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COMPARISON OF STRUCTURAL RELIABILITY PROBLEM FORMULATIONS

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Abstract: The classic formulation of structural reliability problems, such as that used in the GRG (Generalized Reduced Gradient) and FORM (First Order Reliability Method) methods, is defined as an optimization problem, where the random variables are the design variables, the reliability index is the objective function and the equality constraint is given by the performance function, calculated as the safety margin. The reliability index can be defined as the smallest distance, in the space of reduced variables, between the performance function and the origin of the system. So the reliability problem is usually formulated as: determine the design variables (random variables) that minimize the objective function (reliability index) subject to the equality constraint (safety margin). As the objective function is the distance from the project to the origin, in the space of the reduced variables, it doesn't matter in the equation whether these variables have positive or negative values. This can cause problems for the solution, as the sign of these variables significantly interferes with the calculation of the probability of failure of the model being analyzed. Examples will be shown where this formulation is not valid. The paper concludes that the most appropriate formulations are those based on defining the reliability index as the ratio between the mean and standard deviation of the performance function. Formulations such as the Monte Carlo (MC) process use this definition and therefore do not affect the results obtained and are more reliable, especially in more complex problems with a significant number of random variables. Examples using the GRG method and the Monte Carlo process will be presented and the discrepancies between the GRG method and the coherent results given by the Monte Carlo process in some classical problems will be shown. Suggestions for future studies will also be presented.

Keywords: Structural Reliability, Reliability Index, GRG Method, Monte Carlo Process**.**

INTRODUCTION

The analysis of the problems presented here aims to show the limitations of some extremely common methods for determining structural reliability. The GRG and Monte Carlo methods will be used. The former, based on an optimization problem, has random variables as design variables, the objective function is the reliability index and the equality constraint is the performance function, defined here as the safety margin of the structural system under analysis. The Monte Carlo process is defined by generating random values for the design variables and then calculating the mean and standard deviation of the performance function. In this case, the reliability index is defined as the ratio between the mean and the standard deviation of the performance function.

In the case of the GRG method, the reliability index is defined as the distance between the current design and the origin of the system in the space of the reduced random variables, respecting the restriction that the safety margin must be zero. To calculate the distance, the reduced variables are squared, added together and then the square root of this sum is computed. By squaring the reduced variables, you lose information on whether they are positive or negative. For example, in a case where the design variable is the load acting on a beam, you want this load to be increased by an average value. In this case, the reduced variable must be positive to ensure adequate safety. In the case of the random variable being the strength of the beam material, it is desirable for the strength considered in the calculation to be a value lower than the average, therefore with the reduced variable being negative. It can be seen that this design offers a certain degree of safety, since the external force considered is higher than the average and the resistance considered is lower than the average.

Now imagine a situation where the beam has an unsafe design, where in order to achieve a balance between the external load and the resistance, the force (negative reduced variable) is reduced and the resistance (positive reduced variable) is increased in relation to the average. In this case, note that the project is not safe, but by squaring the reduced variables, you lose the information about whether they are positive or negative and therefore you can have a reliability index equal to that of the project in the previous paragraph, but which does not reflect the probability of failure of the model, quite the opposite, it provides erroneous information that the project is safe, but in fact it is not. With the Monte Carlo process, because the average of the performance function can be negative or positive, the information on the sign of the reduced variables is not lost, giving the analysis a correct reliability index that can clearly indicate whether the project is safe or not.

A literature review was carried out on the main methods for calculating structural reliability. The works read range from a general review of the main tools and basic concepts to advanced applications with specific algorithms and problems. Despite the large number of references available, an attempt was made to focus on methods and structures similar to those analyzed in this work.

Tao's work (2017) presents a new model in which tools from Fuzzi logic are combined to represent epistemic uncertainties. A Markov Chain Monte Carlo (MCMC) is used to solve reliability problems by introducing intermediate events that represent the breakdown of the structural system. The use of Monte Carlo Simulation was carried out by Li (2015) in whose work the internal components of the gears of a wind turbine are analyzed in order to improve the calculation of their reliability. The relationship between the different internal components is addressed through fault trees. The results obtained in this formulation were validated using the Bayesian network methodology.

