v. 5, n. 6, 2025

Journal of
Engineering
Research

Acceptance date: 26/06/2025

DEVELOPMENT OF A DIGITAL IMAGE THINNING SYSTEM ON CELLULAR COMPLEXES

Alfredo Trejo-Martinez
TecNM-Tecnológico de Estudios Superiores del Oriente del Estado de México.
Web of Science E-5531-2012
0000-0001-6555-2285

Andriy Kryvko
IPN ESIME ZAC.
Web of Science HGC-5326-2022
0000-0001-8544-699X

Miguel Angel Martinez-Cruz
IPN ESIME ZAC.
Web of Science ADX-7792-2022
0000-0002-4431-9262

Laura Leonor Mira Segura
TecNM-Tecnológico de Estudios Superiores del Oriente del Estado de México.
Web of Science KPB-4924-2024
0000-0002-8131-2689

Humberto Dorantes Benavidez
TecNM-Tecnológico de Estudios Superiores del Oriente del Estado de México.
Web of Science KFQ-2120-2024X
0000-0003-8544-699X

 DOI https://doi.org/10.22533/at.ed.317572526067

Abstract: The present work focuses on the development and implementation of the algorithm proposed by Kovalesky, to digitally identify an object with its digital skeleton. The proposal will be performed on two cell complexes: quadratic and hexagonal. Once the digital skeleton is generated, it will be compared with the geometric representation defined by Harry Blum in 1967, called Blum's Skeleton. Finally, an analysis of the results obtained is performed to conclude which cellular complex preserves the geometric and topological properties of the initial object, such as bifurcation points, branching and connectivity. It is located in two areas of mathematics, related to Digital Image Processing, in particular, with skeletonization methods: Digital Topology and Digital Geometry. A thinning algorithm proposed in 2001 by Kovalevsky, for binary digital images of dimension two, modeled by cell complexes, is experimented on the hexagonal and quadratic cell complexes. For the hexagonal and quadratic complex, Kovalevsky's algorithm is developed and implemented as a pattern-mapping method within this work. The skeletons obtained in various experiments are analyzed with respect to some topological and geometrical properties, and are compared with Blum's skeleton.
Keywords: Pixel, digital image, skeleton, thinning, cell complex, digitization, connectivity, hole, boundary.

INTRODUCTION

Mathematically a digital image is modeled as a function whose domain is a discrete bounded subset of D of R^n, with domain of defined on a bounded subset of Z. The elements belonging to the set D are named pixels.

In the present work, only digital images of dimension R^2were considered. The need for digitization arises from the desire to process images for preservation, organization, editing, space saving, reuse, analysis, etc. Digital Image Processing (DIP) was originally an area of Engineering, but nowadays this field is deeply related to many areas of mathematics, especially Digital Geometry and Digital Topology. Its objective is to contribute to the analysis and understanding of information present in images, with the help of a computer.

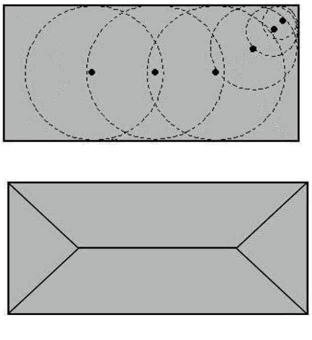
In 2001, Kovaleksky proposed a thinning algorithm [1], for binary digital images modeled by cell complexes, which is very general and brief. Thus, in the present work we aim to develop and implement Kovalevsky's thinning algorithm for the quadratic and hexagonal cell complex, and thus make a comparison between the achieved skeletons and Blum's skeleton, for the same object.

BLUM'S SKELETON

The definition of Blum's skeleton was proposed and developed by Blum (also called as transformation to the medial axis) in 1967 [2], a transformation that maps R (closed and bounded nonempty connected subset of R^2) onto *S* (skeleton of R).

