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APPLICATION OF TOOLS AND TECHNIQUES FOR DETERMINING THE DURATION OF TASKS AND BUFFERS IN THE CRITICAL CHAIN METHOD

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Abstract: The Critical Chain Method (CCM) is a method widely used in Project Management to take into account the resource constraints and uncertainty that may arise when developing a project schedule. For this purpose, this method uses buffers, which absorb the delays that project activities could suffer during execution. This article proposes a methodology for determining the duration of tasks and buffers of a project by combining several tools and techniques commonly used in project management, including expert judgment, three-valued duration estimation (analyzing the best alternative based on the maturity of the project), Delphi methodology for collecting the opinions of the expert panel on the duration of tasks, and Monte Carlo simulation to optimize the durations of tasks and buffers of the project.
Keywords: Critical Chain Method; Monte Carlo; Buffers; Delphi Method.

INTRODUCTION

The Critical Chain Method (CCM) is a widely used method in Project Management to manage the resource constraints and uncertainty that may arise when developing a project schedule. To do so, the CCM method uses buffers, which are responsible for absorbing the delays that project activities could suffer during their execution (Ghaffari & Emsley, 2015).

This article proposes a methodology for the determination of the duration of the tasks and buffers of a project, combining several tools and techniques used in Project Management (IPMA, 2015). The tools considered are expert judgment, duration estimation by three values (analyzing the best alternative according to the maturity of the project), Delphi methodology for collecting the opinions of the expert panel on the duration of the tasks and Monte Carlo simulation to optimize the durations of the tasks and buffers of the project.

This methodology is applied to a solar photovoltaic plant with a nominal capacity of 50 MW, located in the province of Badajoz (Spain).

PROPOSED MODEL FOR THE DETERMINATION OF TASK DURATION AND BUFFERS

A model is proposed to determine the duration of the tasks and buffers of a project, using several of the tools and techniques widely used in project management such as the Delphi Method and Expert Judgment.

The model consists of five stages:

1. Establish a panel of experts.

2. Estimate the duration of tasks by applying the PERT method (Program Evaluation and Review Technique).

3. Collect the expert panel's opinions on the duration of the tasks.

4. Determine the duration of the project buffer and feed buffers by applying the Goldratt or 50% method and the method of the square root of the sum of the squares.

5. Apply Monte Carlo simulation to determine project duration and buffer sizes.

ESTABLISH A PANEL OF EXPERTS

Regarding the composition of the expert panel, according to Jinrong & Enyi (2011), its members should be chosen from among those who meet the following minimum requirements.

- Possess a high degree of knowledge and previous experience in projects similar to the one to be carried out. In this case, renewable energies.

- Possess a high degree of knowledge and experience in project management and leadership.

- That, as a whole, cover the widest possible scope of project areas. To this end, the panel should include professional profiles such as the following: project managers, members of project development teams, developers and/or subject matter experts from outside the project team or stakeholders.

ESTIMATE THE DURATION OF TASKS USING THE PERT METHOD.

The PERT method is a widely used method for planning projects where it is necessary to coordinate a large number of activities. In addition, this method allows graphically represent the different activities that make up the project and calculate the execution times of the tasks that make up the project. For this purpose, the application of the method can be summarized in four stages:

1. List the activities or tasks that are necessary to bring the project to a successful conclusion.

2. Establish the precedence relationships between activities, i.e., for each activity it is established which activities should precede it. In the PERT method, all precedence relationships are "End to Start" (FC).

3. Represent the project network. For the graphical representation of the tasks and their precedence relationships, three rules must be followed:

    - A knot can only be numbered once all the knots preceding it have been numbered.
    - There must be a single starting knot and a single ending knot.
    - Two arrows starting from the same node cannot have the same destination node.

4. Calculate durations. The PERT network is used to calculate the project duration and to evaluate the importance of the different tasks:

    - Early time= minimum time required to reach a knot.
    - Last" time= maximum time to reach a node without delaying the project.

The expected durations of the tasks are set individually. The early and last times depend on the relationship between the different tasks, and are determined from the PERT network. From this process, it is determined that "slack" exists in a node when the early time is less than the last time. The existence of slack in a given task implies that it can be executed with a certain delay without compromising the total duration of the project. If at a given node, the early time is equal to the last time, there is no slack. The "critical path" is the path defined by the nodes that have no slack. This path is important because it indicates all the activities where there should be no slack.

