SATELLITE ACS DESIGN USING SDRE METHOD FOR ORBIT INJECTION PHASE

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ABSTRACT: - Depending on the objectives of the space mission, after the satellite's separation phase from the launcher, the satellite's angular velocities may need to be increased or decreased. Besides, the precision of controlling the satellite's attitude during this phase is of fundamental importance for the success of the mission. When the satellite starts this phase with high angular velocity the Attitude Control System (ACS) needs to manoeuvre the satellite to its normal mode of operation characterized by an attitude of small angles. One way to achieve this transition between these two modes of operation is through the use of gas jets followed by reaction wheels. In this paper one investigates by simulation the ACS system in order to minimize space mission costs by reducing the number of errors transmitted to the next phase of the satellite prototypes project. Due the satellite high angular velocities its dynamics have nonlinear equations of motion. As a result, application of linear control technique cannot be able to design the ACS with performance and robustness to reach the required level of appointment. In order to mitigate this problem, one applies the State-Dependent Riccati Equation (SDRE) method which can deal with nonlinear system. The SDRE controller design algorithm is based on gas jets and reaction wheel torques to perform large angle manoeuvre in order to reduce the high angular velocities to attitude with small angles. The energy of the system is used as the criterion for the transition between these two modes. An important result of this investigation is the numerical validation of the simulator model based on the control algorithm designed by the SDRE method. It is planned to test this control algorithm in the National Institute for Space research (INPE) prototype which supplies the conditions for implementing and testing the SDRE ACS in terms of hardware and software.
KEYWORDS: - Satellite attitude control system, nonlinear dynamics

1 | INTRODUCTION

The design of a satellite Attitude Control System (ACS), that involves plant uncertainties [1] and large angle manoeuvres followed by stringent pointing control, may require new nonlinear attitude control techniques in order to have adequate stability, good performance and robustness. Experimental ACS design using nonlinear control techniques through prototypes is one way to increase confidence in the control algorithm. Experimental design has the important advantage of representing the satellite dynamics in laboratory setting, from which it is possible to accomplish different simulations to evaluate the satellite ACS [2]. However, the drawback of experimental testing is the difficulty of reproducing zero gravity and torque free space conditions. A Multi-objective approach [3] has been used to design a satellite controller with real codification. An investigation through experimental procedure has been done in [4] for simulator inertia parameters identification. An algorithm based on the least squares method to identify mass parameters of a rotating space vehicle during attitude manoeuvres has been done in [5]. A method with the same objectives, but based on Kaman filter theory also has been investigated in [6]. The H-infinity control technique was used in [7] to design robust control laws for a satellite composed of rigid and flexible panels. The SDRE method was applied to control the satellite attitude with nonlinear dynamics using the State Dependent Coefficients (SDC) procedure in [8]. In this paper the SDRE technique [9, 10] is applied to design a nonlinear controller for a nonlinear simulator plant. The ACS designed is able to deal with large angle manoeuvres. The control strategy is based on reaction wheel and gas jets as actuators which allow the design of two control algorithms related to the transition from high angular velocity mode to the normal mode of operation with stringent pointing. The optimal switching control algorithm is based on minimum system energy. Several simulations have proven the computationally feasibility. It is planned for real time execution of the SDRE control algorithm to use the INPE prototype taking in consideration the satellite´s on board computer [11].

2 I THE SDRE DESIGN METHOD

The Linear Quadratic Regulation (LQR) approach is well known and its theory has been extended for the synthesis of nonlinear control laws for nonlinear systems. This is the case for satellite dynamics that are inherently nonlinear [12]. A number of methodologies exist for the control design and synthesis of these highly nonlinear systems; these techniques include a large number of linear design methodologies [13] such as Jacobian linearization and feedback linearization used in conjunction with gain scheduling [14]. Nonlinear design techniques have also been proposed including dynamic inversion and sliding mode control [15] and recursive back stepping and adaptive control [16]. The Nonlinear Regulator problem [17] for a system represented in the State-Dependent Riccati Equation form with infinite horizon, can be formulated by minimizing the cost functional given by

$$J(x_0,u) = \frac{1}{2}\int_{t_0}^{\infty} (x^T Q(x)x + u^T R(x)u\,)dt \qquad (1)$$

