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METHOD OF INTEGRATION BY PARTS, OBTAINING PATTERNS AS A DIDACTIC STRATEGY FOR LEARNING INTEGRAL CALCULUS

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Citlalin Aurelia Ortiz Hermosillo

Abstract: In this article you will find an analysis of the solutions of some integrals using the method of integration by parts (MIP) as a didactic strategy for learning calculus, different functions of the same family were integrated, varying the exponents, in order to find patterns in the results of the indefinite integral, until reaching a generalization of the solution. This was done with the intention of generating learning and the development of mathematical analysis in the students who took the Integral Calculus course at the Tecnológico Nacional de México (TecNM) / Instituto Tecnológico de Matamoros (ITM).
Keywords: learning, calculus, piecewise integral, pattern

INTRODUCTION

The use of patterns in the solution of integrals refers to the technique of recognizing common shapes in integrals to apply already known integration methods, facilitating the resolution process. These patterns can be identified in a variety of situations, and can be used as a didactic strategy to solve integrals more efficiently in the teaching of integral calculus.

The Instituto Tecnológico de Matamoros (ITM) is one of the 254 institutions that belong to the higher education system of the Tecnológico Nacional de México (TecNM) in Mexico. The ITM offers educational services in higher education, distributed in its 11 educational programs, two undergraduate, nine engineering and one graduate degree (González, 2022). Of the nine engineering educational programs offered by the ITM, as part of its curriculum, the subject of integral calculus is offered. This subject is divided into four topics, one of them is: methods of integration and indefinite integral, which has as subtopic 2.3.3 Integration by parts. It seeks to develop learning activities such as: presenting a group of integrals to select the most appropriate solution method, thus solving integrals that cannot be solved directly (TecNM, 2010).

Among the different methods of integration established in the syllabus of the course is the method of integration by parts, which is a technique based on the formula of the derivative of the product of two functions. This technique is very useful when it comes to integrating the product of two functions whose direct product is difficult to integrate. The integration by parts formula is derived from the product rule for derivatives:

$$\frac{d}{dx}(uvv) = u'\,v + uvv'$$

If we clear uv', we obtain the integration by parts formula:

$$\int u\,dv = uv - \int v\,du\,v\,du$$

Where:

- u is a function that we choose to derive.
- dv is the part of the integral that we are going to integrate.
- du is the derivative of u
- v is the integral of dv

For the solution of the integrals by IPM, the following steps must be followed:

1. Select u and dv, select them so that the resulting integral $\int v\,du$, easier to solve than the original.
2. Derive u to obtain du and calculate the integral of dv to obtain v.
3. We substitute u, du and v in the integration by parts formula.
4. Solve the new integral, to conclude.

According to (Márquez, 2010), for the solution of integrals through IPM, the following should also be taken into account:

1. u is an easy function to derive.
2. dv is an easy-to-integrate function.
3. $\int v\,du$ is simpler than the initial integral.

The solutions found under the IPM, will allow you to make an analysis to reach a generalization, as stated by Zarapeta (2022) is one of the most important cognitive processes of mathematical activity and has a relevant role in the classroom. Also the authors Mason, Burton and Stacey (1992), state that generalizations constitute the real nerve of mathematics.

METHODOLOGY APPLIED

The analysis that this work presents, consists of the solution of an integral family as shown in Table 1, with the purpose of determining patterns in its solution of the integral as a didactic strategy for learning calculus, as established by Campos (2021) in his work on GeoGebra as a means to identify patterns, where he mentions that it is desirable to focus part of the mathematics classes to the identification of patterns, the posing of conjectures and their possible justification/ generalization, as this activity is consistent with thinking mathematically.

Analysis case	
Family of functions to be integrated	solution
$\int x^1 \ln x\, dx$	?
$\int x^2 \ln x\, dx$	?
$\int x^3 \ln x\, dx$	?
$\int x^4 \ln x\, dx$	?
⋮	⋮
$\int x^n \ln x\, dx$	?

Table 1: Family of integrals of the form $\int x^n\, ln\, x\, dx\, dx$.

The methodology was carried out through the following steps:

1. Theoretical explanation for IPM solution in the classroom

2. Solution is integral of the form $\int x^n \ln x\, dx\, dx$ for positive integer values and of n≠0.

3. Use of GeoGebra to validate the solutions found.

4. Generalization through analysis of the results.

5. Analysis of the pattern found.

In addition to the steps carried out in the classroom, the geometric representation of the product family of functions was analyzed using GeoGebra, which is a software for all educational levels, dynamically brings together the various branches of Mathematics classification, can be used online on a computer or through the application available for the cell phone (Hermosillo, 2022). As shown in Figure 1.

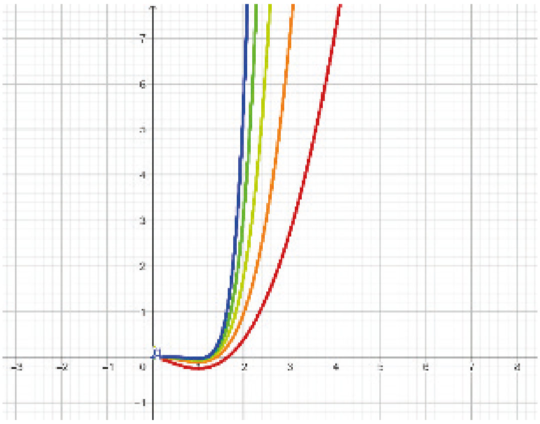

Figure 1: curves of the product of functions

Below is the solution to several integrals of the form $\int x^n \ln x\, dx$, considering the equation $\int u\, dv = uv - \int v\, \boldsymbol{du}$, used in the integration by parts method.