A comparison between the Monte Carlo process and the FORM method was carried out by Sciuva (2003). Five different variations of these methods were studied and applied to two examples: an isotropic material structure and a laminated composite material structure. The random variables considered were the forces, geometry and material properties, all with an uncorrelated normal distribution. The main objective of this work was to analyze the performance of the methods used. Jin (1993) used the Monte Carlo Process together with the Finite Element Method to calculate the reliability of a complex bearing system working under pressure, used in the oil industry. The author's conclusion was that it took only a few simulations to obtain the reliability value of the structural system, demonstrating the applicability of the methods.

Foliente et al (2000) applied the Monte Carlo process, together with a modified version of the BRANZ procedure, to assess the lateral capacity of timber retaining walls. One of the recommendations of this work is that seismic forces should be treated by introducing uncertainties into the system. Along the same lines, Yen (1987) proposes that the safety analysis of structures subjected to loads from geophysical sources should be carried out from a stochastic point of view, introducing uncertainties in both the structure and the loads. The author applied the MVFOSM method and the Advanced First-Order Method, together with a fault tree to determine the probability of failure of the structural system. Hwang et al (1979) have successfully used the augmented Lagrangian method (GLF) and GRD to solve various reliability and non-linear optimization problems and claim that these are the best methods among the many existing algorithms.

According to Akpan (2015), the traditional design of naval structures is based on a combination of experience, common sense and deterministic approaches and usually ignores the potential for design improvement and other benefits offered by the use of reliability methods and structural optimization strategies. In this work, two structures were optimized: (1) a simple ship structure and (2) a more complex ship structure, in an attempt to achieve weight reduction in the face of constraints on ultimate strength and buckling capacity. A weight reduction of 5.6% was obtained in case (1) and 2.0% in case (2). According to the authors, these results highlight the potential benefits of reliability methods and structural optimization strategies and encourage their implementation during the initial phase of ship structural design.

The study of the reliability over time of prestressed box girders, taking into account slow deformation over time, was carried out by Guo (2016). He studied various types of reinforcement and their respective reliability. The analysis over time was simulated using an incremental static analysis. Yanaka (2016) also studies bridges built with prestressed beams, but focuses on their durability by analyzing the reliability of the reinforcement when it is subjected to attacks by agents that cause corrosion. This article deals with the development of recommendations for the durability design of structures in marine environments from the point of view of reliability, taking into account the life cycle cost of a structure. Steinberg (1997) studies the reliability of haunch-type connections used in prestressed concrete beams. The results presented in this article show that the reliability index for these types of connections is relatively low compared to the reliability levels found in most of the design standards currently in use.

The reliability analysis of metallic pipes under a corrosion process was carried out by Gong (2017) using the FORM method. The methodology involves first constructing two linearized equivalent limit state functions for the pipe segment in standard normal space and then evaluating the probabilities of leakage and explosion of the segment incrementally over time based on the equivalent limit state functions. Makhduomi et al (2017) study three first-order reliability method (FORM) algorithms using the steepest descent search direction. The results are compared to evaluate the reliability index of structural steel problems that are designed by the Iranian national building code. The steel components designed by the Iranian code showed good levels of reliability with the reliability index in the range of 2.5 to 3.0. The study of towers to support wind generators installed offshore was carried out by Kim (2015). A dynamic analysis is carried out where the response is expressed as the static response multiplied by the peak response factor. The reliability index is found using the first-order reliability method (FORM). Fracture failure analysis with a reliability approach was carried out by Beom-Jun et al (2016). Due to the uncertainties related to failure assessment parameters such as flaw size, fracture toughness, load spectrum and so on, the concept of probability is preferable to deterministic in failure assessment. In this study, efforts were made to develop the reliability-based failure assessment procedure, which combines failure assessment procedures and the first and second order reliability methods (FORM/SORM). The validity of the results obtained was checked by comparing them with those obtained by Monte Carlo simulation. It was confirmed that the methodology developed worked perfectly for calculating reliability without the time-consuming Monte Carlo simulation, according to the authors.