Considering R^2as a metric space we can obtain the elements of S as follows:

In $R^2$with the Euclidean metric, if p is a skeleton point of R, then p is the center of a circle of maximum radius that fits inside R. This means that p is the center of a disk contained in R, and p does not belong to any larger disk (of different radius) contained in R.



a) (b)

Fig. 1 In figure (a) we have some of the circumferences of maximum radius contained in the object. In (b) we show the set of all the centers of the circles as in (a), such set is the Blum skeleton of the given rectangle.

Journal of Engineering Research ISSN 2764-1317 DOI https://doi.org/10.22533/at.ed.317572526067

It should be noted that to obtain the Blum skeleton, for a Euclidean object, no geometric construction (with ruler and compass) or algorithm is known. We only have the definition [[3], p. 26], which does not provide a method for constructing the skeleton, and it is known that the skeleton is contained in the set of centers of maximal circles that fit inside the object. So, in the experiments performed in this paper, Blum's skeleton was constructed in an approximate way, by determining such a set of centers of circumferences, in some experiments manually (with pencil and paper), in another, with experiments in a simulated manual way, as in Figure 1.

One of the purposes is to argue by means of examples that the definition of Blum's skeleton or mean axis is not applicable for objects digitized on the digital plane.

In this paper we take as digital plane the topological subspace Z^2, which as a topological subspace of R^2, is a discrete topological space. That is, the Euclidean plane is decomposed into squares, where each unit square center is identified by a mark, each square is called a pixel, this representation is also known as the pixel plane ([4], p. 21).

The elements of the digital plane are related by one of the following neighborhood relations:

Let $x = (x_1, y_1)$ and $y = (x_2, y_2)$ be elements of Z^2, then:

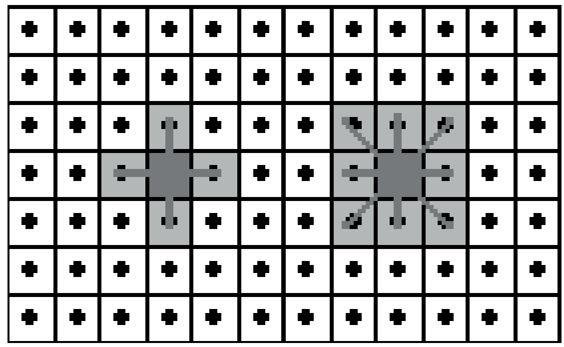
1) y is called 4-neighbor of x if y belongs to the set $\{(x_1+1, y_1), (x_1-1, y_1), (x_1, y_1+1), (x_1, y_1-1)\}$. The set $N_4(x) = \{y \in Z(2): y = x$ or y is 4-neighbor of x$\}$ is called 4-neighborhood.

2) y is called 8-neighbor of x if y is 4-neighbor of x or y belongs to the set $\{(x_1+1, y_1+1), (x_1-1, y_1-1), (x_1+1, y_1-1), (x_1-1, y_1+1)\}$. The set $N_8(x) = \{y \in Z^2: y= x$ or y is 8-neighbor of x$\}$ is called 8-neighborhood.

On the digital plane the following metrics are considered:

1) for x, $y \in Z(2)$, $d_{(4)}(x, y) = |x_{(1)} - x_2| + |y_{(1)} - y_{(2)}|$, it is the metric called "Manhattan metric".

2) for x, $y \in Z(2)$, $d_{(8)}(x, y) = \max\{|x_{(1)} - x_{(2)}|, |y_{(1)} - y_{(2)}|\}$, is the metric called "Chess metric".



a) 4 - neighborhood b) 8 - neighborhood

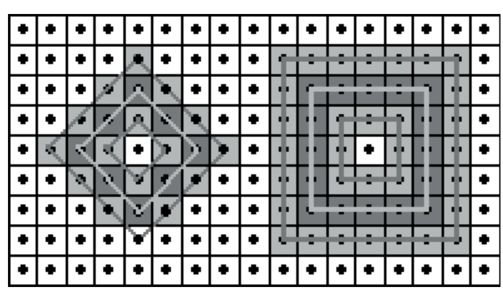
Fig. 2 Neighborhood relations

Based on the relations d_4 and d_8 we can define the following figures:

Let $r > 0$ and $x \in Z(2)$;

We define the closed disk with radius r around x as the set $D_{r,k}(x) = \{y \in Z(2): d_k(x, y) \leq r\}$, for $k \in \{4, 8\}$.