With the state $x \in \Re^n$ and control $u \in \Re^m$ subject to the nonlinear system constraints given by

$$\dot{x} = f(x) + B(x)u$$
$$y = C(x)x \qquad (2)$$
$$x(0) = x_0$$

Where $B \in \Re^{nxm}$ and C are the system input and the output matrices, and $y \in \Re^{s}$($\Re^{s}$ is the dimension of the output vector of the system). The vector initial conditions is x(0), $Q(x) \in \Re^{nxn}$ and $R(x) \in \Re^{mxm}$ are the weight matrix semi defined positive and defined positive. Applying a direct parameterization to transform the nonlinear system into State Dependent Coefficients (SDC) representation, the dynamic equations of the system with control can be write in the form

$$\dot{x} = A(x)x + B(x)u \qquad (3)$$

With $f(x)=A(x)x$, where $A \in \Re^{nxn}$ is the state matrix. By and large $A(x)$ is not unique. In fact there are an infinite number of parameterizations for SDC representation. This is true provided there are at least two parameterizations for all $0 \leq \alpha \leq 1$ satisfying

$$\begin{aligned} \alpha A_1(x)x + (1-\alpha)A_2(x)x \\ &= \alpha f(x) + (1-\alpha)f(x) \\ &= f(x) \qquad (4) \end{aligned}$$

The choice of parameterizations to be made must be appropriate in accordance with the control system of interest. An important factor for this choice is not violating the controllability of the system, i.e., the matrix controllability state dependent $[B(x)+A(x)B(x)... A^{n-1}(x)B(x)]$ must be full rank. The nonlinear control law fed back by the states has the following form

$$u = -S(x)x\,, \quad S(x) = R^{-1}(x)B^{T}(x)P(x) \qquad (5)$$

For some special cases, such as systems with little dependence on the state or with few state variables, Eq. (5) can be solved analytically. On the other hand, for more complex systems the numerical solution can be obtained using an adequate sampling rate. It is assumed that the parameterization of the coefficients dependent on the state is chosen so that the pair (A (x), B (x)) and (C (x), A (x)) are in the linear sense for all x belonging to the neighbourhood about the origin, point to point, stabilizable and detectable, respectively. Then the SDRE nonlinear regulator produces a closed loop solution that is locally asymptotically stable. An important factor of the SDRE method is that it does not cancel the benefits that result from the nonlinearities of the dynamic system, because, it is not a required inversion and not a dynamic feedback linearization of the nonlinear system.

3 I THE SATELLITE ATTITUDE SIMULATOR

Figure 1 shows the INPE 3D simulator which has a disk-shaped platform, supported on a plane with a spherical air bearing. Considering that the INPE 3D simulator is not complete build, one assumes that there are three reaction wheel and the gas jets configuration set capable to perform maneuver around the three axes and that there are three angular velocities sensor, like gyros. Apart from the difficulty of reproducing zero gravity and torque free condition, modeling the 3D simulator, basically, follows the same step of modeling a rigid satellite with rotation in three axes free in space.

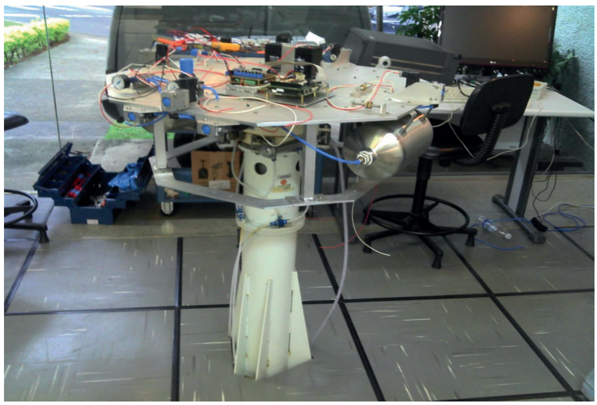

Fig. 1- INPE 3D simulator with gas jets and three reaction wheels.