Reddy (1994) carried out the optimal design of various types of structure by imposing a minimum reliability index. In this case, the reliability index is obtained by interpolating values around a designated mean value. Along the same lines, but working on the design of hydraulic channels, Adarsh (2013) imposes a certain reliability index and varies other design parameters to obtain an optimized solution. In this case, an advanced first-order second moment method and Monte Carlo Simulation are used, and it was found that the results of both approaches show good agreement. Also working with Monte Carlo Simulation and a first-order second moment method, Kareem (1990) made several reliability estimates for a concrete chimney. Multiple potential failure modes are represented by exceeding the admissible moment at any level of the chimney height. Limits are set based on existing theory, taking into account not only the probability of failure of individual modes, but also the joint probabilities of failure in any two modes. This author suggests that the first-order second-moment approximation and simulation methods, which combine the Monte Carlo technique with variance reduction techniques, can provide accurate results for reliability analysis in practical cases of wind-excited structures. According to Saydam (2013), the reliability index of a given system can be calculated using the first order second moment method (FOSM). According to this author, this method is accurate if both the load and resistance effects of the system follow normal or lognormal distributions. However, the amount of error introduced can be significant when the random variables follow distributions other than normal or lognormal.

Greiner (2012) presents an approach for the simultaneous optimization of structural mass and reliability in truss structures. In addition to member sizing, the selection of an optimal topology from a pre-specified structure is a feature of the methodology proposed by the author. To enable a global search, optimization is carried out using a multi-objective evolutionary algorithm. In the work by Meng et al (2016) a new method for calculating the probability of failure of a structure subjected to fatigue is proposed, dealing with problems with uncertainties and random variables. The method is based on calculating the moments of the performance function by developing it in Taylor series. Two numerical examples of increasing complexity are used to demonstrate the viability of the proposed approach.

Bian (2015) developed a new reliability-based approach for the analysis and design of piles, incorporating the Serviceability Limit State and Ultimate Limit State requirements of the LRFD standard. Three methods for reliability-based analysis and design were adopted, namely the MVFOSM method, the AFOSM and the Monte Carlo simulation method. This study recommends the AFOSM method for performing reliability analysis. Continuing with work applied to pile foundations, it is worth highlighting the work of Kwak (2010), who as part of a study to develop parameters for determining load and resistance factors for design (LRFD) for foundation structures in South Korea, calibrated the resistance factors for the static load capacity of steel piles in the context of reliability theory. A database of 52 static load test results was compiled, and the data was classified into two cases: a standard pile tip penetration (SPT) N-value (i) less than 50 and (ii) greater than or equal to 50. Reliability analyses and resistance factor calibration were carried out using the first-order reliability method (FORM) and Monte Carlo simulation (MCS). The reliability indices and resistance factors calculated by MCS are statistically identical to those computed by FORM. Target reliability indices were selected as 2.0 and 2.33 for the case of a group of piles and 2.5 for the case of an isolated pile. The author points out that the resistance factors recommended by this study are specific to pile foundation design and construction practices and subsurface conditions in South Korea.

The use of two methods together for optimization and reliability determination was used by Meng (2014). The methods are Mean-Value First-Order Saddlepoint Approximation (MVFOSA) for the reliability calculation and Collaborative Optimization (CO) Method for the optimization process. The authors showed that the combination of these methods provided good accuracy in two of the examples analyzed. According to Haug (2008), due to the use of complete distribution information, MVFOSA is generally more accurate than MVFOSM with the same computational effort. He adds that it is also more efficient than FORM because there is no need for an iterative search process for the so-called Most Likely Point (optimum point). These conclusions are confirmed by the author through four numerical examples.

