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CONTROL OF THE ANGLE OF THE KNEE JOINT USING ERP SYSTEMS, CONSIDERING UNCERTAINTY IN THE PLANT

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Abstract: In this work, a controller was designed based on the study of conditions in terms of Linear Matrix Inequalities, (in English, Linear Matrix Inequalities, LMIs), for the synthesis of Strictly Real Positive Systems (ERP), with applications aimed at control with variable structure, in order to apply the conditions obtained in a mathematical model for controlling the knee joint angle of paraplegic patients through electrical stimulation. In the project carried out, time-varying polytopic uncertainties were considered in the plant. Thus, the ERP synthesis was based on a Lyapunov $V(x) = x^T P_0(\alpha)x$, and matrix $P_0(\alpha)$ depends on the uncertainties present in the system, with α having a limited rate of variation. The conditions obtained were used in the mathematical model for controlling the electrical stimulation of a paraplegic patient, associated with a dynamic compensator, the use of which is necessary to obtain a system. Matlab software was used to solve the LMIs, as well as to simulate the systems with the results obtained. However, feasible solutions to the problem were found only in the case where the rate of variation of α is zero. The project developed was extremely important for learning, enabling the understanding of Lyapunov's stability theory with time-varying uncertainties, as well as the modeling and construction of controllers.
Keywords: ERP Systems, LMI, Polytopic Uncertainties, Electrostimulation.

INTRODUCTION

Strictly Real Positive Systems (ERP) are passive, asymptotically stable systems, whose transmission zeros have a negative real part. Furthermore, the negative feedback of this type of system is internally stable (Covacic et al., 2010, Covacic; Gaino, 2014). Consider a time-invariant linear plant: $G_p(s)$ represented by:

$$\dot{x} = A_p x + B_p u,$$

$$y = C_p x, \tag{1}$$

Therefore: $x \in \mathbb{R}^n$ *the state vector*, $u \in \mathbb{R}^m$ the control entry, $y \in \mathbb{R}^m$ *the system output*, $A \in \mathbb{R}^{n\times n}$

the characteristic matrix of the system, $B \in \mathbb{R}^{n\times m}$ *the system input matrix* and $C \in \mathbb{R}^{m\times n}$ the system output matrix (Aguirre, 2007). ERP systems were defined in (Anderson, 1968). The necessary and sufficient condition for ERP system is given in Theorem 1 (Anderson, 1968):

Theorem 1: (Anderson, 1968): The transfer matrix of a plant: (1), given by $G_p(s) = C_p(sI - A_p)^{-1}B_p$, it is ERP if and only if there is a matrix: $P = P^T$, therefore:

$$PA_p + A_p^T P < 0,$$

$$B_p^T P = C_p,$$

$$P > 0.$$

Next, necessary and sufficient conditions are established for the existence of a matrix K that makes ERP the system of Figure 1, with entry: $V(s)$ and exit: $Y(s)$.

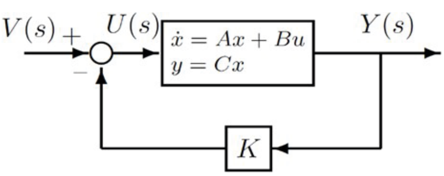

Figure 1: System with output feedback (Covacic et al., 2008).

Theorem 2 (Teixeira, 1989; Kaufman et al.,1994): Consider Figure 1. There is a constant matrix K, such that the system in Figure 1, with input V(s) and output Y(s), is ERP, if and only if the following conditions are satisfied:'

i. $C_p B_p = (C_p B_p)^T > 0$.

ii. All plant transmission zeros: A_p, B_p, C_p have a negative real part.

iii. The transmission zeros of a system are the values of $s \in C$ and:

$$posto\left(\begin{bmatrix} sI-A & B \\ -C & 0 \end{bmatrix}\right) < n+m. \quad (2)$$