The orientation of the platform is given by the body reference system F_b with respect to inertial reference system F_I considering the principal axes of inertia and using the Euler angles (θ_1, θ_2, θ_3) in the sequence 3-2-1, to guarantee that there is no singularity in the simulator attitude rotation. The equations of motions are obtained using Euler´s angular moment theorem given by

$$\dot{\vec{h}} = \vec{g} \qquad (6)$$

Where $\vec{g}$ and $\vec{h}$ are the torque and the angular moment of the system, which is given by

$$\vec{h} = I\vec{\omega} + I_w\left(\vec{\Omega} + \vec{\omega}\right) \qquad (7)$$

Where I= diag (I_{11}, I_{22}, I_{33}) is the system matrix inertia moment, $\vec{\omega}$ is the angular velocity of the platform, I_w = diag (I_{w1}, I_{w2}, I_{w3}) is the reaction wheel matrix inertia moment and Ω = (Ω_1, Ω_2, Ω_3) are the reaction wheel angular velocity. Differentiating Eq. (7) and considering that the angular velocity of F_b is $\vec{\omega}$ and that the external torque is equal to zero, one has

$$\dot{\vec{h}} + \vec{\omega}^x\vec{h} = 0 \qquad (8)$$

Substituting Eq.(7) into Eq.(8), the angular velocity of the system is

$$\dot{\vec{\omega}} = \left(I + I_w\right)^{-1}\left[-\vec{\omega}^x\left(I + I_w\right)\vec{\omega} - \vec{\omega}^x I_w \vec{\Omega} - I_w \dot{\vec{\Omega}}\right] \qquad (9)$$

The simulator attitude as function of the angular velocity is

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \begin{pmatrix} 0 & \sin\theta_3/\cos\theta_2 & \cos\theta_3/\cos\theta_2 \\ 0 & \cos\theta_3 & -\sin\theta_3 \\ 1 & \sin\theta_3\sin\theta_2/\cos\theta_2 & \cos\theta_3\sin\theta_2/\cos\theta_2 \end{pmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \qquad (10)$$

In order to design the attitude control system based on reaction wheel and gas jets actuators to perform a large angle maneuver, it is important to have in mind that each control algorithm is designed based on in two different a set of equations of motions. In other words, the gas jets are applied to reduce the high angular velocity and the reaction wheel is used to control in the fine pointing accuracy mode. As a result, for each operation mode one has different matrices A(x) and the respective matrix B associated with it. The C matrix, although depend on the sensor type is assumed unity for simplicity.

In the fine pointing mode where the reaction wheel is the actuator, the state's x are (θ_1 θ_2 θ_3 ω_1 ω_2 ω_3) and the control u are ($\dot{\Omega}_1$ $\dot{\Omega}_2$ $\dot{\Omega}_3$), the matrices A(x) and B are given by

$$(x) = \begin{pmatrix} 0\,0\,0 & 0 & \frac{\sin\theta_3}{\cos\theta_2} & \frac{\cos\theta_3}{\cos\theta_2} \\ 0\,0\,0 & 0 & \cos\theta_3 & -\sin\theta_3 \\ 0\,0\,0 & 1 & \frac{\sin\theta_3\sin\theta_2}{\cos\theta_2} & \frac{\cos\theta_3\sin\theta_2}{\cos\theta_2} \\ 0\,0\,0 & 0 & \frac{I_{22}\omega_3 - I_w\Omega_3}{(I_{11}+I_w)} & \frac{-I_{33}\omega_2 + I_w\Omega_2}{(I_{11}+I_w)} \\ 0\,0\,0 & \frac{-I_{11}\omega_3 + I_w\Omega_3}{(I_{22}+I_w)} & 0 & \frac{I_{33}\omega_1 - I_w\Omega_1}{(I_{22}+I_w)} \\ 0\,0\,0 & \frac{I_{11}\omega_2 - I_w\Omega_2}{(I_{33}+I_w)} & \frac{-I_{22}\omega_1 + I_w\Omega_1}{(I_{33}+I_w)} & 0 \end{pmatrix} \qquad (11)$$

$$B = \begin{pmatrix} \frac{-I_w}{(I_{11}+I_w)} & 0 & 0 \\ 0 & \frac{-I_w}{(I_{22}+I_w)} & 0 \\ 0 & 0 & \frac{-I_w}{(I_{33}+I_w)} \end{pmatrix} \qquad (12)$$