Al-Harthy and Frangopol (1994) present a reliability-based procedure for the design of prestressed concrete beams. Loading, material properties and prestressing force levels are treated as random variables. Reliability methods, based on the second moment, are used to calculate the failure probabilities of the beams in the initial and final stages. Some examples are solved and design charts are provided by the authors to facilitate the implementation of the proposed approach. Rackwitz and Flessler (1978) proposed an algorithm for calculating structural reliability under a combination of loads. Loads or any other actions on structures are modeled as independent random variables. The performance function is approximated in points by a tangent hyperplane. The iteration algorithm looks for a point where the probability of failure given by the tangent hyperplane reaches its maximum. Any type of performance function and any type of probability distribution for the loads can be handled. The method is illustrated in an example with a wall beam with no tensile strength, loaded by a bending moment and a normal force.

One work that attracted a lot of attention during the course of this research was that by Pachás (2009), where a version of the FORM method is presented in great detail and allows for good computational implementation. In addition, a summary of the main statistical tools involved in the process of calculating structural reliability is presented. The applications are in the area of slope stability. In addition to this work, teaching materials ranging from basic concepts to more sophisticated approaches, such as the Finite Element Method, can be found in the works by Melchers and Beck (2018), Nowak and Colloins (2012) and Ditlevsene Madsen (2005).

MATERIALS AND METHODS

The probability of failure of a model can be calculated using a reliability method. Usually we have the random variables designated by $\boldsymbol{x}$ and the reduced variables given by $\boldsymbol{y}$. The relationship between them is given by

$$y_i = (x_i - \mu_i\,)/\sigma\,_{,i} \qquad (1)$$

where μ is the vector of averages and $\boldsymbol{\sigma}$ is the vector of standard deviations. Since the mean is a positive number, note in (1) that if the value of the random variable x_i is greater than the mean, y_i is positive, and if it is less y_i is negative. In this paper, for didactic reasons, all random variables are assumed to have a normal distribution.

A classic reliability problem can be defined as an optimization problem:

determine $\boldsymbol{x}$ Î $\Re^n$ that minimizes

$$f(\boldsymbol{x}) = h(\boldsymbol{y}) = (y_1{}^2 + y_2{}^2 + \ldots + y\ )_n{}^{20,5} \qquad (2)$$

subject to

$$g(x) = F(x) = 0 \qquad (3)$$

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In this problem, the components of the vector of design variables are the random variables $\boldsymbol{x}$ and $F(\boldsymbol{x})$ is the performance function that can represent, in structural reliability, the system's safety margin. The reliability index is $\beta = f(\boldsymbol{x}^*) = \min f(\boldsymbol{x})$, where $\boldsymbol{x}^*$ is the solution to the problem defined by Equations (2) and (3). The higher the value of β, the lower the probability of failure. Note that in Equation (2) the fact that y_i is positive or negative, having the same absolute value, does not affect the result of the equation. This fact can lead to errors when calculating β.

Another definition for β is the one used in the Monte Carlo Process:

$$\beta = \mu_F / \sigma_{,F} \tag{4}$$

where m_F and s_F are the mean and standard deviation of the performance function $F(\boldsymbol{x})$, respectively. In the Monte Carlo process, a large number of numbers are randomly generated for the random variables and the function $F(\boldsymbol{x})$ is computed for each set of data generated. Finally, the mean and standard deviation of F are computed and β is calculated. Now note that β can be either positive or negative. A negative value of β means that the system has more than a 50% chance of failure, while when β is positive the system has less than a 50% chance of failure. There are situations when the safety of the structure is low, for example, where β is negative. Figure 1 shows a graph relating β to the probability of failure.

In summary, β can be determined either by the formulation of equations (2) and (3), where the GRG method was used in this work, or by the formulation given by (4), where the Monte Carlo Process was adopted for the resolution.