COLLECT EXPERT PANEL OPINIONS ON THE DURATION OF THE TASKS

Expert group members are required to establish the duration of tasks. According to the PMBOK® Guide (Project Management Institute, 2017), to account for uncertainty and risk in estimating duration, the PERT method uses three estimates to define an approximate range of duration for an activity:

Optimistic time (*To*)= shortest time estimate (best value of historical).

Pessimistic time (*Tp*)= longest estimate (worst historical value).

Most probable time (*Tm*)= the most probable value.

Activity durations, which are random variables, can be fitted either to a probability distribution of type β (Beta) or to a triangular distribution.

In the case of adjusting the duration of activities to adistribution, the "average" duration of the three estimates mentioned above is calculated by the formula:

$$E(D) = \frac{To + 4 \cdot Tp + Tm}{6} \quad (1)$$

In the case of adjusting the duration of activities to a triangular distribution, the "average" duration of the three estimates mentioned above is calculated by the formula:

$$= \frac{To + Tp + Tm}{3} \quad (2)$$

The choice of one or another distribution to determine the average duration of the activities will depend on the degree of knowledge of the chosen panel of experts: if the panel of experts is composed entirely of professionals with clearly accredited knowledge, an estimate with a triangular distribution will be chosen (Xu & Zhang, 2015). If, on the other hand, all or part of the experts that make up the panel have little or insufficiently contrasted experience, the use of the β distribution is recommended.

The critical path, like the activities, will have a random duration modeled as a Gaussian function whose mean is obtained as the sum of the durations of the activities composing the critical path, whose variance is the sum of the variances of the activities composing the critical path.

DETERMINE THE DURATION OF THE PROJECT BUFFER AND THE FEED BUFFERS

Buffers are used to maintain project progress in the face of uncertainty (which is the cause of most project delays and cost overruns) by protecting the scheduled completion date. To manage the uncertainty associated with the duration of activities, buffer protection is added at specific points in the project, not at individual activities. These buffers are fictitious activities of fixed duration, responsible for absorbing the uncertainty, i.e. any delay in the execution of the activities.

Depending on the location of the buffers in the project plan, a distinction is made between three types:

1. Project buffers, to absorb any delays in critical chain activities.

2. Feed buffers, to protect the critical chain from delays in secondary chains.

3. Resource buffers, to warn the resources that are going to be needed next in the critical chain below; generally, these buffers do not have a specific duration.

Considering the three types of buffers identified above, the present work focuses on the application of the first two.

The project buffer is inserted between the last activity in the critical chain and the project completion date, in order to absorb the uncertainty inherent in projects. Its size depends on the desired probability of completing the project on time, and will vary depending on the duration of the tasks that make up the critical path. Feeder buffers are inserted where non--critical activity chains merge with the critical chain (Hoel & Taylor, 1999), absorbing the potential delays of these activities in order to prevent them from having an impact on the project duration.

Goldratt's method or 50% of the chain.

This is a rule introduced by Goldratt (1997) in his book "Critical Chain", and is the simplest and most widespread method for sizing buffers, since it establishes that the buffer must be as long as half the duration of the protection times of the set of activities where it is placed. This rule allows to obtain the size of the buffers following the next four steps:

1. Estimate the duration of the chain.

2. Separate individual protections for each activity.

3. Move those protection times to the end of the chain.

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4. Reduce the duration of protection times to 50%.

To make the calculation of the buffer size, the formula (Luiz et al., 2017) is used:

$$Buffer = 0.5x \sum_{i=1}^{n} (dcPi - dsPi) \quad (3)$$

where *n* is the number of activities in the critical chain, *i* identifies the activity in question, *dcPi* is the duration of activity *i* with buffer and *dsPi* is the duration of activity *i* without buffer.

Square root of sum of squares (SSQ) method

Also known as the Newbold formula (Newbold, 2018), it determines the buffer size as a function of the variation in the duration of the activities, calculating this variation as the duration of the activity with protection (*dcPi*), minus the duration of the activity without it (*dsPi*), i.e., the individual protections (Bie et al., 2012). Thus, the buffer size will be the square root of the sum of the individual protections, squared (Ashtiani et al., 2007).

$$Buffer = \sqrt{\sum_{i=1}^{n} (dcPi - dsPi)^2} \quad (4)$$

APPLY MONTE CARLO SIMULATION

Once the PERT technique has been applied and the task durations have been estimated, the project duration is calculated as the sum of the average durations (expected time) of the critical path activities. As a complement to the PERT technique, simulation allows taking into account all the variables and performing iterations by experimenting with different situations to know what results to expect in the face of certain causes (Ogunlana & Dey, 2019).

