v. 5, n. 3, 2025

International Journal of Human Sciences Research

Acceptance date: 30/01/2025

GEOGEBRA IN THE TEACHING AND LEARNING OF 2ND DEGREE FUNCTIONS IN A PUBLIC ELEMENTARY SCHOOL - REPORT OF A REVEALING EXPERIENCE

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Abstract: This report presents an application, and contributions, of the GeoGebra software[1] in the teaching and learning of quadratic functions, in a school of the State Public Education Network of Bahia, in the year 2022. Guided by the question: *How can GeoGebra* be used *in the construction of graphs of quadratic functions?* the project used digital technologies in pedagogical mediation to make student-knowledge interaction effective, based mainly on Papert's constructionism, and on the guidelines of the BNCC[2] (BRASIL, 2022), for which it used guided investigative activities, providing new means of learning related to the construction of graphs of quadratic functions. The results obtained showed that the use of GeoGebra *software* is effective for teaching and learning mathematics because, in addition to specific learning, it aroused students' interest, active participation, motivation and curiosity.
Keywords: GeoGebra; Quadratic function; Teaching; Mathematics education.

INTRODUCTION

Dynamic geometry *software*[3] has become increasingly popular in the teaching and learning process, sometimes replacing in a practical and efficient way old construction tools such as the ruler and compass, basic in Euclidean geometry, and the grid on paper, in the teaching of function graphs. Dynamic geometry *software* are working tools that have enhanced the teaching and learning of various objects of knowledge, such as functions, challenging teachers to seek out new methodologies for teaching and learning mathematics.

The development of a systematic study plan that culminated in the learning of the essential elements for constructing the graph of a polynomial function of the second degree, with the active participation of the student, was the proposal presented in 2021, under the title "*The Use of GeoGebra in the Construction of Graphs of Quadratic Functions:* analysis and contributions", as a Course Conclusion Work (TCC), of the "Specialization Course in Mathematics Teaching: Matem@Tica na Pr@Tica", taught by the Federal Institute of Education, Science and Technology of Bahia (IFBA). The roots of the research project go back to 1985, when, as a high school student, I started programming and soon realized that this new knowledge had the potential to help me learn school content. At that time, I started researching (obviously in the common sense of the term) on the subject, having my first contacts with Papert's texts[4] , and implementing programs that helped me advance and better understand mathematical objects, especially functions.

During my undergraduate degree in mathematics in the 1990s at the Federal University of Bahia (UFBA), I made some significant advances in the knowledge needed to apply digital technologies to the teaching and learning of mathematics. My knowledge of higher mathematics, theories of education and the popularization of computers and the internet led me to two courses that updated my knowledge of the use of *software* for education, the first offered by the mathematics institute and the second by the faculty of education, which coincided with the publication of the book "*Cyberculture*" by Lévy (1999), which was introduced to me and suggested as reading by my late teacher Ivana.

In 2005, as a teacher in the state school system of Bahia, at the Colégio Polivalente de Aratu, in the municipality of Simões Filho, I

1. Dynamic mathematics *software* created by Markus Hohenwarter in 2001 at the Universität Salzburg.
2. Common National Curriculum Base.
3. Software that allows you to "use, manipulate, combine, visualize and virtually construct geometric objects" (BORBA, SILVA, GADANIDIS. 2023, p.31).
4. Seymour Papert (1928-2016), a pioneer in the use of computers in education and creator of the LOGO programming language.

developed what became the basis of the project presented to IFBA in 2021. Using *software of* my own design, I developed and applied a didactic sequence for teaching functions of the first degree, from which I obtained the first practical data, with this type of teaching methodology, which served as a guide in the development of this proposal.

With the advent of remote teaching, caused by the COVID-19 pandemic, I once again dedicated myself to methods for teaching with digital devices, and was obliged to research contemporary authors who could contribute new knowledge. After the resumption of face-to-face classes, education system managers made timid investments in digital resources in schools, but enough for me to continue with my 2005 proposal.