RESULTS AND DISCUSSIONS

Consider the problem of a bar (Figure 2) subjected to a tensile force S (= x_1) and which has an internal resistance equal to R (= x_2). Then the random variable x_1 is the acting force, while the random variable x_2 is the strength of the material, with a normal probability distribution, as shown in Figure 3.

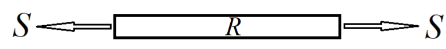

Figure 2 - Bar subjected to traction

The performance function is

$$F = R - S = x_2 - x_1 \tag{5}$$

Note in Figure 3 that μ_1 = 1.00 and μ_2 = 2.65. The values of $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = 0.5$. The reduced variables are calculated using expression (1).

Solving the problem described in equations (2) and (3) using the GRG method, with the above data, the optimum solution is

$$\boldsymbol{x}^* = [1.825 ; 1.825]^T \text{ and } \boldsymbol{y}^* = [1.65; -1.65].^T \tag{6}$$

The value of β is

$$\beta = [\ 1.652 + (-1.65)\ ]2\ 0,5 = 2.333 \tag{7}$$

and the associated probability of failure is

$$Pf = 0.98\%. \tag{8}$$

It can be seen from (8) that the project has a certain degree of safety.

Solving the same problem with the Monte Carlo Process (MC), generating 1,000,000 projects, gives us

$$F\ m = 1.65 \text{ and } sF = 0.71 \tag{9}$$

which determines a value of β equal to

$$\beta = \mu_F / \sigma_F = 2.34 \tag{10}$$

and the associated probability of failure is

$$Pf = 0.96\%. \tag{11}$$

Comparing the results of GRG (7) and (8) with those of MC (10) and (11) shows that the results are practically identical, converging on the same value. In this case, it is worth remembering that the average demand is lower than the average resistance, so a safe design has been obtained.

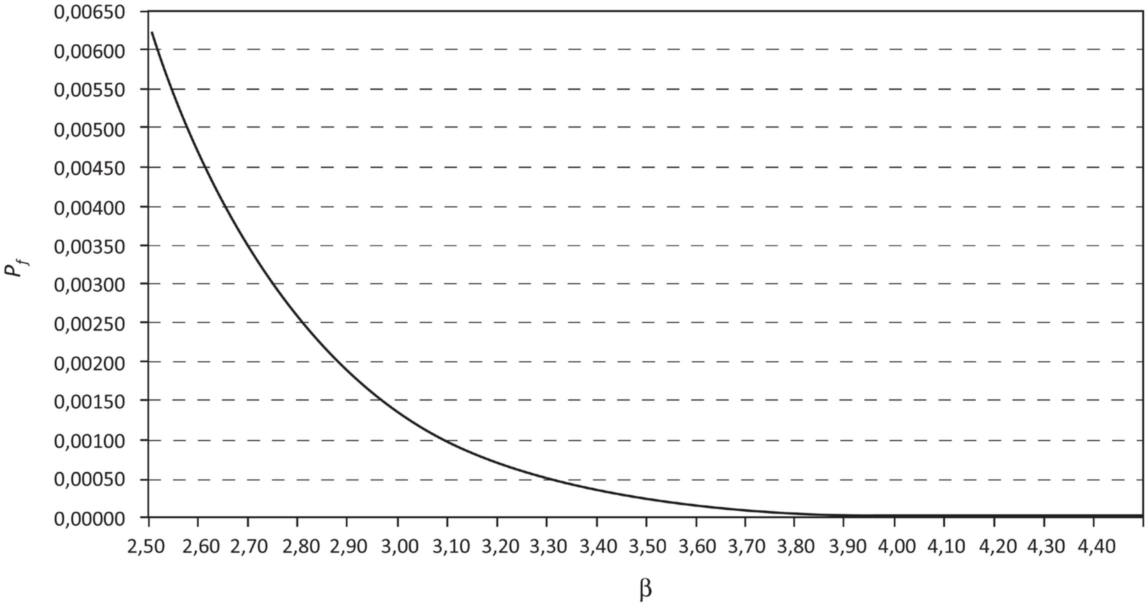

Figure 1 - Probability of failure P_f as a function of the reliability index *b* (Brasil e Silva, 2019)