One knows that the reaction wheel generates internal torques and the attitude control is performed by exchange of angular moment between the reaction wheel and the satellite. On the other hand, gas jets generates external torque M given by

$$M_{Pi} = -T_i . d_i \qquad (13)$$

Where M_{Pi} is the torque generated around the "i" axis due to the force T_i applied at distance d_i from the rotation axis.

In the angular reduction mode using gas jets the reaction wheel is locked, therefore, its acceleration and angular velocity are zero and the satellite angular velocity is given by

$$\dot{\omega} = (I + I_w)^{-1}[-\omega^x(I + I_w)\omega \; - T_i . d_i \,] \qquad (14)$$

The states x are $(\theta_1 \; \theta_2 \; \theta_3 \; \omega_1 \; \omega_2 \; \omega_3)^T$ and the control u are $(T_1 \; T_2 \; T_3)^T$, therefore, the matrices A(x) and B are given by

$$(x) \; = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{sin\theta_3}{cos\theta_2} & \frac{cos\theta_3}{cos\theta_2} \\ 0 & 0 & 0 & 0 & cos\theta_3 & -sin\theta_3 \\ 0 & 0 & 0 & 1 & \frac{sin\theta_3 sin\theta_2}{cos\theta_2} & \frac{cos\theta_3 sin\theta_2}{cos\theta_2} \\ 0 & 0 & 0 & 0 & \frac{I_{22}\omega_3}{(I_{11}+I_w)} & \frac{-I_{33}\omega_2}{(I_{11}+I_w)} \\ 0 & 0 & 0 & \frac{-I_{11}\omega_3}{(I_{22}+I_w)} & 0 & \frac{I_{33}\omega_1}{(I_{22}+I_w)} \\ 0 & 0 & 0 & \frac{I_{11}\omega_2}{(I_{33}+I_w)} & \frac{-I_{22}\omega_1}{(I_{33}+I_w)} & 0 \end{pmatrix} \qquad (15)$$

$$B = \begin{pmatrix} \frac{-d_1}{(I_{11}+I_w)} & 0 & 0 \\ 0 & \frac{-d_2}{(I_{22}+I_w)} & 0 \\ 0 & 0 & \frac{-d_3}{(I_{33}+I_w)} \end{pmatrix} \qquad (16)$$

4 | CRITERION FOR CHANGING THE ACTUATOR

The implementation of the SDRE algorithm in real time has becomes more realistic because of the commercial microprocessor is getting faster. Here, the control system has to deal with two operation modes where the first one is the reduction of high angular velocity using gas jets and the second one is the control in three axes with fine pointing accuracy using reaction wheel. As a result, it is necessary to establish a criterion to change from one actuator to another. This criterion of course is function of the satellite space mission and the control system equipments. For example, from the angular velocity reduction mode to the normal mode of operation the criterion could be associated with the amount of energy that the reaction wheel can support before being saturated or with the minimum and maximum values of the gas jets capacity. The criterion used here is based on the total potential and kinetic energy of the system, which means that when the system reaches a certain level of energy the control algorithm change the type of actuator.

The potential energy associated with the angular displacement is

$$U = K_u \Delta\theta^2 \qquad (17)$$

Where K_u is a constant and $\Delta\theta$ represent the angular displacement of the simulator.

The simulator kinetic energy is given by

$$K = K_c w^2 \qquad (18)$$

Where K_c is a constant and is the angular velocity of the simulator. It is important to say that the constants K_u and K_c must be such to maintain the total system energy compatibles. Besides, the level of energy can be changed according with the kind of control system to be evaluated. Here one assumes certain level of energy just for simulation purpose.