GEOGEBRA IN THE TEACHING AND LEARNING OF 2ND DEGREE FUNCTIONS IN A PUBLIC ELEMENTARY SCHOOL - REPORT OF A REVEALING EXPERIENCE

The project "*The Use of GeoGebra in the Construction of Graphs of Quadratic Functions*: analysis and contributions" was applied in a school of the State Education Network of Bahia, with 6 classrooms, a library, a sports court, a small teachers' room and a secretariat, located in the municipality of Simões Filho, in the metropolitan region of Salvador, state of Bahia.

The project was guided by the question: how can GeoGebra be used to construct graphs of quadratic functions? It was based on the thinking of Papert (2008, p.30), "(...) there are (...) areas of knowledge in which the epistemic transition is much more brutal (...) and in which a machine that provides a context for smoothing it is much more at hand. One of these areas is mathematics." and in the search for a process of transition from the paradigm of mathematics teaching, "based on teacher transmission, student memorization and competitive, individualistic learning" (BEHRENS, 2013, p.79), to a more progressive method, recognizing the digital age "as a new way of categorizing knowledge [...]" (BEHRENS, 2013, p.80), facing "electronic resources with discretion" (BEHRENS, 2013, p.80). The digital device used in the pedagogical practice were the *Chromebooks*[5], through which GeoGebra smoothed the process of teaching and learning functions of the second degree, connecting the original 2005 project, which was located in what Borba (2023, p.30) classifies as the *second phase* of digital technologies in mathematics education in Brazil, which began in the first half of the 1990s, with the *fourth phase* (BORBA, 2023, p.42), which began around 2004, favoring the student's protagonism, collaboration and initiative.

The project was presented to the school management and promptly accepted at the end of 2021, with implementation scheduled for February to April 2022. The challenge was great: the school didn't have any free spaces, the internet signal was terrible, it didn't reach many meters beyond the office and the teachers' room, the school underwent a renovation and the library, which was the most suitable space for this purpose, was closed.

SOLVING THE PROBLEM OF SCHOOL SPACE AND SUPPORT MATERIAL

I initially set out to help clean and organize the library, but factors within the school made it difficult to do so. The school management tried several times to make the space available, but were unsuccessful in their attempts, which were limited, among many factors, by the school's infrastructure, which had no space to move some of the material to.

5. Chromebook is the name given to a series of notebooks that use the Chrome OS operating system.

With the deadline looming and no provision for the library space, I decided to change the format of the project and carry it out in the teachers' room. I obviously counted on the solidarity of my colleagues, whose resting space was compromised, but new problems had to be solved, such as the number of students who could be served, the lack of conditions for using a projector or television to facilitate some general guidance.

The teachers' room was the most unlikely place to carry out this didactic proposal: a small space, with lots of materials, heavy traffic and no possibility of support equipment. The solution was to divide the class into groups of 6 students, meet them on the day of the Mathematics Complementary Activities (CA), prepare handouts to guide the activities and support posters.

I arranged 6 school desks, of the table and chair type, at the back of the teachers' room, in an L-shape, placed side by side. The proximity of the desks was not a problem, as the project had already been designed with this organization, so that the students could collaborate with each other during the activities. The most important thing was the individual use of the computers. The proposal was based on student-machine, student-student and student-teacher interaction (figure 1), thus establishing the format for building knowledge.

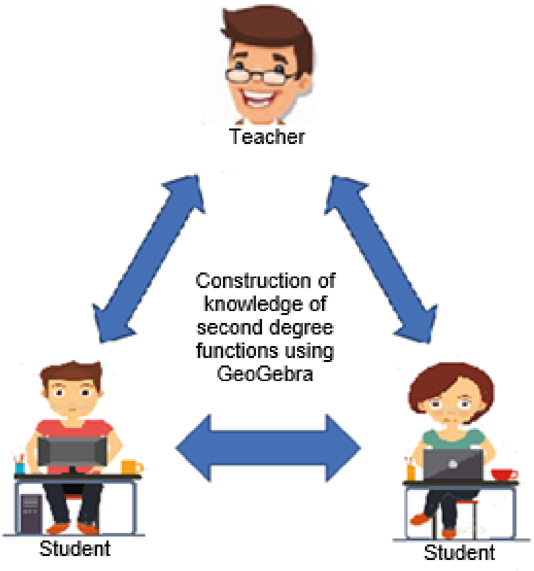

Figure 1: Didactic design project

Source: Own composition based on *cliparts* available at *graphicriver.net* and *tpsearchertool.com.*

Initially, these tables were only going to be placed in the teachers' room when the project was being implemented, but the team's collaboration was fundamental and everyone agreed that these materials should remain there until the end of the project, thus solving the space problem collectively.