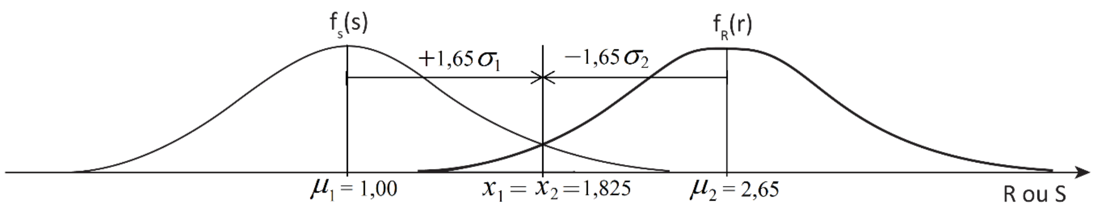

Figure 3 - Problem where $S < R$

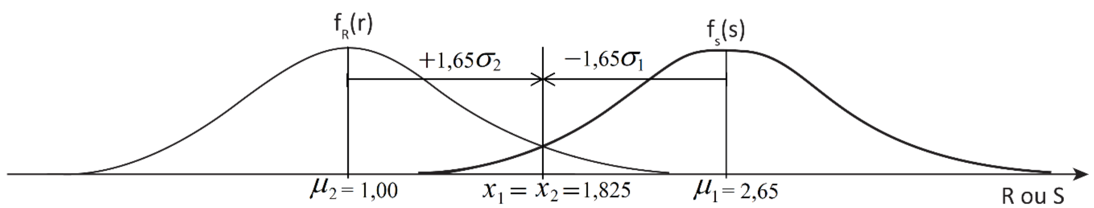

Figure 4 - Problem where $R < S$

Consider the same bar as in Figure 2, with the probability distributions shown in Figure 4. Now note that μ_2 (average resistance) is lower than the average request μ_1. The positions on the graph in Figure 3 have been swapped.

Note in Figure 4 that $\mu_1 = 2.65$ and $\mu_2 = 1.00$. Consider that $\boldsymbol{\sigma}_1 = \sigma_2 = 0.5$. The reduced variables are calculated using expression (1).

Solving the problem described in equations (2) and (3) using the GRG method, with the above data, the optimum solution is

x* = [1.825 ; 1.825]T and y* = [-1.65; 1.65]. T (12)

The value of β is

$$\beta = [\,(-1.65)^2 + 1.65\,]^{2\;0,5} = 2.333 \qquad (13)$$

and the associated probability of failure is

Pf = 0.98%. (14)

It can be seen from (14) that the project, in this formulation, has a certain degree of security.

Solving the same problem with the Monte Carlo Process (MC), generating 1,000,000 projects, gives us

$$\mu_F = -\,1.65 \text{ and } \boldsymbol{\sigma}_F = 0.71 \qquad (15)$$

which determines a value of β equal to

$$\beta = \mu_F / \sigma_F = -2{,}34 \quad (16)$$

and the associated probability of failure is

$$Pf = 99.04\%. \quad (17)$$

Comparing the results of GRG (14) with MC (17) shows that the results are totally different. Although the absolute value of β is the same in both cases, they show opposite signs, which completely changes the probability of failure of the model. It is intuitive that when the average resistance is lower than the average stress, the design is unsafe and has a high probability of failure. In this case, the formulation given by equations (2) and (3) is not adequate.

Consider a hollow circular pile, with a diameter of 42 cm, a wall of 9 cm, longitudinal reinforcement of 8 bars of 12.5 mm (CA-50), with a *d'* = 3.625 cm and fck = 40 MPa. The interaction diagram for this pile is shown in Figure 5, where *N* is the axial force and *M* is the bending moment, with their respective indices indicating resistance or demand, according to NBR-6118 (ABNT, 2014).