Monte Carlo simulation is a set of statistical methods that combines randomness and determinism, and allows solving mathematical problems using random numbers as a basis for simulations. John von Neumann and Stanislaw Ulam designed this idea to facilitate decision making in uncertain environments (Delgado et al., 2011), since it allows to see all possible outcomes and evaluate the impact of risk. This method allows predicting a set of outcomes, using values between a minimum and a maximum number, from a set of fixed input values.

It should be noted that the results obtained with this methodology are estimates that give an idea of how the system would behave under certain initial conditions and with the interaction of different variables, but it is not an exact calculation and should not be understood as such. The steps to follow to use Monte Carlo methods are (IBM, 2020):

1. Configure the model, identifying both the dependent variable to be predicted and the independent variables or input variables that will allow its prediction.

2. Specify the probability distributions of the independent variables and, experience or historical data, define a range of possible values and their probability.

3. Repeatedly perform simulations to obtain the random values of the independent variables until there are enough of them.

In this paper the @Risk® software (Palisade LLC, Ithaca, NY, USA) will be used to perform the Monte Carlo simulation of the PERT diagram. The use of the @Risk tool allows for increased size accuracy over results that would be obtained with other software, by allowing not only a large number of iterations to be performed for a simulation (up to 100,000), but also to perform a multitude of simulations (up to 1,000), with affordable computational costs.

CASE STUDY

The methodology presented in section 2 is applied to a real project of a 50 MW photovoltaic plant, located in the province of Badajoz (Spain). The general technical characteristics of the project are:

- Surface area occupied by the solar field: 98 ha.
- Number of photovoltaic panels: 91,728 bifacial panels of 545 Wp.
- Support structure for the photovoltaic panels: solar tracker on an azimuthal axis, with a monoposte driving system.
- Power inverter: 326 150 kVA/c.u. inverters.
- First voltage elevation level: 8 transformer stations, with a total power of 48,500 kVA.
- Subway internal distribution networks connecting the transformer stations of the first voltage elevation level.
- Park transformer substation: consisting of a 50 MVA position, where the voltage is raised from 30 kV to 66 kV.
- Overhead 66 kV evacuation line, 8 km long, connecting to the electrical substation of the distribution company in the area.

ESTABLISH A PANEL OF EXPERTS

For the development of this article, a group of four experts in the development of projects for the construction of renewable energy generation facilities was used.

- E1: Project Manager, with extensive experience in the design and construction of renewable energy facilities.
- E2: Promoter of electric energy production facilities from renewable sources.
- E3: Manager of a construction company of renewable projects in the engineering, procurement and construction modality.
- E4: Responsible for the development of energy production facilities renewable sources.

ESTIMATE THE DURATION OF TASKS USING THE PERT METHOD.

For the case study presented in this section, thirteen tasks have been identified. Table 1 shows the description, duration (in months) and precedence relationships established for the completion of the project in question.

The network resulting from applying the PERT technique to the case study, taking into account the specifications in Table 1, is shown in Figure 1.

COLLECT EXPERT PANEL FEEDBACK ON TASK DURATION

Taking into account the curricula of the members of the expert panel, it was decided to use a triangular distribution for the estimation of task durations. As an example, as a result of applying the Delphi method, the following expert opinions on task B are obtained:

To aggregate the experts' opinions, the geometric average is chosen over the mean or the arithmetic average, since it is less sensitive to the externalities of the sample. This criterion considers the entire sample.

DETERMINE THE DURATION OF THE PROJECT BUFFER AND THE FEED BUFFERS

As determined in Section 3.2, the critical path is composed of tasks A-B-C-D-D-G-I-J--K-M, with a duration of 31.159 months. As for the feed buffers, the project presents three secondary chains:

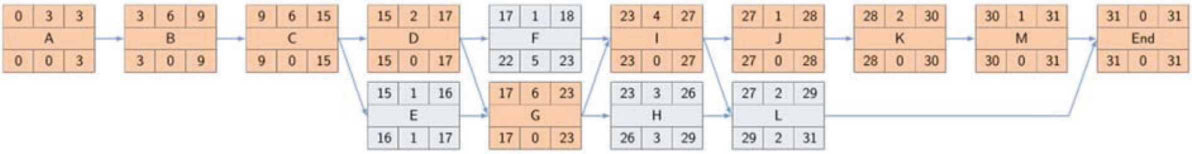

Figure 1: PERT Network

Task	Description	Duration	Background
A.- Basic Engineering	Elaboration of Basic Projects for start of processing	3	-
B.- Administrative Procedures	Legal processing of projects (building permits works and administrative authorization)	6	A
C.- Detail Engineering	Elaboration of Execution Projects	6	B
D.- Procurement	Bidding and awarding of contracts for supply and contracting	2	C
E.- Topography	Topographic stakeout of the work to be executed.	1	C
F.- Receipt of material	On-site stockpiling of electro-electronic material mechanic	1	D
G.- (Civil works) civil	Execution of civil works: pipelines, ditches, fences, ...	6	D, E
H.- Buildings	Supply and erection of buildings prefabricated (substation, CT, ...)	3	G
I.- Electro-mechanical work	Installation of solar trackers, solar panels and electrical wiring	4	F, G
J.- Cold tests	Mechanical and electrical checks prior to start-up	1	I
K.- Act of start-up	Administrative procedures required for the energization of the installation.	2	J
L.- Roads	Final execution/refining of roads external and internal to the facility	2	H, I
M.- Network	Energization, final testing and commissioning operation	1	K

Table 1: List of tasks and precedence relationships

The critical path is the sequence of the following tasks A-B-C-D-D-G-I-J-J-K-M, with a duration of 31 months.

						Average		
Task	Background	Expert	Optimistic	More likely	Pessimistic	Optimistic	More likely	Pessimistic
B	A	E1	5	6	12	4,6058	6,0000	9,1652
		E2	3	6	7			
		E3	5	6	7			
		E4	6	6	12			

Table 2: Experts' opinions for task B

- Task E, with a duration of 0.916 months.
- Task F, with a duration of 0.988 months.
- H-L tasks, with a combined duration of 4,937 months.

Goldratt's or 50% of the chain method

Taking into account the Goldratt or 50% method, the size of the project and feed buffers is calculated. Table 3 summarizes the results obtained:

Camino	Project Buffer	Buffer feeding	Duration	Buffered duration
A-B-C-D-D-G-I-J-K-M	2,181	-	31,159	33,340
E	-	0,039	0,916	0,955
F	-	0,043	0,988	1,031
H-L	-	0,445	4,937	5,382

Table 3: Determination of project and feed buffers and total durations. 50% method

Square root of sum of squares (SSQ) method

From the application of the method of the square root of the sum of the squares, results shown in Table 4 are obtained for the size of the project and feed buffers and for the total durations:

Camino	Project Buffer	Buffer feeding	Duration	Buffered duration
A-B-C-D-D-G-I-J-K-M	1,832	-	31,159	32,991
E	-	0,078	0,916	0,994
F	-	0,086	0,988	1,074
H-L	-	0,665	4,937	5,602

Table 4: Determination of project and feed buffers and total durations. SSQ method

Comparing the results in Tables 3 and 4, it can be seen that the size of the project buffer is smaller by applying the SSQ method. However, in the case of the feed buffers, since they are buffers with a very small number of tasks, their size is smaller with the 50% method.

APPLY MONTE CARLO SIMULATION

For the application of the Monte Carlo method, we start from the averages of the experts' opinions for the pessimistic, most probable and optimistic durations, and consider a triangular distribution for these three durations.

Introducing the above durations as triangular distribution functions in the @Risk tool, 100 Monte Carlo simulations are performed, each of them with 50,000 iterations, in order to have representative results of the sample. In this case, the most probable duration will be the mode of the resulting distribution function, since it is the value that appears most frequently within the set of values.

Figure 2 shows the distribution function of the project duration resulting from the Monte Carlo simulation. It shows that the distribution that best fits the results is a "BetaGeneral" distribution.

Considering the above, Table 5 summarizes the results obtained for buffer sizes and total durations:

Determination of project and feed buffers and total durations. Monte Carlo simulation

Camino	Project Buffer	Buffer feeding	Duration	Buffered duration
A-B-C-D-D-G-I-J-K-M	1,774	-	30,905	32,679
E	-	0,079	0,999	1,078
F	-	0,086	1,000	1,086
H-L	-	0,889	4,999	5,888

RESULTS: COMPARISON OF METHODS

A comparison of the three methods applied (Goldratt method, SSQ method and Monte Carlo simulation) shows that both the duration considered and the project buffer are more optimized in the case of Monte Carlo simulation, as shown in Table 6.

