2.0.

Equation 10.0 shows how the error mentioned in the previous paragraph was calculated.

$$e = \frac{|\delta_{A0} - \delta_{D0}|}{\delta_{D0}} * 100$$

δ_{D0} refers to the exit angle of the 4-bar system.

δ_{A0} corresponds to the exit angle of the Ackerman system.

Angular variation of the steering wheel (degrees)	Angle of the inner wheel to the curve with the normal (degrees)	Angle of the outer wheel to curve with the normal (degrees)
1	0.4993	0.8146
2	1.1511	1.4765
3	1.7994	2.1417
4	2.4441	2.8100
5	3.0852	3.4814
6	3.7226	4.1559
7	4.3562	4.8334
8	4.9860	5.5138
9	5.6118	6.1970
10	6.2336	6.8830
11	6.8514	7.5718
12	7.4650	8.2632
13	8.0743	8.9573
14	8.6794	9.6539
15	9.2801	10.3530
16	9.8764	11.0546
17	10.4681	11.7585
18	11.0552	12.4648
19	11.6377	13.1734
20	12.2153	13.8841
21	12.7881	14,5970
22	13.3560	15.3121
23	13.9189	16.0292
24	14.4767	16.7483
25	15.0293	17.4693
26	15.5767	18.1922
27	16.1187	18.9170
28	16.6553	19.6435
29	17.1864	20.3718
30	17.7119	21.1018
31	18.2318	21.8333
32	18.7458	22.5665
33	19.2541	23.3012
34	19.7564	24.0373
35	20.2527	24.7748
36	20.7429	25.5138
37	21.2270	26.2540
38	21.7048	26.9954
39	22.1762	27.7381
40	22.6413	28.4819
41	23.0998	29.2267
42	23.5517	29.9726
43	23.9969	30.7195

44	24.4355	31.4672
45	24.8671	32.2158
46	25.2919	32.9652
47	25.7097	33.7153
48	26.1204	34.4661
49	26.5239	35.2175
50	26.9203	35.9694
51	27.3093	36.7217
52	27.6910	37.4744
53	28.0652	38.2274
54	28.4319	38.9807
55	28.7911	39.7340
56	29.1426	40.4875
57	29.4864	41.2409
58	29.8224	41.9942
59	30.1505	42.7472
60	30.4708	43.4999
61	30.7831	44.2522
62	31.0874	45.0039
63	31.3836	45.7549
64	31.6717	46.5051
65	31.9515	47.2544
66	32.2232	48.0025
67	32.4866	48.7494
68	32.7417	49.4948
69	32.9883	50.2385
70	33.2266	50.9804
71	33.4565	51.7202
72	33.6778	52.4577
73	33.8906	53.1926
74	34.0949	53.9246
75	34.2906	54.6533
76	34.4777	55.3785
77	34.6562	56.0996
78	34.8259	56.8164
79	34.9871	57.5282
80	35.1395	58.2345
81	35.2831	58.9347
82	35.4181	59.6280
83	35.5443	60.3138
84	35.6617	60.9910
85	35.7704	61.6588
86	35.8703	62.3158
87	35.9614	62.9609
88	36.0438	63.5924

89	36.1173	64.2086
90	36.1821	64.8076
91	36.2380	65.3870
92	36.2852	65.9441
93	36.3236	66.4760
94	36.3532	66.9791
95	36.3741	67.4493
96	36.3862	67.8821
97	36.3895	68.2722
98	36.3841	68.6139
99	36.3699	68.9007
100	36.3470	69.1258
101	36.3154	69.2818
102	36.2751	69.3615
103	36.2262	69.3579
104	36.1685	69.2650
105	36.1022	69.0780

Source: The authors.

Angle of the outer wheel of the Ackerman system (degrees)	Outer wheel angle of the 4-bar system (degrees)	Error
0.497134	0.8146	38.97
1.139653	1.4765	22.81
1.771590	2.1417	17.28
2.393089	2.81	14.84
3.004388	3.4814	13.70
3.605628	4.1559	13.24
4.196949	4.8334	13.17
4.778581	5.5138	13.33
5.350569	6.197	13.66
5.913137	6.883	14.09
6.466505	7.5718	14.60
7.010713	8.2632	15.16
7.545886	8.9573	15.76
8.072323	9.6539	16.38%
8.590054	10.353	17.03
9.099283	11.0546	17.69
9.600036	11.7585	18.36
10.092508	12.4648	19.03
10.576890	13.1734	19.71
11.053118	13.8841	20.39
11.521459	14.597	21.07
11.982007	15.3121	21.75
12.434856	16.0292	22.42

12.880095	16.7483	23.10
13.317812	17.4693	23.76
13.748167	18.1922	24.43
14.171163	18.917	25.09
14.586953	19.6435	25.74
14.995611	20.3718	26.39
15.397206	21.1018	27.03
15.791881	21.8333	27.67
16.179546	22.5665	28.30
16.560489	23.3012	28.93
16.934615	24.0373	29.55
17.302053	24.7748	30.16
17.662852	25.5138	30.77
18.017134	26.254	31.37
18.364869	26.9954	31.97
18.706097	27.7381	32.56
19.041003	28.4819	33.15
19.369476	29.2267	33.73
19.691620	29.9726	34.30
20.007466	30.7195	34.87
20.317180	31.4672	35.43
20.620575	32.2158	35.99
20.917882	32.9652	36.55
21.209050	33.7153	37.09
21.494092	34.4661	37.64
21.773024	35.2175	38.18
22.045992	35.9694	38.71
22.312867	36.7217	39.24
22.573791	37.4744	39.76
22.828699	38.2274	40.28
23.077659	38.9807	40.80
23.320736	39.734	41.31
23.557860	40.4875	41.81
23.789090	41.2409	42.32
24.014417	41.9942	42.81
24.233832	42.7472	43.31
24.447455	43.4999	43.80
24.655203	44.2522	44.28
24.857127	45.0039	44.77
25.053207	45.7549	45.24
25.243487	46.5051	45.72
25.427880	47.25	46.19