CONDITIONS BASED ON LMIS IN ELECTROSTIMULATION OF PARAPLEGIC PATIENTS

The study of control systems to control movements of paraplegic patients through electrical stimulation is addressed, for example, in Riener; Fuhr (1998), who uses a Mamdani-type fuzzy controller, and in articles such as Teixeira et al. (2006), Teixeira et al. (2006a), in which work was carried out on controlling the leg angle of a paraplegic, with electrical stimulation considering Takagi-Sugeno (T-S) fuzzy models. The control methods presented and developed by them are based on Linear Matrix Inequalities (LMIs). The advantage of using LMIs is that, when feasible, they can be solved using computational mathematical software, such as Matlab (Gahinet et al., 1995). This method also allows other specifications related to the project such as pole allocation, decay rate, input and output restrictions, which will eventually be specified and analyzed for the obtained ERP system (Covacic et al., 2014).

POLYTOPIC UNCERTAINTIES

In general, practical situations, there are uncertainties in plant parameters and these uncertainties must be considered in the design of control systems. The conditions proposed in this project apply to plants similar to $G_p(s)$ in (1), with uncertainties in the characteristic matrix A and in the input matrix B. Consider, then, the plant Gp(s) whose representation in state space is described by (Covacic; Gaino, 2014):

$$\dot{x} = A_p(\alpha)x + B_p(\alpha)u,$$

$$y = C_p x, \quad (2)$$

With: $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ e $\alpha \in \mathbb{R}^r$ given by:

$$\alpha = [\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_{r-1} \quad \alpha_r]^T, \quad (3)$$

Therefore: $\alpha_i \geq 0$, $i = 1, 2 \ldots, r-1, r$, unknown variables that vary over time with a limited rate of variation, so that (Covacic; Gaino, 2014):

$$\alpha_1 + \alpha_2 + \cdots + \alpha_{r-1} + \alpha_r = 1. \quad (4)$$

Polytopic uncertainties can be understood as a margin of error that accompanies the elements of the Ap(α) and Bp(α) matrices, which have all their parameters known.

In Covacic; Gaino (2014), the following conditions are also considered:

A1) The state vector x is not available for feedback, but the output vector y is.

A2) Matrixes with uncertainty: $A_p(\alpha)$ and $B_p(\alpha)$ are unknown and described by:

$$A_p(\alpha) = \sum_{i=1}^{r} \alpha_i A_{pi},$$

$$B_p(\alpha) = \sum_{i=1}^{r} \alpha_i B_{pi},$$

Therefore: A_{pi} e B_{pi}, $i = 1, \cdots, r$, known and constant matrices (polytopic uncertainties).

A3) The variation rates of: α_i, $i = 1, 2, \cdots, r-1, r$, *they* are limited, therefore:

$$|\dot{\alpha}_i| \leq \phi_i < \infty.$$

Still in Covacic; Gaino (2014), it is proposed to obtain an ERP system through the project of a control law of the type:

$$u(t) = -K\,y(t) \quad (5)$$

and a matrix F connected in series with the output, according to the system illustrated in Figure 2:

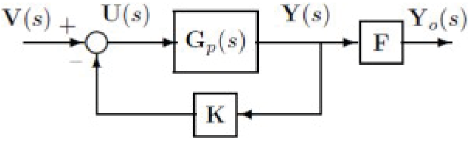

Figure 2: ERP system.

Theorem 3: (Mozelli et al, 2009): Consider the plant in Figure 2, with the control law (5). A sufficient condition for the existence of matrices F and K that make the Figure 2 ERP system is the existence of matrices: $P_{0i} = P^T$, R and F, that meet the following LMIs:

$$\frac{1}{2}\left(A_{pi}^T P_{0j} + P_{0j} P A_{pi} + A_{pj}^T P_{0i} + P_{0i} P A_{pj}\right) -$$

$$C_p^T R C_p - C_p^T R^T C_p + \sum_{i=1}^{r} \emptyset_i P_{0i} < 0, \tag{6}$$

$$P_{0i} B_p = C_p^T F^T, \tag{7}$$

$$P_{0i} > 0, \tag{8}$$

to $i, j = 1, \cdots, r$, *when the above conditions are satisfied, the matrix K is given by*:

$$K = (F^T)^{-1} R. \tag{9}$$

FUNCTIONS OF LYAPUNOV

The idea of stability of a mechanical system, according to Lyapunov, is directly related to the "energy" of the system, which can be conserved, dissipated or even expanded over time. From the theory of classical mechanics, for example, it is known that a vibrating system is stable if its total energy is continuously decreasing until an equilibrium point is reached (Prado, 2010). The following definitions and Theorems show the definitions and conditions for stability and asymptotic stability in Lyapunov's concept.