Example 1:

Calculate the integral $\int x \ln x \, dx \, dx$

$$\int x \ln x \, dx = (\ln x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\left(\frac{1}{x}\right) dx$$

$$\int x \, lnx \, dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int xdx$$

$$\int x \, lnx \, dx = \frac{x^2}{2}\ln x - \frac{1}{2} * \frac{x^2}{2} + c$$

$$\int x \, lnx \, dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$$

Considering a:

$$u = \ln x \qquad dv = x \, dx$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int xdx$$

$$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$$

Example 2:

Calculate the integral $\int x^2 \ln x \, dx \, dx$

$$\int x^2 \ln x \, dx = (\ln x)\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3}\left(\frac{1}{x}\right) dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 \, dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \frac{1}{3} * \frac{x^3}{3} + c$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + c$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + c$$

Considering a:

$$u = \ln x \qquad dv = x^2 \, dx$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int x^2dx$$

$$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$$

Example 3

Calculate the integral $\int x^3 \ln x \, dx \, dx$

$$\int x^3 \ln x \, dx = [\ln x]\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4}\left(\frac{1}{x}\right) dx$$

$$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 \, dx$$

$$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{1}{4} * \frac{x^4}{4} + c$$

$$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + c$$

Considering a:

$$u = \ln x \qquad dv = x^3 \, dx$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int x^3dx$$

$$du = \frac{1}{x}dx \qquad v = \frac{x^4}{4}$$

RESULTS

Considering the examples developed in the previous section by varying the exponent n, which are type exercises that students solve in the topic of the method of integration by parts, in the subject of integral calculus for all engineering curricula of the 254 institutes of the TecNM, it is obtained that by applying $\int u\, dv = uv - \int v\, du$, we obtain a generalization that leads us to the pattern for the solution of all integrals of the form $\int x^n \ln x\, dx$, as shown in the following Table 2.

Integral	u	dv	solution
$\int x \ln x\, dx\, dx$	$\ln x$	xdx	$\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$
$\int x^2 \ln x\, dx$	$\ln x$	${}^2x\, dx$	$\frac{x^3}{3}\ln x - \frac{1}{9}x^3 + c$
$\int x^3 \ln x\, dx$	$\ln x$	${}^3x\, dx$	$\frac{x^4}{4}\ln x - \frac{1}{16}x^4 + c$
$\int x^4 \ln x\, dx\, dx$	$\ln x$	${}^4x\, dx$	$\frac{x^5}{5}\ln x - \frac{1}{25}x^5 + c$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\int x^n \ln x\, dx\, dx$	$\ln x$	${}^nx\, dx$	$\frac{x^{n+1}}{n+1}\ln x - \frac{1}{(n+1)^2} x^{n+1} + c$

Table 2: Solution of the integrals.

CONCLUSIONS

In order for the student to understand the generalization of the solution of a family of integrals of the same type and to obtain a mathematical pattern, it was necessary to approach the subject of the method of integration by parts from the particular to the general, thus developing their capacity for abstraction, analysis and synthesis, as well as their ability to identify, pose and solve problems, which are competencies that the subject establishes that the student must achieve.

To find the solution of the integral of the form $\int x^n \ln x\, dx$ by using the method of integration by parts, it has a pattern for positive values of n, it will only be necessary to solve it using the equation $\int \boldsymbol{u\, dv = uv} - \int \boldsymbol{v\, du}$, it is worth mentioning that it does not apply for n=0, the pattern is as follows, as shown in Table 3:

Integral	solution	solution
$\int x \ln x\, dx\, dx$	$\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$	$\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + c$
$\int x^2 \ln x\, dx\, dx$	$\frac{x^3}{3}\ln x - \frac{1}{9}x^3 + c$	$\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + c$
$\int x^3 \ln x\, dx\, dx$	$\frac{x^4}{4}\ln x - \frac{1}{16}x^4 + c$	$\frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + c$
$\int x^4 \ln x\, dx\, dx$	$\frac{x^5}{5}\ln x - \frac{1}{25}x^5 + c$	$\frac{x^5}{5}\left(\ln x - \frac{1}{5}\right) + c$
$\vdots$	$\vdots$	$\vdots$
$\int x^n \ln x\, dx$	$\frac{x^{n+1}}{n+1}\ln x - \frac{1}{(n+1)^2} x^{n+1} + c$	$\frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) + c$

Table 3: Generalization of solutions.

The use of patterns in solving integrals is a fundamental tool to speed up the calculation process. The key is to recognize these standard forms, whether in elementary functions or complex combinations, and apply the appropriate techniques. Constant practice with different types of integrals will allow you to identify these patterns quickly and more easily. When the student manages to understand and assimilate the pattern for solving this family of integrals, he/she will be able to give an immediate answer without the need to solve it.

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