We define the circumference with radius r around x as the set $C_{r,k}(x) = \{y \in Z(2): d_k(x, y) = r\}$, for $k \in \{4, 8\}$.



a) Circle with center x, y radius r = 1,2,3 y metric d_4 b) Circle with center x, y radius r = 1,2,3 y metric d_8

Fig. 3. The circumference

Journal of Engineering Research ISSN 2764-1317 DOI https://doi.org/10.22533/at.ed.317572526067

The digitization of a subset of the Euclidean plane, which we can also call an object, is the capture of it by some mapping to the digital plane. The result is a subset of the digital plane, called a digital object. In [4], two digitization mappings, developed by C.F. Gauss and C. Jordan, have been established.

1. Let M be a subset of the Euclidean plane. The Gaussian digitization G (M) is the union of the squares (pixels) whose center is contained in M.
2. Let M be a subset of the Euclidean plane. Let $J(-)\ (h_{)\ (}M)$ be the union of all squares (pixels) which are completely contained in M, and let $J^{+}_{(h)\ (}M)$ be the union of all squares which intersect M. $J^{(-)\ (}{}_{h)\ (}M)$ is called the inner Jordan digitization of M and $J^{+}_{(h)\ (}M)$ is called the outer Jordan digitization of M.

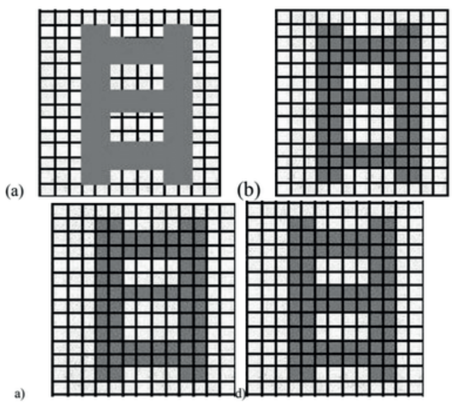


Fig. 4 Figure a) shows the Euclidean object and in figure b) we have the result of the Gaussian digitization, as well as in c) we have Jordan's inner digitization and in d) Jodan's outer digitization.

Taking into account some of the digitization processes we can apply the definition of Blum's skeleton to the digital plane since we have defined the concept of circumference in the digital plane. Let's look at figure 5.

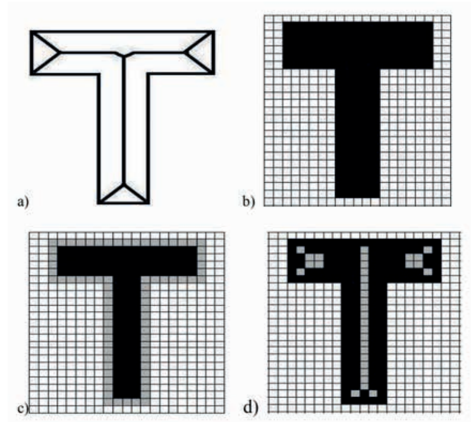


Fig. 5. a) shows the Blum's skeleton of the object in the Euclidean plane. In b) we have the digitization of the object using Gaussian digitization. In c) the contour of the digitized object is identified, and d) we have the Blum skeleton for the digitized object.

As we can see the definition of Blum's skeleton is not adequate to identify digital objects, because the skeleton should allow us to determine topological and geometric properties of the object, such as connectivity and shape, and in the example of Figure 5 these properties are lost.

