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SOLVING PROBLEMS OF SYSTEMS OF LINEAR EQUATIONS BY PROGRAMMING WITH OCTAVE

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Abstract: The present work is proposed as a teaching proposal for the training of students, focused on competencies for the management of technological tools that will allow them to develop effectively in their professional field. In particular, the Octave software allows, among other things, to create solutions consisting of user-coded programs that meet the user's needs, as much as he/she wishes. From a concrete example of electrical circuits worked through Systems of Linear Equations (SEL), the versatility of Octave software as a pedagogical resource and as a calculation and programming tool will be shown.

Keywords: Competencies. Octave. Programming. Electrical Circuits. Systems of linear equations.

INTRODUCTION

The vertiginous advance of technologies in recent years, even more so in post-pandemic times, motivated us to rethink the way in which we carry out our teaching practices and thus reflect on the competencies that future graduates of different careers, particularly future engineers, should have. It is evident that many of the tasks that used to be performed by humans are now carried out by technological resources. This makes it essential to train new professionals with skills in the use and application of computational tools that allow them to optimize not only results, but also the time and procedures to achieve them. In this sense, the application of specific software is extremely effective in achieving these objectives.

Continuing along this line, one of the most appropriate ways to achieve the purposes described above is to *approach the concepts through application problems*, modeling from the particular to the general. It is here, where programming can take a leading role, which will allow making the right decisions, automating processes and making results effective.

For these reasons, the present work seeks to provide teachers with contributions that favor their teaching practices and strengthen the competencies they wish to develop in the students of the courses in which they work. In particular, we will work with the OCTAVE software, which is a powerful computational tool not only to be used in the classroom but also in the work of the graduate professional.

Octave or GNU Octave is a program and programming language for performing numerical calculations. It is characterized by:

- Its language, based on matrices, is the most natural way to express computational mathematics. The integrated graphs make it easy to visualize the data and obtain information from them.
- Since it is a free software, it is easily accessible and distributed to any user.
- It uses a language that is compatible with the one used by MATLAB.
- It allows the development of complex applications with the help of the window editor, menus and controls of the GUI (Graphics User Interface) utility.

On the other hand, we consider that it is important to work from the different subjects with examples that allow visualizing the application of the contents taught. One of them, fundamental for linear algebra, is the resolution of Systems of Linear Equations, which has innumerable applications in the field of engineering, as well as in many other disciplines. In particular, a concrete example will be presented to work this content starting from electrical circuits, with the intention of linking different aspects: content to be taught (to learn), acquisition of technological competences through the use of Octave and application problems.

DEVELOPMENT

First of all, we consider that it is necessary to remember some concepts related to the application problem we wish to work on.

ELECTRICAL CIRCUITS

<u>Electrical circuits</u> are a specialized type of electrical network that provides information about energy sources, such as batteries, and devices powered by those sources, such as light bulbs or motors. The following symbols are used to graphically represent an electrical circuit:

In addition, in an electrical circuit can be distinguished:

- **Junction:** is the point where three or more conductors meet. Junctions are also called nodes.
- **Branch:** is the fragment between two consecutive nodes.
- Mesh: is any closed conduction path.

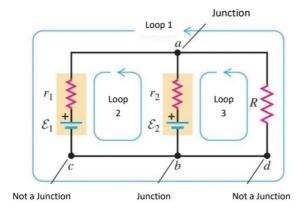


Figure 2: Graphical representation of an electrical circuit

OHM'S LAW:

Ohm's law states exactly how much potential difference V is needed to drive a current I through a resistor R:

V = I.R

The force is measured in volt (V), the resistance in ohm (Ω) and the current in amper (A).

KIRCHHOFF'S LAWS

Kirchhoff's Laws are applicable to the calculation of voltages, currents and resistances in an electrical grid and are widely used in electrical engineering to obtain the values of the current intensity in the branches of an electrical circuit and the electrical potential at each point of the circuit. They arise because in many occasions it is not possible to reduce the electrical grids when the resistors are in series or in parallel.