Definition 1: (Slotine; Li, 1991): An equilibrium state x = 0 is said to be stable if, for any R > 0, there exists an r > 0, such that, if:$||x(0)|| < r$, therefore $||x(t)|| < R$ for every $t \geq 0$. Essentially, stability means that the system's trajectory can be maintained arbitrarily close to the origin, if started sufficiently close to it (Slotine; Li, 1991).

Definition 2: (Slotine; Li, 1991): An equilibrium point of 0 is asymptotically stable if it is stable and there is still some: $r > 0$, so: $||x(0)|| < r$ imply that $x(t) \to 0$, insofar as $t \to \infty$, therefore:

$$\lim_{t \to \infty} x(t) = 0.$$

Lyapunov starts from the generalization of the concept referring to spring mass systems, covering it to more complex systems. Given a set of nonlinear differential equations, the procedure for Lyapunov methods is the generation of a system analogous to the energy function of a dynamical system (Slotine et al., 2009; Prado, 2010).

Definition 3: (Slotine; Li, 1991; Prado, 2010): Seeing that: $U \subseteq \mathbb{R}^n$ *is a neighborhood of an equilibrium point, a continuous function:* V: $U \to \mathbb{R}$, which has continuous partial derivatives, and if its time derivative along any system space trajectory is negative, it will be a strict Lyapunov function for a field of vectors: $X : \Omega \to \mathbb{R}^n$ in $x_0 = 0$ if:

$$V(0) = 0, \text{ com } V(x) > 0 \ \forall x \in U \setminus \{0\},$$

$$V(\varphi(t_1, x)) \geq V(\varphi(t_2, x)) \ \forall x \in \Omega,\ t_1, t_2 \in \mathbb{R}\ t_1 < t_2 \text{ e}$$

$$\varphi(t_1, x),\ \varphi(t_2, x) \in U,$$

$$\dot{V}(x) \leq 0.$$

Theorem 4: (Prado, 2010): Let X be a vector field with X (0) = 0. If there is a Lyapunov function: V: $U \to \mathbb{R}$ that is positive definite around the equilibrium point then 0 is stable, 0 being an n-dimensional vector, according to the field.

Theorem 5 (Prado, 2010): Let X be a vector field with X (0) = 0. If there is a strict Lyapunov function: V: $U \to \mathbb{R}$ *that is* positive, defined around the equilibrium point then 0 is asymptotically stable, with 0 being an n-dimensional vector, according to the field. Therefore, if we represent the Lyapunov function as:

$$V(x) = x^T P_0(\alpha) x,$$

where x is the vector of the system state variables and P0(α) is the parameter-dependent Lyapunov matrix, with α given in (3), if the derivative of the function, at all times greater than zero, is negative, it can It can be stated that the system is asymptotically stable.

Thus, polytopic uncertainties will be included in the calculation of the P0(α) matrix, so that the decay rate can guarantee speed and robustness in the system when its stability is verified by the Lyapunov function.

FUNCTIONAL NEUROMUSCULAR ELECTROSTIMULATION

Functional Electrical Simulation (FES) is used as a form of physiotherapeutic treatment, in motor changes, to promote a functional contraction through electrical stimulation of muscles devoid of normal stimulation (Costa et al., 2013). Currently, FES has been used to improve the execution of daily activities and increase the functional independence of patients with spinal cord injuries, such as paraplegics.

Electrical stimulation applied via the surface, intramuscularly or through implanted electrodes can produce artificial contraction of paralyzed muscles, as long as fiber depolarization is achieved (Ferrarin; Pedotti, 2000).