MODELING OF THE DIGITAL IMAGE AS A CELLULAR COMPLEX.

Abstract cell complexes are well known in combinatorial topology and polyhedra geometry. The use of this model to describe the geometry and topology of digital objects was first discussed by Kovalevsky in the 1980s [1].

To understand the structure of a cell complex let us analyze figures 6.a, 6.b and 6.c. These consist of elements of different dimensions on the Euclidean plane: faces (without edges), edges (without endpoints) and vertices. In addition, we have that an edge a_1 bound or border the faces c_1 and c_2; in turn the vertices v_1 and v_2 bound or border the edge a_1 and the faces c_1 and $c_{(2)}$. The set formed by all these elements is known as a cell complex and its

Journal of Engineering Research ISSN 2764-1317 DOI https://doi.org/10.22533/at.ed.317572526067

elements are called cells. It is important to emphasize that an element of dimension 2 is the interior of a convex polygon, an element of dimension 1 is a line segment, which is considered without its two endpoints. An element of dimension 0 is a single point.

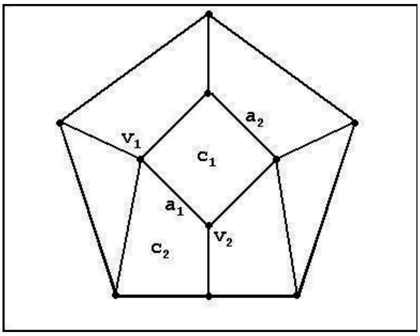


Fig. 6. Cell complex.

By decomposing the Euclidean plane into regular convex polygons (all its angles and sides equal), it turns out that the only possible regular convex polygons are: the square, triangle and hexagon [[3], p. 45-46]. Thus we have three distinct complexes to consider: quadratic cell complex, triangular cell complex and the hexagonal cell complex.

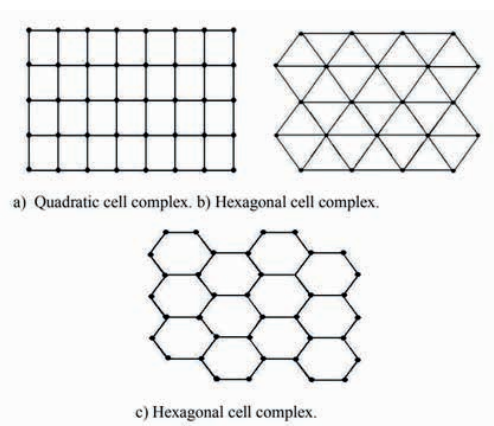


Fig. 7. Cell complexes.

All the experiments, from the digitization of objects to the study of the Kovalevsky skeletons, will be performed exclusively on the quadratic cell complex and the hexagonal cell complex.

When digitizing Euclidean objects on the mentioned cell complexes we generate cell subcomplexes, applying the following method:

In the digital plane we consider the pixel as a unit square whose center belongs to Z^2. Generalizing this idea to the quadratic or hexagonal cellular complex, we have: we consider as pixel the cells of higher dimension, in order to make use of the Gaussian or Jordan digitization mappings, where each cell is identified with its center. Once we have information of the cells of dimension two that make up the object, we define which cells of smaller dimension are added to the digital object. It was considered that for each cell of dimension two that conforms the digital object, this will be accompanied by all its edges and vertices, to achieve the preservation of the connection of the original object.

In order to visualize each of the methods mentioned above, the following table explains how each of the complexes is handled within a PC.

Element of the quadratic complex.	Representation in the Euclidean plane	Representation on the monitor
Cell dimension 0. (quadratic and hexagonal)	Point	Pixel
Cell dimension 1.	Edge	Set of two pixels
Cell dimension 2.	Square	Set of four pixels
Element of the hexagonal complex.	Representation in the Euclidean plane	Representation on the monitor
Cell dimension 1.	Edge	Pixel
Dimension cell 2.	Square	Set of four pixels

TABLE I. Visualization of the quadratic and hexagonal cellular complexes in the PC.