THE AVAILABILITY OF THE PARTICIPATING STUDENTS

Obviously, the need to move the application to the days of the mathematics CA and to attend to only six students at a time, would imply the problem of releasing students from classes other than mathematics. When I spoke to my fellow teachers about this problem, everyone was collaborative and agreed to release the students involved.

THE WI-FI SIGNAL

Education departments still seem to lack the organization of competent personnel to choose digital equipment suitable for applications related to the reality of their schools. Many Education Departments purchase

Chromebooks for educational use, but this equipment needs to be connected to the internet in order to be used, and most public schools don't have an internet connection or the network doesn't support many connected devices, in many cases not even reaching the entire school space.

The problem of range was overcome when only the teachers' room was left as an option, but many times we were unable to connect all six devices simultaneously, making it necessary to share my mobile internet. In this combination of poor school infrastructure and personal goodwill, we solved the problem of internet access, thus completing the organization of the entire structural part of the project.

THE DEVELOPMENT OF ACTIVITIES

The meetings began on July 21, 2022, and the participants attended regularly until they finished on October 6 of the same year. The curiosity about the development of the work was remarkable, as 100% of the participants said they had never taken part in math classes using computers, although 80% of them said they had already used some *software* to study math.

All the participants had some level of lag in relation to the formal curriculum, since they were returning from the COVID-19 pandemic, but only 10% were in an age-grade gap. All of them said they had good expectations of the work they were starting and 60% found the study of mathematics interesting and 40% thought it was sometimes interesting. 10% did not have access to any digital devices and 90% accessed the internet via their cell phones.

The activities were planned for a period of seven weeks, with only one meeting a week, on the morning shift. Each meeting lasted 1 hour 40 minutes, corresponding to a total class load of 8 hours 20 minutes. At each meeting, the minimum amount of instruction was given, i.e. "teaching in such a way as to produce the greatest amount of learning from the minimum amount of teaching" (PAPERT, 2008, p.134), the minimum necessary for the student's theoretical and practical development was presented at each stage of the proposal.

The study of the construction of graphs of quadratic functions using GeoGebra involved two stages: the first was training in the use of GeoGebra, which helped to introduce the Cartesian plane and the marking of points using their coordinates in a very natural way, without the need for theoretical formulations; the second was the use of GeoGebra to construct and analyze graphs of quadratic functions.

Each meeting took place in two or three stages: the construction of the graphic environment needed for analysis (based on initial instructions), the promotion of analysis through the manipulation of objects linked to the graphics, guided by a handout, encouraging self-study, and the realization of activities (list of activities).

TRAINING IN THE USE OF GEOGEBRA

The GeoGebra training involved two meetings, the students received handouts containing information about the GeoGebra interface, a list of proposed activities, and a poster was fixed in front of everyone to help guide the use of GeoGebra, with guidelines for marking points on the Cartesian plane using ordered pairs, meeting skill EF06MA16 of the BNCC (BRASIL, 2018, p.303). The GeoGebra training made it possible to introduce the essential concepts for studying the graphs of quadratic functions, such as locating and marking points on the plane, perceiving the two-way relationship between ordered pairs and points, and graphically interpreting the variation in the coordinates of a point, without formalizing these objects conceptually.

The activities proposed for learning the relationship between ordered pairs and points on the Cartesian plane did not focus on merely introducing ordered pairs, but on observing the behavior of the point by manipulating the coordinates and proposing tasks based on these observations, taking advantage of the student's immersion in the dynamic geometry environment and the pleasure of using digital devices (Photo 01).