5 I SIMULATION RESULTS

The superiority of the SDRE method to perform a regulation and tracking large angle manoeuvre over the LQR method has been demonstrated in [10]. Here, the simulation is to demonstrate the ability of the SDRE method to control a nonlinear plant based on switching control algorithm using the previously criterion of energy to change from the gas jets to reaction wheel action. The simulator platform can accommodate various satellites components; like sensors, actuators, computers and its respective interface and electronic. Therefore, the inertia moments of the simulator depend on the equipment's distribution over it. One uses the following typical inertia moment for the simulator: $I_{11} = I_{22} = 1.17$ Kg·m^2 and $I_{33} = 1.13$ Kg·m^2; and for the reaction wheel $I_x = I_y = I_z = 0.0018$ Kg·m^2. The maximum and minimum gas jet torque used is +/- 10Nm and the total amount of system energy to change from gas jets to reaction wheel is 0.5J. In the fine pointing mode the typical sensor used are theta = 0.2 (deg) and rate theta = 0.1 (deg/s).

To demonstrate the performance of the SRRE controller one imposes a severe large angle maneuver which starts with 0 (deg) and ends having to track an angular reference of 100, 50 and 70 (deg). The controller performance requirements are small overshoot and quick time of response. The controller robustness is associated with its ability to perform big tracking maneuver apart from the perturbations due to sensor noise and plant nonlinear terms. Besides, it is important to say that this performance is a function of the weighting matrices of the SDRE controllers. After some trial and error one gets the following values for matrices R= diag [0.0001 0 0;0 0.0001 0;0 0 0.0001] and Q= diag [1 0 0 0 0 0;0 1 0 0 0 0;0 0 1 0 0 0;0 0 0 100 0 0;0 0 0 0 100 0;0 0 0 0 0 100].

Figures 2 and 3 show the angular displacement and angular velocity when the SDRE controller performs the simulator large maneuver from 0º and it has to track the previously angular reference. One observes that the SDRE ACS has good performance despite of the nonlinear terms of the plant and it is able to get the reference in about 250s.

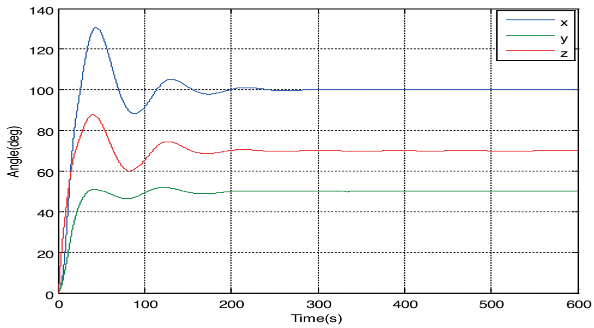

Fig. 2- Simulator angular displacement in x, y and z.

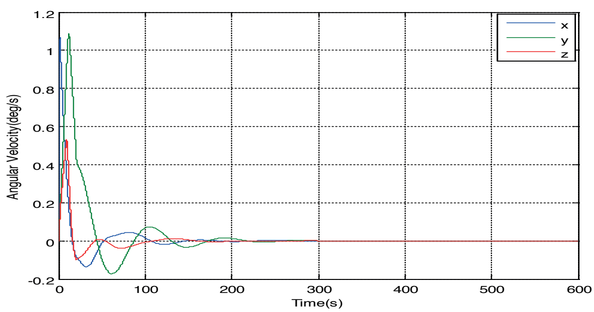

Fig. 3 – Simulator angular velocities in x, y, and z

Figure 4 show the SDRE controller the transition phase of the previously manoeuvre where the torque initially only due to the gas jets and Fig. 5 the torque is only due to the reaction wheel, respectively.