According to Table I, we present the visualization of a bounded region of the quadratic and hexagonal complex.

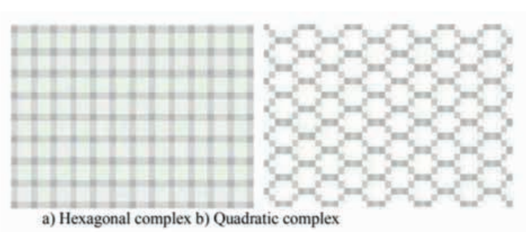
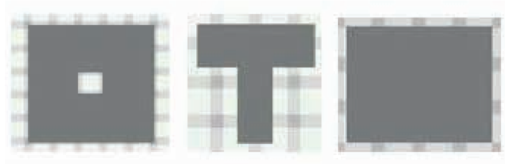
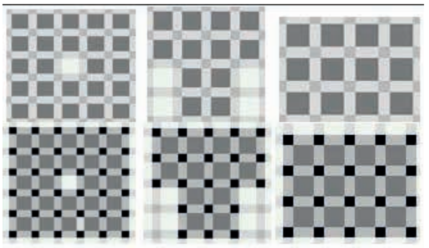


Fig. 8 Cell complexes.

Figure 9 shows the results obtained as a result of the programming of the digitizing algorithm on the cell complexes. First the Euclidean object to be digitized is shown and then all the cells of dimension two belonging to the digitization of the object are shown, so that finally all the cells of smaller dimension that structure the digitization are shown.



a) Series of Euclidean objects.



b) Series of digital objects (Gaussian digitization, Jordan exterior and Jordan interior, respectively) on the quadratic cell complex.

Fig. 9. Digitization over the quadratic complex.

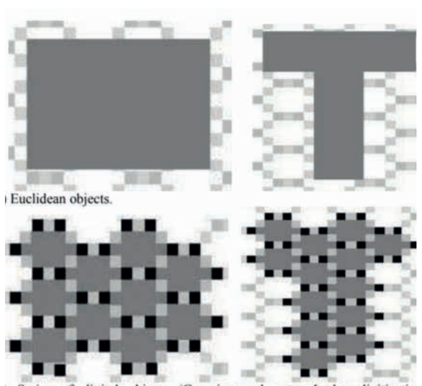


Fig. 10. Digitization on the quadratic complex.

THINNING ON CELL COMPLEXES AND ITS IMPLEMENTATION.

Historically, Listing already in 1862 used the term linear skeleton to describe the subset resulting from continuous deformations of a connected subset of a Euclidean space without change in the connectivity of the original set, until a set of points and lines is obtained [[4], p. 215]. Many image analysis algorithms are based on this process of deformations, called thinning.

In the literature the term thinning has no single interpretation, although it describes reduction operations that preserve connectivity when applied on digital objects, involving iterations to transform boundary elements into background elements.

A thinning algorithm must satisfy the following conditions as has been stated in several publications, see for example [5]:

1. The resulting object from the original object must be thin.
2. The subset must approximate the mid--axis.
3. The endpoints must be preserved. End points are those points that are ends of digital lines or curves. By not preserving them in a thinning process, the digital lines or curves degenerate into points.
4. The connectivity of both the object and the background must be preserved. The pixels of the border or open border of a digital object whose removal preserves the connectivity of the object and its background, is known as single pixels. There are different ways to place these pixels in the digital object, among which are: templates and Boolean expressions.
5. The algorithm must be robust against noise.

We speak of Frontier when the outline of the digitized object on some cell complex is structured of cells of dimension zero one (points and segments). This set is denoted as Fr. And we speak of open border when the outline of the digitized object on some cell complex is structured of cells of dimension zero and two (points and squares or hexagons). This set is denoted as Of.