The average resistance is shown in green on the graph (Figure 5), while the average stress is shown in red. It can be seen that the average resistance is higher than the average demand, which indicates a project with a certain degree of safety.

In this problem, consider that the random variables are N_{sd} (= x_1) and M_{sd} (= x_2). The values of the means and standard deviations of the random variables are μ_1 = 110 kN, σ_1 = 11 kN, μ_2 = 85 kN.m and σ_2 = 8.5 kN.m. Note that in this case

$$M_{rd} = M_{rd} (N),\ _{sd} \quad (18)$$

i.e. the moment resistance is a function of the axial load. The performance function is

$$F = M_{rd} - M_{sd} \quad (19)$$

Solving the problem described in equations (2) and (3) using the GRG method, with the above data, the optimal solution is

$$x^* = [106 \; ; 100]^T \text{ and } \quad y^* = [-0.32; 1.77]^T \quad (20)$$

The value of β is

$$\beta = [(-0.32)^2 + 1.77]^{2\ 0,5} = 1.8 \quad (21)$$

and the associated probability of failure is

$$Pf = 3.6\%. \quad (22)$$

It can be seen from (22) that the project, despite having a low value for β, compared to those indicated in the literature (Brasil e Silva, 2019) which is of the order of 3, presents a certain degree of safety.

Solving the same problem with the Monte Carlo Process (MC), generating 1,000,000 projects, gives a value of β equal to

$$\beta = 1{,}8 \quad (23)$$

and the associated probability of failure is

$$Pf = 3.6\%. \quad (24)$$

Comparing the results of GRG (22) with MC (24), it can be seen that the results are identical. In this case, it is worth remembering that the average stress is lower than the average resistance, so a safe design has been obtained.

Considering the same pile, the interaction diagram is shown in Figure 6. Again, the graph shows the average resistance in green and the average stress in red. It can be seen that the average resistance is lower than the average stress, which indicates a design with very low safety and certainly a high probability of failure.

In this problem, consider again that the random variables are N_{sd} (= x_1) and M_{sd} (= x_2). The values of the means and standard deviations of the random variables are μ_1 = 70 kN, σ_1 = 7 kN, μ_2 = 120 kN.m and σ_2 = 12 kN.m. Note that in this case $M_{rd} = M_{rd} (N_{sd})$ and that $F = M_{rd} - M_{\cdot sd}$

Solving the problem described in equations (2) and (3) using the GRG method, with the above data, the optimum solution is

$$\boldsymbol{x}^* = [71 \; ; 95]^T \text{ and } \quad \boldsymbol{y}^* = [0.18 \; ; -2.08] .^T \quad (25)$$

The value of β is

$$\beta = [0.18^2 + (-2.08)]^{2\ 0,5} = 2.09 \quad (26)$$

and the associated probability of failure is

$$Pf = 1.8\%. \quad (27)$$

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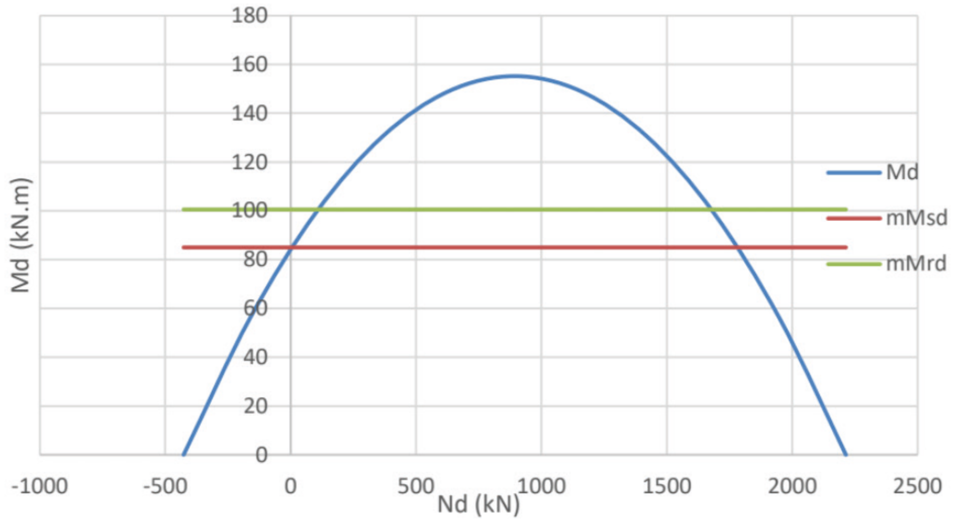