- <u>Kirchhoff's law of junctions</u>: it states that the algebraic sum of the currents in any junction is equal to zero (the sum of the incoming currents to a node is equal to the sum of the outgoing currents). In other words,

$$\sum I = 0$$

- <u>Kirchhoff's law of turns</u>: it states that the algebraic sum of the potential differences in any grid, including those associated with the fem and those of elements with resistance, must be equal to zero (in any grid the sum of all potential differences in the resistors is equal to the sum of all the voltages of the sources). In other words,

$$\sum V = 0$$

SIGNING CONVENTIONS FOR LOOPS

First we assume a direction of the current in each branch of the circuit and indicate it in the corresponding diagram. Then, starting from any point in the circuit, we make an imaginary path of the mesh by adding the fem (ε) and the *I. R* as we find them. When passing through a source in the - to + direction, the fem is considered positive; when going from + to -, the fem is considered negative. When

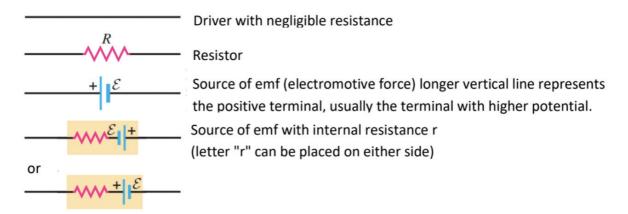


Figure 1: Symbols used in electrical circuit diagrams.

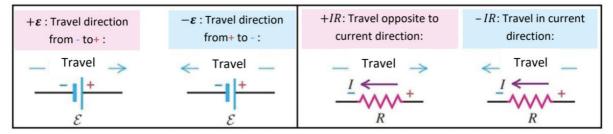
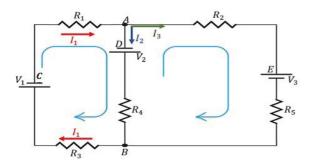


Figure 3: Sign convention for fem and resistances.

going through a resistor in the same direction as that assumed for the current, the term *I*. *R* is negative. When going through a resistor in the opposite direction of the current that was assumed, the term *I*. *R* is positive.

EXAMPLE OF AN APPLICATION PROBLEM:

Determine the current intensities of the branches of the following electrical circuit



where the resistors and voltages are $R_1 = 2\Omega$ $R_2 - 4\Omega$ $R_3 = 4\Omega$ $R_4 = 3\Omega$ $R_5 = 2\Omega$ $V_1 = 10V$ $V_2 = 6V$ and $V_3 = 4V$. The circuit has three batteries and five resistors. The current I_1 flows through the left branch BCA, the current I_1 flows through the middle branch AB and the current I_3 flows through the right branch AEB.

We apply **Kirchhoff's law of junctions** at *A* and *B* respectively.

$$I_1 = I_2 + I_3 \implies I_1 - I_2 - I_3 = 0$$
 in the node A

$$I_1 = I_2 + I_3 \implies I_1 - I_2 - I_3 = 0$$
 in the node B

It can be seen that the equations at A and B are the same.

Next, we apply **Kirchhoff's law of the meshes** for each one, considering clockwise the path of the mesh.

Taking into account the sign conventions we have:

For the grid *CADBC*, the equation

$$-V_1 - I_1R_1 + V_2 - I_2R_4 - I_1R_3 = 0 \implies 6I_1 + 3I_2 = -4$$

and for the mesh *EBDAE*, the equation

$$-I_3R_2 - V_3 - I_3R_5 + I_2R_4 - V_2 = 0 \implies 6I_3 - 3I_2 = -10$$

Then a system of linear equations is determined

$$\left\{ \begin{array}{l} I_1 - I_2 - I_3 = 0 \\ 6I_1 + 3I_2 = -4 \\ -3I_2 + 6I_3 = -10 \end{array} \right. \Rightarrow \left[\begin{matrix} 1 & -1 & -1 \\ 6 & 3 & 0 \\ 0 & -3 & 6 \end{matrix} \right] \left[\begin{matrix} I_1 \\ I_2 \\ I_3 \end{matrix} \right] = \left[\begin{matrix} 0 \\ -4 \\ -10 \end{matrix} \right]$$

PROBLEM SOLVING USING OCTAVE SOFTWARE

Solving the system of linear equations by applying Octave can be done in the following ways:

- a) Through the command window, using Octave library statements, it is possible to obtain the solution set. This will give us a solution for this particular problem by virtue of the reduced step form of the SEL extended matrix. The interpretation is left to the user (Figure 4).
- b) By creating a function .m file, the student can code his own solution for any system of linear equations returning the analysis of the same, which facilitates the presentation and explanation. For example, in this case the functions "sistemas.m" and "sc_ind.m" were created by the authors of the paper.

Remark: The program code is made in a general way such that it solves any system of linear equations regardless of the context and the problem situation that generates it. The user will have to adapt the results, according to the notation used.