It is worth mentioning that for the removal of single pixels from the border or the open border, templates will be used. A template is an array of pixels around a pixel of interest (pixel neighborhood) that allow to perform local operations by comparing values present in the templates, with the values of the digital object. These operations serve different purposes, among which is the preservation of the connectivity of the digital object. It is worth mentioning that, in order to detect an endpoint and its preservation in a digital object, templates were also implemented. The established templates that underlie the development of the thinning algorithm are set out in [[3], p. 68 -73].

Based on the information stated we have that the steps that were considered for the development of the thinning algorithm for the quadratic and hexagonal complex are as follows (for the algorithm see [[7], p. 66-74]):

1. The object of interest in the Euclidean plane is digitized into a subcomplex (denoted as T), of the cell complex C, by the established digitization methods.

A new subcomplex, $T_1 = T \cup Fr(T)$, is generated to ensure that the input subcomplex to the algorithm is closed.

3. The elements of $Fr(T(_{1)}) \frown_{T(1)}$, which comply with some of the established configurations of the templates are searched and marked.

4. The elements marked in step 3 are deleted and counted, generating a subcomplex T_2.

5. The elements of $Of(T_2) \frown_{T(2)}$, which comply with some of the configurations of the templates, are searched and marked.

6. The elements marked in step 6 are deleted and counted, generating a subcomplex S.

7. The number of elements counted in steps 4 and 6 is added up. At this point an iteration ends.

8. If the sum of the previous step is zero, the iterations end and the subset S is reported, which represents the skeleton of the subcomplex T. Otherwise, T_1= S is updated, and a new iteration begins, starting at 3, until the end condition of the algorithm is met.

Below is a series of figures showing the results of the programming of the thinning algorithm on the cell complexes, in the following order: first the Euclidean object is shown and then its skeleton on the cell complex is shown.

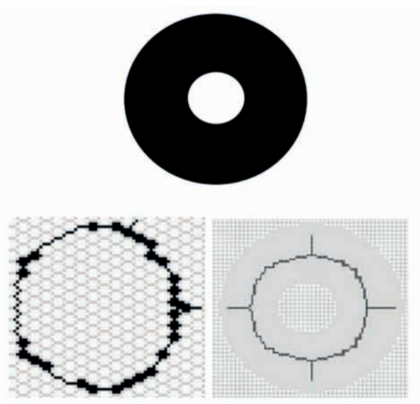


Fig. 11. Digitized skeleton of the letter O.

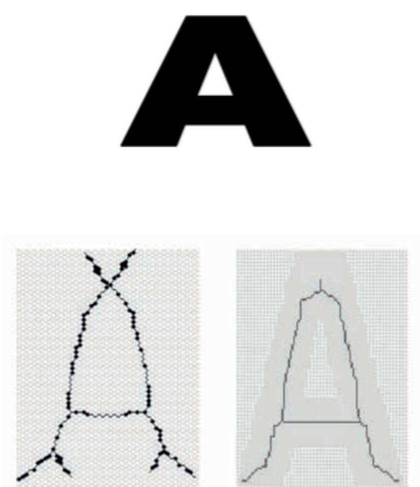


Fig. 12. Digitized skeleton of the letter A.

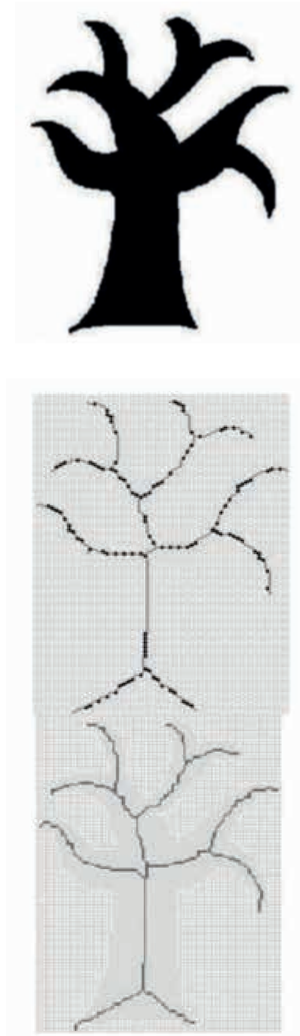


Fig. 13. Digitized skeleton of a series of branches.

