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GEOMETRICAL VISUALIZATION OF THE ANGLE IN INCLINED PLANES

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Abstract: This paper aims to employ concepts of plane geometry to deduce the angle formed in an inclined plane. This deduction is fundamental to solve a wide range of problems in static and dynamic physics. A common difficulty among students is the correct representation of this angle in a free body diagram, regardless of the orientation of the inclined plane.
Keywords: Statics, Dynamics, Inclined plane, Plane geometry, Free body diagram.

INTRODUCTION

The inclined plane, a simple machine widely used throughout history, consists of a flat surface that forms an acute angle with another. This tool facilitates the movement of heavy objects by reducing the force required to move them.

These devices can be related to prehistoric tools such as wedges, axes and arrows, around 2600 B.C. Their most prominent application in antiquity is found in the construction of the pyramids, between 1900 B.C. and 1400 B.C., where ramps were used to manipulate large blocks of stone. Likewise, at Stonehenge, around 1400 BC, inclined planes were used to place the crossbeams, according to Gale [1]. Even in Mesopotamia, around 3500 BC, there is evidence of their use in the construction of stairs, as shown by Duato [2].

The term 'simple machines' was coined by Greek philosophers in the 3rd century B.C. Later, Archimedes, in the 3rd century B.C., made detailed studies on the lever, the pulley and the screw, quantifying their mechanical advantages. In the 1st century A.D., Heron of Alexandria expanded this classification to include the wedge and the lathe. During the Renaissance (14th-17th centuries), the idea that all simple machines are fundamentally derived from the lever, the inclined plane and the pulley was consolidated, according to the work of Nuñes M [3].

Euclid, considered the "Father of Geometry", was the first to make a formal study of the inclined plane in his Elements (325-265 BC). In this work, the properties of various geometric figures, including the inclined plane, are rigorously presented. Although it is possible that some of the results were already known, the systematic organization and proof of theorems are attributed to Euclid. Later, Archimedes made important contributions to plane geometry, according to Barrera [4]. For a more complete analysis of the inclined plane, it is necessary to turn to trigonometry, a branch of mathematics that studies the relationships between the sides and angles of triangles. The Babylonians and the Egyptians had already developed methods for calculating angles and side lengths in right triangles, as evidenced in the construction of the pyramids. The Egyptians, in particular, established the measurement of angles in degrees, minutes and seconds, according to Perez [5]. Finally, for a dynamic study of the inclined plane, Newton's second law is applied, which relates the net force applied on a body to its acceleration, according to Sánchez [6].

METHODOLOGY OR DEVELOPMENT

In this study, plane geometry tools, such as the use of angle brackets, will be used to determine the angle in the free body diagrams. The drawings will be made by hand. Subsequently, a survey will be conducted to assess the students' level of understanding regarding the deduction and representation of this angle in the diagrams.

Angles opposite at the vertex are shown in Figure 1 below.

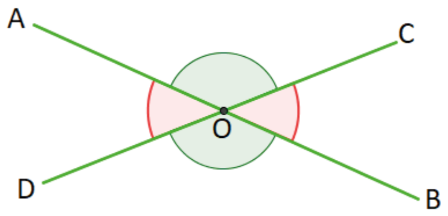
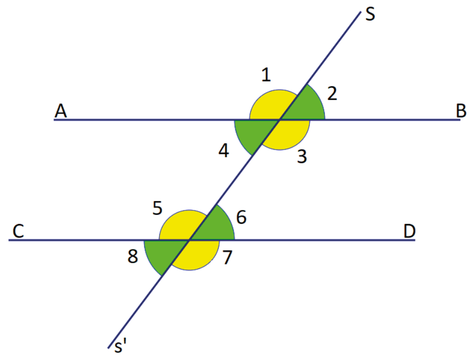


Fig. 1. Shows the opposite angles at the vertex.

Angles formed by two straight lines cut by a secant as shown in Figure 2.



Angles formed by two straight lines cut by a secant line.

The corresponding angles will be used, which are defined as those that occupy the same relative position in each of the parallel lines with respect to the secant. They are congruent (equal in measure).

$$\sphericalangle 1 y \sphericalangle 5, \sphericalangle 2 y \sphericalangle 6, \sphericalangle 3\ y\ \sphericalangle 7, \sphericalangle 4\ y\ \sphericalangle 8$$

Theorem. The sum of the interior angles of a triangle is equal to 180° degrees. For the first development, Figure 3 will be used as a reference.

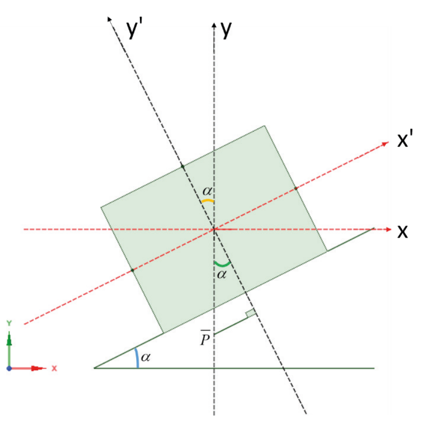


Fig. 3. Inclined plane.

In the figure, we can see a blue inclined plane on which two auxiliary lines have been drawn: a purple one, perpendicular to the base of the plane, and an orange one, parallel to the first one. Due to the arrangement of these lines, corresponding angles are formed between the purple and orange lines. According to the postulate of corresponding angles, these angles are congruent. Furthermore, the angle formed by the purple and orange lines is opposite at the vertex to the angle to be determined. Therefore, since all these angles are congruent, the value of the angle of the inclined plane can be obtained for its representation in the free body diagram, using figure 4.