Method	Project Buffer	Duration	Buffered duration
Goldratt	2,181	31,159	33,340
SSQ	1,832	31,159	32,991
Monte Carlo simulation	1,774	30,905	32,679

Table 6. Comparison between methods for project duration and project buffers

Table 7 shows the comparison between the three methods used to determine the size of the feed buffers and non-critical paths. In this case, it is observed that, in the case of feed-in buffers for non-critical paths with a low number of affected tasks, the buffers determined with Monte Carlo simulation are similar to the buffers obtained with the SSQ method, so that, in situations with this casuistry, it is not necessary to resort to Monte Carlo simulation to obtain optimal results and the direct application of the SSQ method is recommended.

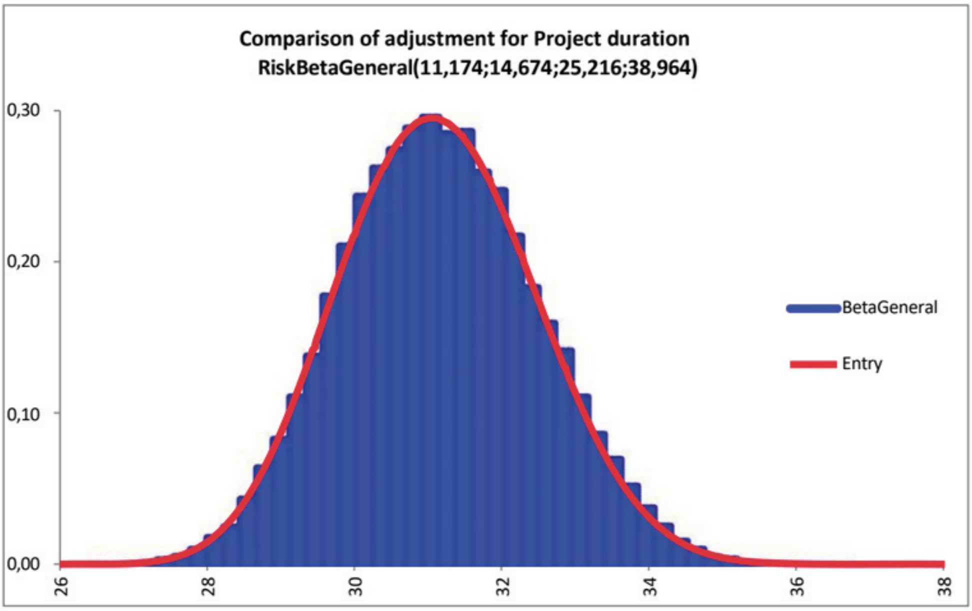

Figure 2: Monte Carlo simulation output and fit graph

Camino	Method	Buffer feeding	Duration	Buffered duration
	Goldratt	0,039	0,916	0,955
E	SSQ	0,078	0,916	0,994
	Monte Carlo simulation	0,079	0,999	1,078
F	Goldratt	0,043	0,988	1,031
	SSQ	0,086	0,988	1,074
	Monte Carlo simulation	0,086	1,000	1,086
H-L	Goldratt	0,445	4,937	5,382
	SSQ	0,665	4,937	5,602
	Monte Carlo simulation	0,889	4,999	5,888

Table 7. Comparison between methods for duration and feeding buffers

CONCLUSIONS

In this work, a new methodology has been proposed for the determination of the size and duration of a project, applied to the practical case of a 50 MW solar photovoltaic power plant. The methodology proposes the composition of a panel of experts and the application of the Delphi method to collect their opinions on the duration of the project tasks, aggregating them by means of the geometric average, which allows considering all the values of the sample without penalizing the extreme values, as it happens in the case of using the mean values or the arithmetic average.

For the sizing of the buffers based on the durations established by the panel of experts, two classic methods were used, the Goldratt method and the SSQ method, and they were compared with the results obtained from applying Monte Carlo simulation. The Monte Carlo simulation was performed with the @ Risk® software, which allows working directly with the distribution functions, and which provides, among other statistical values, the value of the mode of the result distribution function, which has been chosen for the determination of the task durations as it is the representative value of the totality of the data of the result function.

Traditional buffer sizing methods have limitations in environments with uncertainty, whereas the use of Monte Carlo simulation allows to consider all the uncertainty associated with the duration of each activity. For the determination of the

In the case of the project duration and the size of the project buffer, the optimal method is the one proposed in the methodology of this article based on Monte Carlo simulation. However, in the case of feedforward buffers, for paths with small number of tasks, no improvement in optimization is observed between the SSQ method and Monte Carlo simulation, so it is recommended the application of the SSQ method for these cases of small entity paths, recommending Monte Carlo simulation for larger feedforward buffers.

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