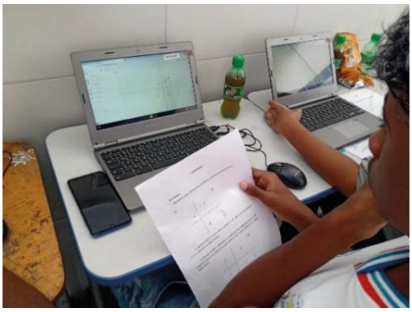

Photo 01 - Marking points on the Cartesian plane
Source: author

From the very first moment, the students showed themselves to be autonomous, participative, collaborative and curious. They were at ease with the new class format and showed their own initiative in using the equipment, such as turning it on, *logging* into the system, guiding their classmates, choosing between using the *TouchPad*[6] or the *mouse* and accessing the GeoGebra *website*. These characteristics are in stark contrast to the day-to-day life of the traditional classroom, where attention to the teacher competes with attention to the cell phone and various games. Although there is no prohibition, the cell phone was not the protagonist of these moments, the interaction between the students was centered on resolving doubts inherent in the classroom process and many of them were resolved between them, leaving the teacher in charge of only the most inherent guidelines of the pedagogical process.

USING GEOGEBRA TO CONSTRUCT AND ANALYZE GRAPHS OF QUADRATIC FUNCTIONS

For the study of second-degree polynomial functions and their graphs, a problem situation was taken as a starting point, which modeled the throwing of a ball using an algebraic expression. The students used GeoGebra to calculate some numerical values for this expression and produced data to fill in two tables, the values of which formed ordered pairs representing the points used to construct the first graph of a quadratic function.

The first graph (Photo 02) used the potential of dynamic geometry to understand the mathematical modeling of an oblique throw and the definition of the numerical value of an algebraic expression. The students were able to introduce a generic, moving point on the graph, reproducing the movement of the ball from its starting point to its finishing point, allowing them to reflect on the relationship between the dependent and independent variable through observation, which would not be possible in traditional classes.

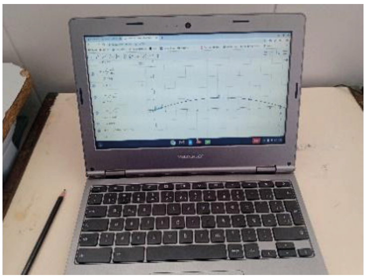

Photo 02 - Analysis of throwing a ball

Source: author

The final weeks focused on the relationship between the coefficients of the law of the quadratic function and the graph, as well as determining the zeros of the function. All these studies were done in a qualitative way, through constructions, observations and reflections, based on the dynamic property of graphs.

6. Touch-sensitive device, common in *notebooks*, used to perform various functions, including controlling the cursor.

International Journal of Human Sciences Research ISSN 2764-0558 DOI: https://doi.org/10.22533/at.ed.5585325230115

To study the relationship between the coefficients and the graphs, GeoGebra's "sliders" were used, which make it easy to change the numerical values of these variables (Photo 03) and their immediate effects on the graphs. In order to determine the zeros of the functions, the use of a moving point on the graph was used, as in the contextualized study of throwing a ball, thus inducing the student to observe that the zeros of the function are obtained when the graph intersects the abscissa axis.

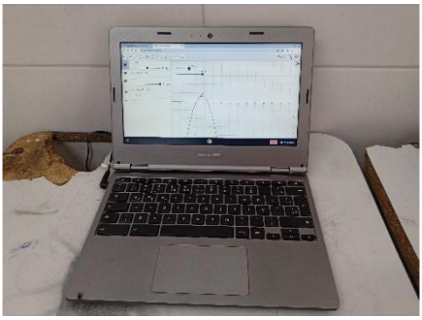

Photo 03 - Study of function coefficients

Source: author

RESULTS AND CONCLUSIONS

The use of GeoGebra motivated students to study and carry out activities at home. Some said at various times that they had accessed GeoGebra at home to redo the activities and get to know it better. An indication of the veracity of this information were the requests to take the guidance handouts and the proposed activities home. A quick mastery of GeoGebra was also observed, agreeing with the statement that "We learn more easily when we realize the purpose, the usefulness of something, when it brings us perceptible advantages." (MORAN, 2013, p.28).

Dealing with digital equipment, which is part of their daily lives, or at least of the *cyberculture* society[7] , in which the students are immersed, made them more motivated. There was a lot of sharing of personal results between the students, a lot of willingness to help each other, to demonstrate the knowledge acquired in class or brought from outside, and even authorial creations were developed from the proposed activities, in line with what is proposed by Papert's constructionism (2008, p.134), one of the participants said he had decided to study programming.