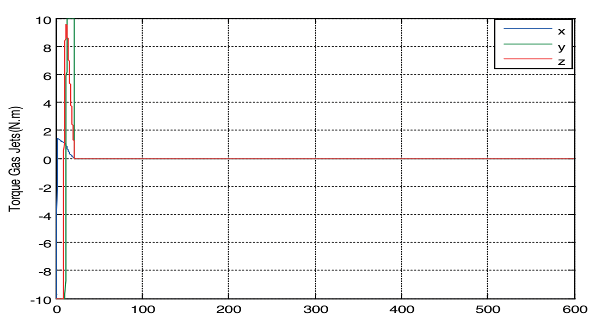

Fig. 4 - SDRE controller gas jets torque.

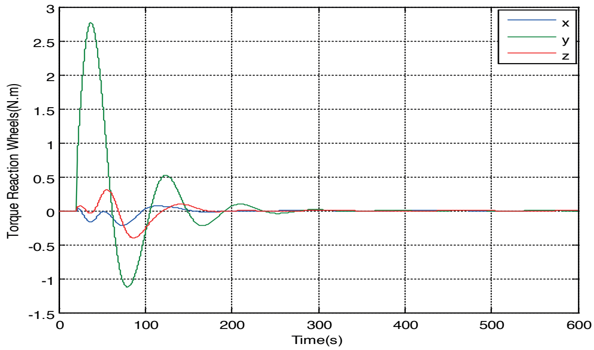

Fig. 5 - SDRE controller reaction wheel torque.

Figure 6 shows how the switching control algorithm works. That is, the gas jets stop acting and the reaction wheel starting acting when the criterion for changing actuators is achieved. That is, the system total energy equal to 0.5J.

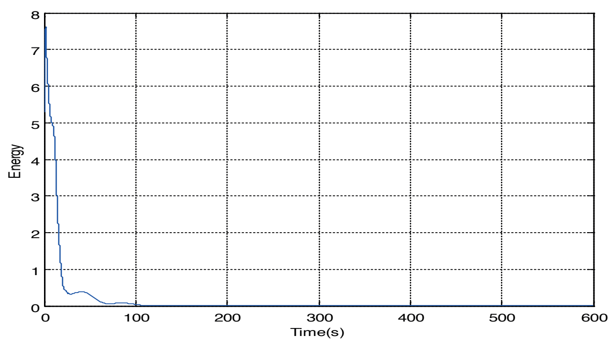

Fig. 6 – Actuators behaviours when achieves - 0.5J.

From simulations one observes that at the beginning of the manoeuvre the level of energy is high, because the simulator is far from the final attitude to be follow. As a result, the switching control algorithm selects as actuator the thrusters in order to deal with high angular velocities reduction. On the other hands, when the simulator reaches the small reference attitude the total energy decreases rapidly. So the switching control algorithm selects as actuators the reaction wheels, in order to perform fine pointing adjustment of the simulator. Finally, it is important to say that the criterion for changing the actuated based on the energy value defined in the program was established just to provide a good visualization of the torques from the two actuators during the simulation. However, further study of this actuated change can be done based in other criterion of optimization like minimum time manoeuvres or fuel, reaction wheel speed and pointing accuracy.

6 I CONCLUSION

In this paper one develops a general 3-D simulator nonlinear model, once it only depends on the inertia moment of the system. The Matlab/Simulink model is used to investigate large angle tracking manoeuvres in order to design a control algorithm based on the gas jet and reaction wheel, where the first actuator is used to reduce high angular velocity and the second one to perform fine pointing control. The switching control algorithm used to chance from gas jet to reaction wheel action is based on the potential plus the kinetic energy of the system. Therefore, the transition between modes of operation occurs when the system reaches a certain level of energy. The nonlinear controller design uses the conjunction of the SDRE (State Dependent Riccati Equation) and SDRE filter methods to deal with high nonlinear simulator plant and system noise. Simulations have demonstrated the good performance and robustness of the SDRE controller to perform large angle tracking manoeuvres. The investigation has also shown that SDRE control algorithm can be implemented in satellite on board computer. The next step of this work is to compare the SDRE technique here develops with other nonlinear control methodology.

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