FES, due to the principle of operation and the results obtained, produces muscle contraction similar to the contraction generated by a stimulus sent by the Central Nervous System (CNS) (Gaino et al., 2007). The application of FES in physiotherapeutic treatments, especially for paraplegic patients, has proven effectiveness (Gaino et al., 2007).

The use of FES, as a form of treatment, results in long-term changes in the nervous circuit of the spinal cord, and may even recover part of the function lost due to the spinal cord trauma (TRM) suffered by the patient. Their results indicate that the spinal cord is capable of adapting to events in the environment that surrounds the individual (including electrical stimulation) and that it is sensitive to the type of stimulus it receives, not just the conductor of information from the brain to and from the brain. (Hook; Grau, 2007).

In Figure 3, it is observed that, when a potential difference is applied to two stimulating electrodes, in this case external electrodes, there is a circulation of current within the tissue, which, as it is predominantly aqueous, occurs through the ordered movement of ions (Gaino et al., 2007).

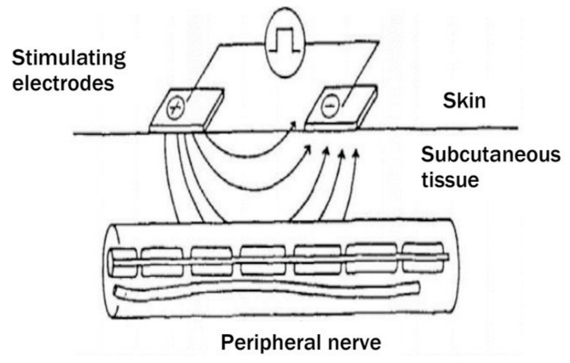

Figure 3: *Electrical nerve stimulation via external electrodes (Gaino et al., 2007).*

When the electric current passes through the cell membrane, it causes its depolarization, which, according to its intensity, may be sufficient to provoke an Action Potential (AP), analogous to that produced by the CNS, which will propagate through of the membrane. The amplitude/intensity and duration of the electrical stimulus must have a minimum value to evoke an AP, that is, above a threshold (Gaino et al., 2007).

The authors have published several works related to electrical stimulation of paraplegic patients (Biazeto et al., 2014; Biazeto et al., 2016; Gaino et al., 2019; Covacic et al., 2020; Gaino et al., 2020), with the wheelchair powered by blowing and suction (Gentilho Junior et al., 2014; Mineo, Covacic and Gaino, 2021; Leôncio Junior, Covacic and Gaino, 2022) and with robotic arm control (Marques et al., 2016; Arruda, Covacic and Gaino, 2019; Covacic, Gaino and Capobianco, 2019;

To the authors' knowledge, regarding the design of ERP systems with LMIs and pole allocation for systems with polytopic uncertainties applied to the angular movement of the paraplegic patient's leg, no published reports were found in the literature and in the state of the art.

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KNEE JOINT MODELING

To study a control system using FES, a mathematical model of the lower limb is necessary, such as the one used in Ferrarin; Pedotti (2000), as shown in Figure 4, which relates the width of the electrostimulating pulse to the torque generated around the knee joint.

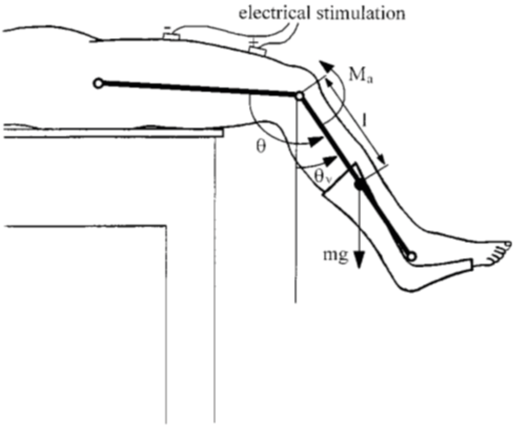

Figure 4: Schematic representation of the lower limb with superficial stimulating electrodes (Ferrarin; Pedotti, 2000).