During the process there were few absences and no procrastination, the proposed activities were all carried out with the active participation of the participants, the teacher was rarely asked, making the pedagogical work very enjoyable.

As far as learning is concerned, the first thing that can be observed is the relationship between an ordered pair and a point, and GeoGebra makes this very natural. The student introduces an ordered pair and its graphical representation is instantly visualized, a coordinate is modified and the point changes place, an ordered pair is excluded and the point disappears, so that a biunivocal association is intuitively built up between the points on the plane and the ordered pairs, so when we introduced quadratic functions, it became easier to understand the relationship between the law of the function and its graph.

Learning the relationship between the coefficients of the function law and the graph was greatly enhanced by dynamic geometry. In just a few minutes, with the help of the handouts and observations of the graphical manipulations, the students were able to understand this relationship, which was verified by completing the proposed activities and through questions from the teacher.

The students' autonomy was detected in the authorial productions, based on controlled alterations to the proposed activities, using commands that were not guided by the teacher, but were the result of their own research. The resources discovered did not in-

7. "[...] a set of techniques (material and intellectual), practices, attitudes, ways of thinking and values that are developing along with the growth of cyberspace." (LÉVY, 1999, p.17)

terfere with the planned objectives, but added to them and indicated new possibilities for approaching them. These discoveries were fundamental in making the students feel motivated and confident in using GeoGebra as a mathematical learning tool. I also observed that these discoveries were motivating because they served as an object of sharing within the group, improving self-esteem, according to Moran (2013, p.18) this element of "affectivity is a basic component of knowledge [...] it dynamizes interactions, exchanges, the search, the results [...] it promotes unity".

With the students' autonomy and active participation, the teacher was less required. I dedicated myself to answering specific questions about the activities and the theoretical content, to giving more individual guidance on GeoGebra, to complementing the students' knowledge of mathematical objects, and also to validating modifications made by the students to the activities. Mutual respect and affection were widely perceived by everyone, validated by the opinions of fellow teachers and the school management.

Based on the results observed, GeoGebra is a fundamental tool for learning how to construct and analyze graphs of polynomial functions of the second degree. In a very natural way, the students assimilate and understand the Cartesian plane and its elements, the relationship between points and ordered pairs, the dependence between variables, the numerical value of an algebraic expression, the relationship between the law of a function and its graph, the nomenclatures, as well as enhancing the possibilities of discovery through the manipulation of the dynamic controls and tools available in GeoGebra. All of this learning took place in a meaningful way, as the previous results show, with a lot of collaboration between the students, their own intuitive initiatives, developed from the little instruction given by the teacher, "Constructionist activity [...] is teaching in such a way as to produce the greatest learning from the least teaching [...]" (PAPERT, 2008, p.134).

FINAL CONSIDERATIONS

Although we are living through the fourth phase of digital technologies in mathematics education in Brazil (BORBA. SILVA. GADANIDIS., 2023, p.42), schools still lack the structure to motivate teachers to implement this type of pedagogical approach. There is a lack of equipment in sufficient condition and quantity, appropriate spaces, stable Wi-Fi networks (when they exist), support staff and equipment and the most diverse types of school materials, leaving the realization of this work up to personal initiatives and dependent on the support of the school community.

School systems are still structured around lectures and written tests, whether they are of the content studied, system evaluations or ENEM, and their schools are geared to this format. Therefore, work aimed at improving understanding of mathematical objects and then formalizing them will always encounter resistance and difficulties, which are often insurmountable.

Despite all the difficulties, the work was successful because the students realized the contemporaneity of their learning, the symbiosis of man and machine being used to build knowledge.

THANKS

I would like to thank the Federal Institute of Education, Science and Technology of Bahia (IFBA), which, through the specialization course "Matem@Tica na Pr@Tica", made it possible to reconstruct this project, especially my advisor, Prof. Jorge Adriano Carneiro Nunes, who was very present and made significant contributions to the preparation of the TCC, my wife, a great motivator, my friend Profa. Joilma Barbosa from the University of Pernambuco (UPE), who helped with suggestions for the organization of the text, the entire team at the Dr. Berlindo Mamede de Oliveira State School in the municipality of Simões Filho, who supported and contributed with all the conditions that favored successful learning with GeoGebra, and all the participating students, who engaged and made each class happen.

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