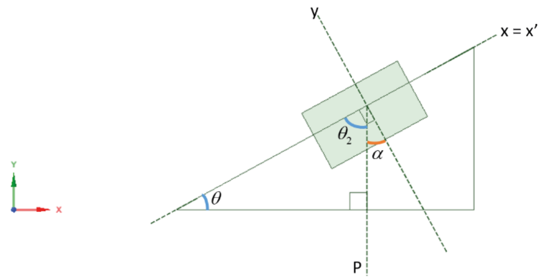


Fig. 4. Inclined plane showing three angles.

In this case, we will apply the theorem that states that the sum of the interior angles of a triangle is equal to degrees, as well as the definition of a right angle, which measures degrees. These properties will be fundamental for the analysis of the triangle formed by the dotted line, the base and the height, represented in blue.

$$\theta + \theta_2 + 90^o = 180^o \qquad (1)$$

$$\theta + \theta_2 = 90^o \qquad (2)$$

The triangle formed by the base, the axis, the axis and the axis where they intersect to form a right angle (90^0 degrees), satisfies the following equation

$$\theta_2 + \alpha = 90^o \qquad (3)$$

From equations (2-3), a system of two linear equations with two unknowns is constructed.

$$\theta + \theta_2 = 90^o \qquad (4)$$

$$\theta_2 + \alpha = 90^o \qquad (5)$$

To solve the system, multiply the whole equation (4) by minus one. In this way, we obtain

$$-\theta + \alpha = 0 \qquad (6)$$

From which it follows that:

$$\theta = \alpha \qquad (7)$$

What was sought to be demonstrated.

In the following procedure, the same construction lines are used, but the set of squares is used as the main tool. As illustrated in figures (4 - 5), this method allows to determine again the angle required for the free body diagram.

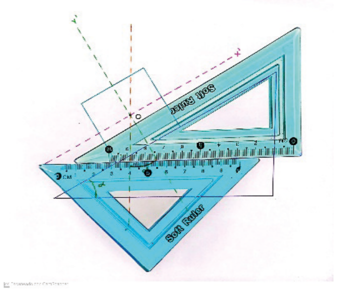


Fig. 4. Formed angle of the plane using angle brackets.

Figure 5 shows the angle formed by the purple line, parallel to the y-axis, and the green dotted line y'. It is observed that this angle coincides with the corresponding angle in the free body diagram, where a new coordinate system has been established.

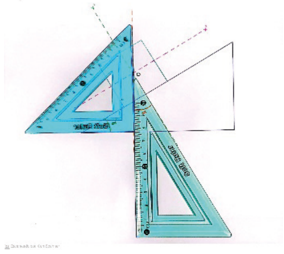


Figure. Angle formed from the plane using angle brackets.

Since the purple line is parallel to the y-axis, the intersection of its perpendicular with the base of the plane and the new y' axis defines the angle to be used. This geometrical construction is visualized in Figure 6, which simplifies the analysis.

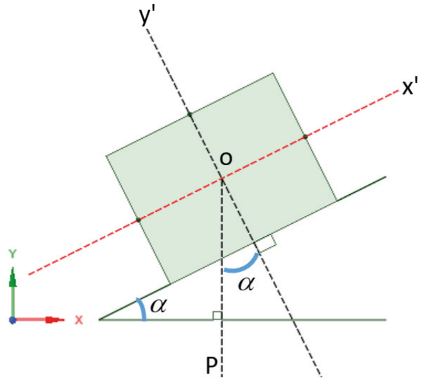


Figure. 6. The angle used is that formed by the intersection of the purple line with the new coordinate axis

In this study, it was determined that the 64 undergraduate students had learned the procedure approximately six months earlier. The assigned task consisted of graphing the angle of an inclined plane and elaborating free body diagrams to evaluate their skills. The following results were obtained from this analysis and are presented in Table 1.

They did not recognize the angle	12 people	18.75%
If the deduction was made	2 persons	3.125%
They do not know how to perform	9 people	14.0625%
If they recognize the angle	41 people	64.0625%

Table 1. represents the data obtained from the survey.

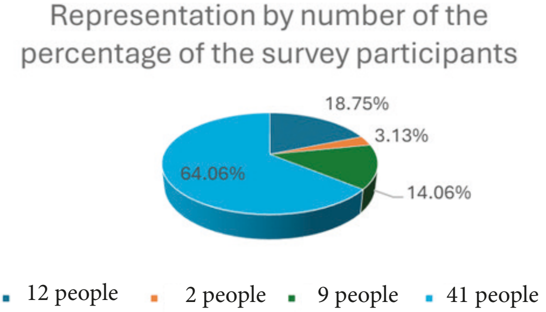


Figure 7. Represents the number of students who: did not recognize the angle, did perform the deduction, do not know how to perform the deduction, and do recognize the angle.

RESULTS AND ANALYSIS

The most laborious procedure consists of analyzing the properties of angles formed by secant lines. However, in this particular case, the identification of right angles and the application of the sum of internal angles theorem simplify the process. The use of angle brackets is faster. According to Table 1, 64.06% of the participants correctly identified the angle in the inclined plane, 3.13% were able to demonstrate it and 32.8125% failed to recognize it.

CONCLUSIONS

The trend observed in Table 1 suggests that, if 100 people were evaluated, approximately 51.26953125% would have difficulty recognizing the angle of the inclined plane. This finding is of concern, considering the academic profile of the participants. Despite the fact that this principle is addressed at previous educational levels, the results indicate a rapid forgetting among engineering students. More comprehensive studies with larger samples are needed to better understand this phenomenon and to design more effective pedagogical strategies.

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