De acordo com esta modelagem a equação de equilíbrio em torno da junção do joelho é:

$$M_i = M_g + M_s + M_d + M_a,$$

which can be expressed with the following second-order nonlinear differential equation:

$$J\ddot{\theta} = mgl\,\text{sen}\,\theta_v - M_s - B\theta + M_a,$$

Therefore:

- J: the inertial moment of the shin-foot complex;
- *Θ:* the angle of the knee joint (angle between shin and thigh in the sagittal plane);
- $\dot{\theta}$: the angular velocity of the knee joint;
- θ_v: the shin angle (angle between the shin and the vertical axis in the sagittal plane);
- $\ddot{\theta}$: the angular acceleration of the shin;
- M: the mass of the cinnamon-foot complex;
- g: the acceleration of gravity;
- l: the distance between the knee and the center of mass of the shin-foot complex;
- B: the coefficient of viscous friction;
- $M_s = -\lambda e^{E\theta}(\theta - \omega)$: the torque due to the stiffness component;
- *M: the active knee torque produced by electrical stimulation;*
- $M_g = mgl\text{sen}(\theta_v)$ torque due to gravity;
- $M_{i:}$ *the total inertial torque.*

According to this model, a distinction is made between the angle θ (the angle of the knee joint, used for the stiffness and damping components) and the angle θv (the angle between the shin and the vertical axis in the sagittal plane). However, as thigh movements can be neglected, the angular acceleration coincides with the relative angular acceleration of the knee (Ferrarin; Pedotti, 2000). The terms λ and E, referring to the torque due to the stiffness component (M_s), are the coefficients of the exponential terms, and ω is the elastic angle of rest of the knee. The negative sign is due to the choice of extensor torque as positive (Ferrarin; Pedotti, 2000). In Ferrarin; Pedotti, (2000), the relationship between active torque to which the muscle is subjected and the width of the electrical stimulation pulses (P) can be described by the following transfer function:

$$H(s) = \frac{G}{1+s\tau},$$

Therefore:

- G static gain;
- τ pole time constant.

In Ferrarin; Pedotti (2000), methods are suggested for experimentally obtaining these two parameters of interest. The resulting equation in state variables from the model

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described in this section, demonstrated in (Teixeira et al., 2006) and (Teixeira et al., 2006a) is given by (2), with:

$$A(\alpha)=\begin{bmatrix}0 & 1 & 0\\ \tilde{f}_{21}(x_1) & -\frac{B}{J} & \frac{1}{J}\\ 0 & 0 & -\frac{1}{\tau}\end{bmatrix},\qquad B(\alpha)=\begin{bmatrix}0\\0\\\frac{G}{\tau}\end{bmatrix},\quad x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},$$

$$C=[1\quad 0\quad 0],\qquad u=P_n,$$

And it is, by definition:

- $x_1 = \Delta\theta_v = \theta_v - \theta_{v0}$;
- $x_2 = \Delta\dot{\theta}_v = \dot{x}_1$;
- $x_3 = \Delta M_a = M_a - M_{a0}$;
- $P_N = P - M_{a0}/G$;

$$\tilde{f}_{21}(x_1)=\frac{1}{Jx_1}\Big[-mgl\,\mathrm{sen}(x_1+\theta_{v0})-$$
$$\lambda e^{-E\left(x_1+\theta_{v0}+\frac{\pi}{2}\right)}\left(x_1+\theta_{v0}+\frac{\pi}{2}-\omega\right)+M_{a0}\Big],$$

with:

$$M_{a0}=mgl\,\mathrm{sen}(\theta_{v0})+\lambda e^{-E\left(\theta_{v0}+\frac{\pi}{2}\right)}\left(\theta_{v0}+\frac{\pi}{2}-\omega\right).$$

As the function $\tilde{f}_{21}(x_1)$ is a nonlinearity of the system with an indeterminacy in $x_1 = 0$, ee can expand it into a Taylor series in order to eliminate this term: x_1, in this case a seventh order expansion. In control design, polytopic uncertainties in system parameters can be considered in order to achieve robust control over uncertainties.