INTERPRETATION OF THE SOLUTION SET

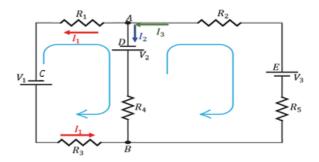
As seen in the screenshot, our **.m** function **file** solves the system of linear equations resulting in the following solution set:

$$S_B = \left\{ \begin{bmatrix} -11/12\\1/2\\-17/12 \end{bmatrix} \right\}$$

That is: I_1 =-11/12A, I_2 =2A and I_3 =-17/12A.

PHYSICAL INTERPRETATION OF RESULTS

It is observed that the currents I_1 and I_3 are negative, this means that the direction of circulation of such currents is opposite to the chosen one, that is to say:



Observation: In an electrical circuit, Kirchhoff's laws and the voltage, current, and resistance relationships provide the equations that describe the behavior of the circuit. However, in some cases, there may be redundancies in these equations due to symmetry or circuit configuration. When there are linearly dependent equations, the resulting system is indeterminate compatible. This is because the redundant equations do not provide additional information and do not completely constrain the unknown variables of the system. In practical terms, this means that there are multiple combinations of current and voltage values that can satisfy the circuit without violating Kirchhoff's laws and the voltage, current and resistance relationships. It is important to note that, in practice, we seek to avoid circuit configurations that generate systems of indeterminate compatible equations, since they make it difficult to determine unique and accurate values for the unknown variables.

```
Ventana de comandos
                                                   >> S=rref(Amp)
>> A=[1 -1 -1;6 3 0;0 -3 6]
A =
                                                   S =
                                                                      0
                                                                                  0
                                                                                        -11/12
                    -1
                                -1
          1
                                                                                  0
                    3
                                                             0
                                                                       1
                                                                                             1/2
          6
                                                             0
                                                                        0
                                                                                          -17/12
          0
                    -3
                                 6
                                                   >> X=S(:,4)
>> B=[0;-4;-10]
                                                  x =
B =
                                                        -11/12
          0
                                                          1/2
         -4
                                                        -17/12
        -10
>> Amp=[A B]
Amp =
          1
                     -1
                                -1
                                             0
          6
                                            -4
                     3
                                 0
                                           -10
          0
                     -3
```

Figure 4: SEL resolution using Octave commands.

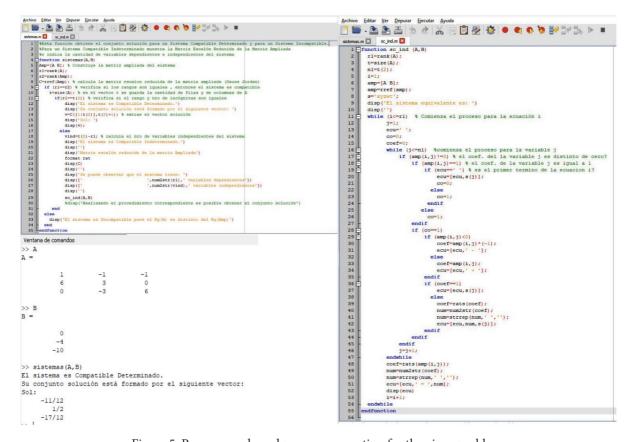


Figure 5: Program code and program execution for the given problem.

CONCLUSIONS

The University is one of the institutions most actively involved in the economic, social and cultural development of society. In order to comply with this social task, it must work for the preservation, development and promotion of culture, conceived as functions that are embodied in the execution of the university processes: teaching, research and extension. It is here, where our task as university teachers begins to provide the society in which we live, all those subjects that will contribute to this development, always thinking about the strategies and methodologies that best fit the current needs.

Competencies are processes, capabilities, abilities and skills performed with suitability in certain contexts, integrating different knowledge. Thus, one of our main tasks as teachers and trainers of engineering professionals is to set ourselves the goal of promoting these competencies by encouraging students motivated to perform activities and solve problems of their professional life in the most ethical, creative, autonomous and efficient way possible.

Clearly, an interesting path is to take advantage of the potential offered by the technological tools that are available and that, in the mathematical discipline, are very useful since the power of representation they offer allows the observation of problems in a more concrete way and thus identify abstract concepts in a clearer way.

Particularly in the field of linear algebra, Octave allows us to work with abstract objects such as matrices and to visualize both their properties and their applications. In addition, this software allows us to perform calculations that would be very laborious manually and thus optimize time in learning and assimilation of the concepts. Another benefit of Octave is that it allows us to modify the teacher-student relationship by putting the teacher in the role of facilitator and guide in the process of building new knowledge and not simply a mere transmitter of concepts.

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