Figure 5 - Interaction diagram with $S < R$

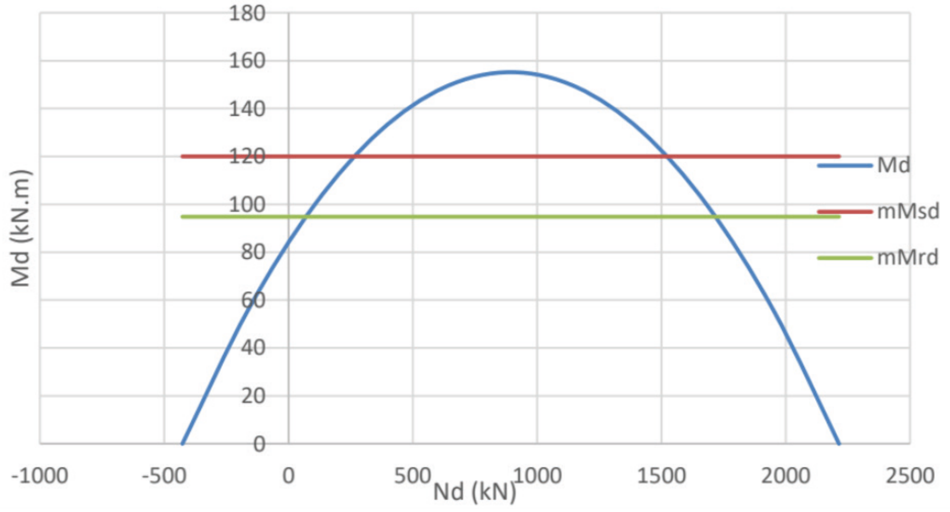

Figure 6 - Interaction diagram with $R < S$

It can be seen from (27) that the project has a certain degree of security.

Solving the same problem with the Monte Carlo Process (MC), generating 1,000,000 projects, gives a value of β equal to

$$\beta = -2{,}09 \tag{28}$$

and the associated probability of failure is

$$P_f = 98.2\%. \tag{29}$$

Comparing the results of GRG (27) with MC (29) shows that the results are totally different. Although the absolute value of β is the same in both cases, they have opposite signs, which completely changes the probability of the model failing. In this example, it can be seen that the formulation given by equations (2) and (3) does not provide adequate results when the resistance is less than the demand.

CONCLUSIONS

Two of the main problem formulations used to calculate the reliability of structural systems were presented in the paper, the first being based on an optimization problem, which was solved here using the GRG (Generalized Reduced Gradient) method. The second is based on the definition of the reliability index and was solved using the Monte Carlo Process (MC). It was shown through two solved examples that the formulation based on the optimization problem was effective in cases where the demand was less than the resistance, but ineffective in cases where the resistance was less than the demand, presenting values totally contrary to intuition and also to the correct mathematical solution of the problem. The MC, on the other hand, was effective in both situations and was able to compute the reliability index values quite accurately. It can therefore be concluded that the use of methods such as GRG and FORM needs to be cautious in problems where the resistance values may be lower than the request. In more complex cases where there are many random variables, the engineer's intuition for solving the problem can be lost, increasing the chance of failure when applying the first formulation. It is suggested that future studies explore problems with a large number of design variables and also compare more methods in order to identify patterns in the behavior of the main reliability methods used.

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