METHODOLOGY AND EXPERIMENTAL PROCEDURES

Given the patient's knee joint angle as the system's output variable and considering that this angle is in the variation range of –30° to 30°. According to Covacic et al. (2010), the model is represented according to (2), with:

$$A(\alpha)=\alpha_1 A_1+\alpha_2 A_2,$$

$$A_1=\begin{bmatrix}0 & 1 & 0\\ a_{211} & -\frac{B}{J} & \frac{1}{J}\\ 0 & 0 & -\frac{1}{\tau}\end{bmatrix},\quad A_2=\begin{bmatrix}0 & 1 & 0\\ a_{212} & -\frac{B}{J} & \frac{1}{J}\\ 0 & 0 & -\frac{1}{\tau}\end{bmatrix},$$

$$B=\begin{bmatrix}0\\0\\\frac{G}{\tau}\end{bmatrix},\qquad C=[1\quad 0\quad 0],$$

$$\alpha_1\geq 0,\qquad \alpha_2\geq 0,\qquad \alpha_1+\alpha_2=1,$$

This way: a_{211} and a_{212} the maximum and minimum values $\tilde{f}_{21}(x_1)$ of , respectively. The numerical values of the system parameters are given in Table 1.

J = 0,362 [kgm²]	m = 4,37 [kg]
l = 23,8 [cm]	B = 0,27 [N.m.s/rad]
λ = 41,208 [N.m/rad]	E = 2,2024 [rad⁻¹]
ω = 2, 918 [rad]	τ = 0,951 [s]
G = 42500 [N.m/s]	θ_{vo} = 30°
M_{a0} = 4,6068 [N.m]	

Table 1: Numerical Values of Parameters (Covacic et al., 2010).

To θ_0 = 30°, −30° ≤ x_1 ≤ 30° and the parameters given in Table 1, the matrices: A_1, A_2 and B are:

$$A_1=\begin{bmatrix}0 & 1 & 0\\ -14{,}3886 & -0{,}7459 & 2{,}7624\\ 0 & 0 & -1{,}0515\end{bmatrix},$$

$$A_2=\begin{bmatrix}0 & 1 & 0\\ -1{,}5002 & -0{,}7459 & 2{,}7624\\ 0 & 0 & -1{,}0515\end{bmatrix},$$

$$B=\begin{bmatrix}0\\0\\44690\end{bmatrix},\qquad C=[1\quad 0\quad 0].$$

As $CB = 0$, according to Theorem 2, it is not possible to obtain an ERP system with a static compensator. Thus, as described in Covacic et al. (2010), the plant was inserted into the Figure 5 system, with: $F_0 = [6{,}2\times10^7]$ and the dynamic compensator of order m = 1 given by:

$$\dot{x}_c=A_c x_c+B_c x_c,$$

$$u=C_c x_c,$$

whose matrices: A_c, B_c and C_c are:

$$A_c=[0],\qquad B_c=[1],\qquad C_c=[1].$$

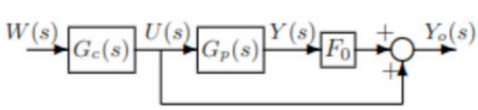

Figure 5: Open loop system described in Covacic et al. (2010).

Thus, the augmented system is described by (2), whose matrices are:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -14{,}3886 & -0{,}7459 & 2{,}7624 & 0 \\ 0 & 0 & -1{,}0515 & 44690 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (10)$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1{,}5002 & -0{,}7459 & 2{,}7624 & 0 \\ 0 & 0 & -1{,}0515 & 44690 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (11)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad (12)$$

$$C = [6{,}2 \times 10^7 \quad 0 \quad 0 \quad 1]. \quad (13)$$

It is observed that, in the LMIs in (5), (6) and (7), given in Covacic; Gaino (2014), the value of φ_i is limited according to condition A3. According to Covacic; Gaino (2014) and (Mozelli et al., 2009), condition A3 guarantees inequality:

$$\sum_{i=1}^{r} \phi_i P_{0i} \geq \sum_{i=1}^{r} \dot{\alpha}_i P_{0i}.$$

However, solutions were found for LMIs (5), (6) and (7) only in the case where P_0 is constant. In this case, we have: $P_{0i} = P_0$ to $i = 1, \cdots, r$. Therefore:

$$\sum_{i=1}^{r} \dot{\alpha}_i P_{0i} = \sum_{i=1}^{r} \dot{\alpha}_i P_0 = P_0 \sum_{i=1}^{r} \dot{\alpha}_i = 0,$$

therefore, deriving expression (4) in time, it appears that:

$$\sum_{i=1}^{r} \dot{\alpha}_i = 0.$$

Therefore, the LMIs in (6), (7) and (8) guarantee an ERP system with the K matrix given in (5), adopting: $\varphi_i = 0$, to $i = 1, \cdots, r$. To obtain the LMIs described in (6), (7) and (8), the Matlab software console version 2012 was used, which has specific functions in its library for LMI resolution environments. However, it is not possible (using Matlab) to obtain the resolution of Linear Matrix Equalities (LMEs), such as the LME (6). Therefore, it was necessary to make an approximation for this LME given by:

$$\| P_{0i} B_p - C^T F^T \| < \lambda,$$

which, according to Schur's complement, is equivalent to (Covacic; Gaino, 2014):

$$\begin{bmatrix} \lambda^2 I & P_{0i} B_p - C_p^T F^T \\ B_p^T P_{0i} - F C_p & I \end{bmatrix} > 0,$$

which in turn is an LMI and is solved by Matlab. In this case it was possible to use: $\lambda = 10^{-4}$. Given the system in Figure 2, with the matrices given in (10)−(13), for the case in which: $P_{0i} = P_0$ to $i = 1, \cdots, r, e$, the F and K matrices necessary to obtain the ERP system were determined, through the resolution of Theorem 3, also considering the LMIs that guarantee the restriction in the signal amplitude: $x_2(t)$ ($\|x_2(t)\| \leq 1$) (Covacic; Gaino, 2014).

In the time-varying Lyapunov function methodology, a case was considered to validate the model. Considering the rate of variation of uncertainties, which characterizes a time-varying Lyapunov function, a simplification is determined for the case of the paraplegic patient's dynamics depending on the feasibility of LMIs (6), (7) and (8), since the feasibility of LMIs was obtained only in the case in which $\varphi_i = 0$, to $i = 1, \cdots, r$.

This means that, in the time-varying system, for the case in question, the system is very conservative, requiring a relaxation of the LMI conditions to obtain other solutions, which are not addressed in this work and are not found in the literature. No reports of this strategy were found in the state of the art.

Using Matlab, the F and K matrices obtained were:

$$F = [2.17065 \times 10^7],$$
$$K = [1.3357 \times 10^7].$$

With the matrices F and K obtained, the eigenvalues of $(A_{p1} - B_p K C_p)$ are:

$$p_1 = -3.3677 + 0.0000\,j,$$
$$p_2 = -0.0004 + 0.0038\,j,$$
$$p_3 = -0.0004 - 0.0038\,j,$$
$$p_4 = -0.0011 + 0.0000\,j$$

and the eigenvalues of $(A_{p2} - B_p K C_p)$ are:

$$p_1 = -1.7407 + 0.0000\,j,$$
$$p_2 = -0.0035 + 0.0118\,j,$$
$$p_3 = -0.0035 - 0.0118\,j,$$
$$p_4 = -0.0109 + 0.0000\,j.$$

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The transmission zeros of $\{(A_{p1} - B_pKC_p),B_p,FC_p\}$ are:

$$z_1 = -1.0567 + 0.0000\,j,$$
$$z_2 = -0.3703 + 3.7753\,j,$$
$$z_3 = -0.3703 - 3.7753\,j$$

and the transmission zeros of $\{(A_{p2} - B_pKC_p),B_p,FC_p\}$ are:

$$z_1 = -1.0922 + 0.0000\,j,$$
$$z_2 = -0.3525 + 1.1790\,j,$$
$$z_3 = -0.3525 - 1.1790\,j.$$

Thus, given the eigenvalues and transmission zeros obtained above for the matrices A1 and A2 presented, it is concluded that the closed-loop poles and transmission zeros all have a negative real part, therefore, the system is called ERP.