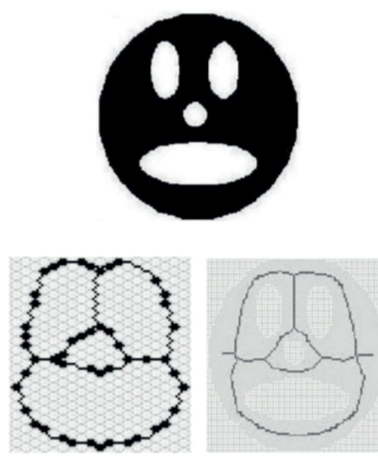


Fig. 14. Digitized skeleton of a series of circles.

Journal of Engineering Research ISSN 2764-1317 DOI https://doi.org/10.22533/at.ed.317572526067

CONCLUSIONS

According to the results obtained, the following conclusions can be drawn:

The digitization methods presented in Section III, applied to Euclidean objects, towards cell complexes, are very sensitive to very small perturbations of the original object.

We also mention that the discretization resolution plays an important role, so that geometrical and topological properties of the Euclidean object are reflected in the digitized object.

It is worth mentioning that other negative effects of digitization are observed when the Euclidean object is translated or rotated. A translation or rotation causes the contour of the digitized object to change.

For objects digitized into a cellular complex, it can only be reported that connectivity is a property that is guaranteed to be preserved under the established algorithm, i.e., this algorithm, for each connected subcomplex, produces a connected skeleton. Moreover, this algorithm does not generate holes in the object, and does not connect parts of the object that were originally unconnected. The reported experiments show that with other properties care must be taken for interpretation, in particular, the number of end elements, as well as the number of branching elements, may change when the object is slightly translated or rotated.

Finally, we can say that the Blum skeletons of each of our tests, when compared with the skeletons generated as a consequence of the application of the program achieved, have a great similarity, which is greater for the hexagonal complex. However, here we are aware of problems caused by digitization effects.

REFERENCES

1. V.A. Kovalevsky, Algorithms and data structure for computer topology, en: *Digital and Image Geometrry,* (G. Bertrad, A. Imiya, R. Klette, Eds.), Lectures Notes in Computer Science, Vol. 2243, p. 37-38, Spinger Verlag, Berlin Heidelberg, 2001.

2. H. Blum, A transformation for extracting new descriptors of shape, en: W. Wathen-Dunn (ed), *Model for the perception of speech and Visual from*, MIT Press, Cambrige, MA, U.S.A., p. 362-380, 1967.

3. A. Trejo Martínez, Geometría del adelgazamiento sobre el complejo celular cuadrático y sobre el complejo celular hexagonal, Tesis de maestría. Departamento de Control Automático, CINVESTAV-IPN, México D.F. Diciembre 2007.

4. R. Klette, A. Rosenfeld, *Digital Goemetry: Geometric Methods for Digital Picture Analysis.* Morgan Kaufmann Publishers, Elsevier, CA, U.S.A., 2004.

5. G. Klette, *Skeletons in Digital Image Processing,* Technical Report CITR-TR-112, Computer Science Depatment of the University of Auckland, Julio 2002.

6. *Manual de Usuario DIAS* (versión - 1998), FSU Jena, Tower-Soft, Berlín. M.R. Zempoalteca, Adelgazamiento en imágenes digitales de dimensión dos, modeladas por complejos celulares cuadrático, Tesis de maestría, Departamento de Control Automático, CINVESTAV-IPN, México D.F. 200