25.606559	48.0025	46.66
25.779430	48.7494	47.12
25.946532	49.49	47.58
26.107769	50.2385	48.03
26.263305	50.9804	48.48
26.413108	51.7202	48.93
26.557077	52.4577	49.37
26.695306	53.1926	49.81
26.827823	53.9246	50.25
26.954589	54.6533	50.68
27.075627	55.3785	51.11
27.190961	56.0996	51.53
27.300482	56.8164	51.95
27.404405	57.5282	52.36
27.502555	58.2345	52.77
27.594949	58.9347	53.18
27.681733	59.628	53.58
27.762793	60.3138	53.97
27.838142	60.991	54.36
27.907859	61.6588	54.74
27.971890	62.3158	55.11
28.030246	62.9609	55.48
28.083002	63.5924	55.84
28.130037	64.2086	56.19
28.171487	64.8076	56.53
28.207231	65.387	56.86
28.237403	65.9441	57.18
28.261943	66.476	57.49
28.280855	66.9791	57.78
28.294207	67.4493	58.05
28.301936	67.8821	58.31
28.304044	68.2722	58.54
28.300595	68.6139	58.75
28.291524	68.9007	58.94
28.276894	69.1258	59.09
28.256703	69.2818	59.21
28.230947	69.3615	59.30
28.199687	69.3579	59.34
28.162789	69.265	59.34
28.120375	69.078	59.29

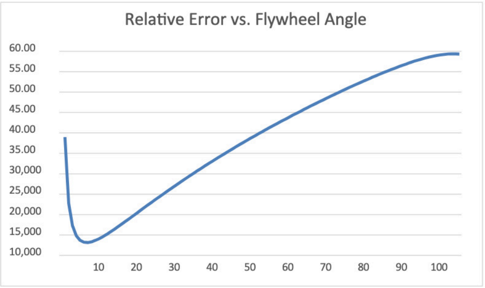
Table 2.0 – Error calculation.

Source: Authors.

ANALYSIS OF RESULTS

Error analysis

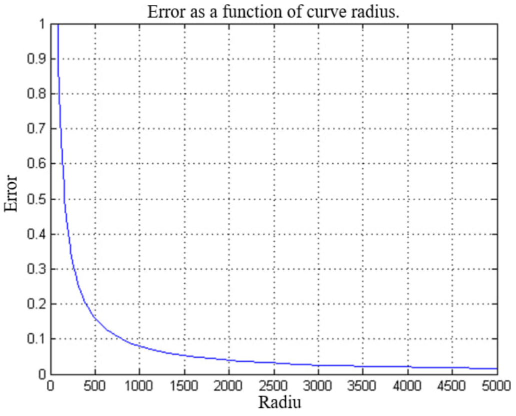
The values found for the percentage error show that a real 4-bar system does not fully comply with the Ackerman effect. The first error found was 38.97%, which decreased until reaching a minimum of 13.17% when the angular variation of the steering wheel was 7°. From this value onwards, the error began to increase with the increase in the variation caused in the steering wheel and reached its maximum, 59.34%, in the penultimate value worked, 104° of variation. Graph 1.0 illustrates this comparison:



Graph 1.0 – Relationship between error and flywheel angle.

Source: authors themselves.

Another interesting aspect to note is the relationship between the error and the radius of the curve that the vehicle makes and the relationship between the steering wheel angle and the error. This relationship shows that the greater the radius of the curve, the smaller the error. This can be seen in graph 2.0 below:



Graph 2.0 - Relationship between curve radius and system error.

Source: authors themselves.

Space required for the vehicle to make a curve

The kinematic steering conditions are used to calculate the space required to make the turn. The maximum radius is the outermost point of the vehicle, while the minimum radius is the innermost point. The difference between these radii is the space required (ΔR).

Equation 12:

$$\begin{aligned}\triangle R &= \sqrt{\left(\frac{l}{\tan\delta_i}+2w\right)^2+(l+g)^2}-\frac{l}{\tan\delta_i}\\ &= \sqrt{\left(\frac{l}{\tan\delta_o}+w\right)^2+(l+g)^2}-\frac{l}{\tan\delta_o}+w.\end{aligned}$$

ΔR means the space required by the car to make a turn. Thus, the minimum, maximum, and average spaces are given in Table 3.0. Using $w= 0.7\ m$, $l= 1.4\ m$, and $g = 0.2\ m$, the table was constructed.

	Minimum ($\delta_o = 0.49°$)	Maximum ($\delta_o = 28.12°$)	Average ($\delta_o = 19.99°$)
Space	1.889 meters	13.12 meters	2.26 meters

Table 3.0 – Minimum, maximum, and average radii and spaces.

Source: authors themselves.

CONCLUSION

The ultimate goal of the work was achieved, and it was possible to analyze the 4-bar system thoroughly. As mentioned earlier, the system presents some errors when compared to Ackerman, but these were expected due to the impossibility of building an ideal system.

After observing the results, it became even clearer with the construction of graph 0.0 that the smaller the radius, the greater the error observed in the system. This shows consistency, since in sharper curves with greater maneuvering difficulty, where the steering angle is also greater, the system presents more failures.

The next step is to analyze the causes of these errors and find ways to avoid them and consequently optimize the steering system so that it responds efficiently to driver input. The study should continue to be detailed due to the importance of this system in the behavior, especially dynamic, of motor vehicles such as karts.

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