In the figures 7, 8 e 9, the transient response of the dynamic compensator simulation applied to the dynamics of the paraplegic patient and the control law u(t) is shown, starting from the initial state $x(0) = [\,-\pi/6 \;\; 0 \;\; -4.6068\,]^T$.

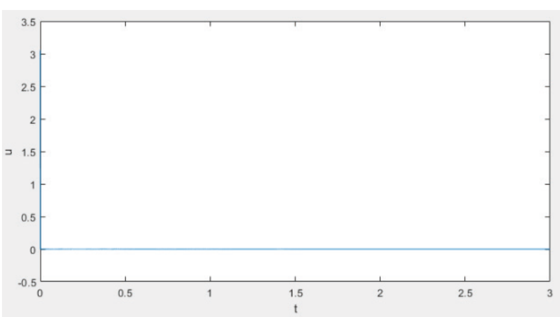

Figure 6 – Entry u(t).

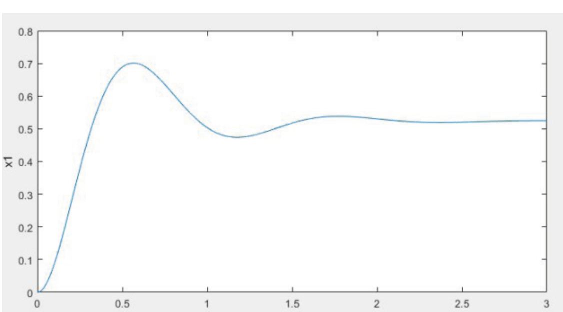

Figure 7 – State response x_1.

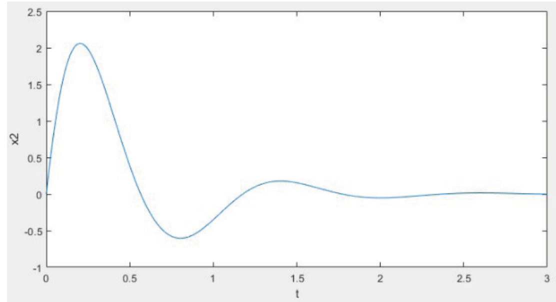

Figure 8 – State response x_2.

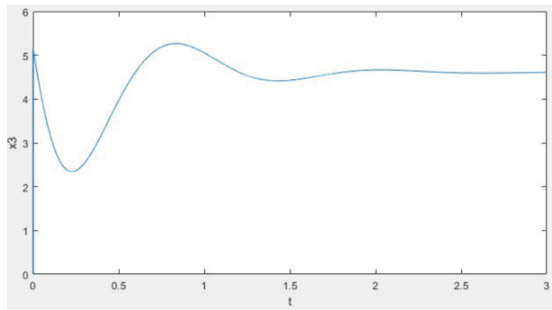

Figure 9: State response x_3.

It is observed that, comparing the results obtained in Figures 7, 8 and 9 with the results obtained in Covacic, Gaino, (2014), even considering polytopic uncertainties, delimiting the conditions obtained through the LMIs, the system obtained satisfies the ERP conditions.

CONCLUSIONS

It is observed that, as presented in this project, it was possible to obtain an ERP system, considering time-varying polytopic uncertainties, and the less conservative quadratic Lyapunov theory was used., $V(x) = x^T P_0(\alpha)x$, thus, through LMIs, conditions were obtained that can be used in a mathematical model of electrical stimulation of a paraplegic patient with the association of a dynamic compensator designed by Covacic et al. (2010). Therefore, it was possible to show stability at the operating point where the trajectory of the leg, in the simulated mathematical model, leaves a state of rest and stabilizes at 30°.

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