EXERCÍCIOS RESOLVIDOS SOBRE INDUTOR DE CORRENTE ALTERNADA

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RESUMO: Neste capítulo, detalhamos o processo passo a passo para resolver exercícos relacionados a um indutor de corrente alternada. Exploramos as interações entre grandezas eletromagnéticas essenciais, como o fluxo magnético em Weber, a indução magnética em Tesla, o campo magnético em Ampere por metro, a corrente elétrica em Ampere, a densidade superficial de corrente em Ampere por metro quadrado e o potencial vetor magnético em Weber por metro.
PALAVRAS-CHAVE: indutor de corrente alternada. Cálculo de campos. Exercícios. Integral de linha. Integral de superfície.

SOLVED EXERCISES ON ALTERNATING INDUCTOR

ABSTRACT: This chapter details the step-by-step process for solving exercises related to an alternating current inductor. We explore the interactions between essential electromagnetic quantities such as magnetic flux in Weber, magnetic flux density in Tesla, magnetic field strength in Amperes per meter, electric current in Amperes, electric current density in Amperes per square meter, and vector magnetic potential in Weber per meter.
KEYWORDS: Alternating current inductor. Field calculation. Exercises. Linear integral. Integral through the surface.

REVIEW OF INDUCTOR THEORY

The inductor is an electromagnetic device composed of a coil of N turns of magnet wire, such as copper, wound on a coil form made of insulating material. Inside the coil form, there is a core made up of ferromagnetic laminations isolated from each other. Inductors can be found wound on polar pieces and installed in the slots of electrical machines. In figure 01, it is possible to observe an inductor with an E-I core, with dimensions in millimeters.

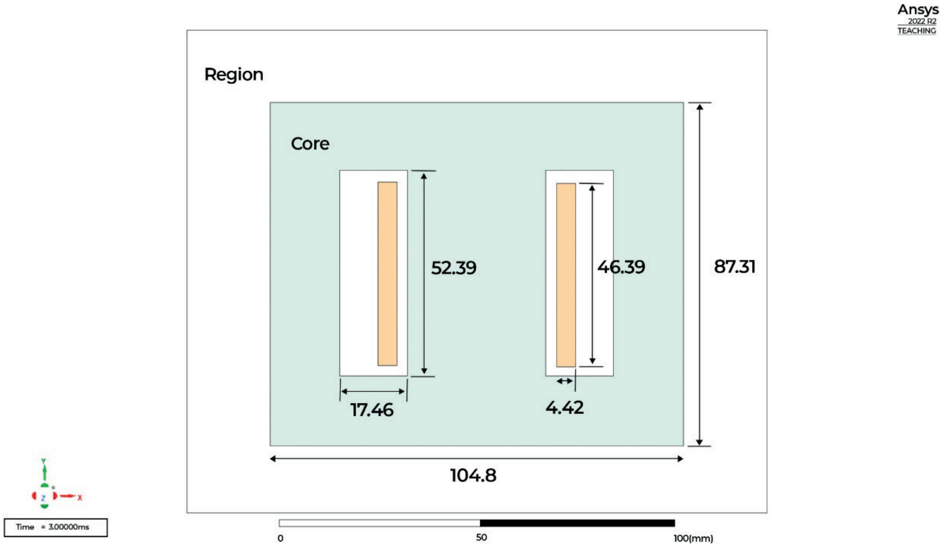

Figure 1 – Two-dimensional drawing of the inductor with E-I core

Source: Own authorship.

When an alternating voltage in volts is applied to an inductor, an alternating current in amperes flows in the coil, generating a magnetic field strength in Amperes per meter. This field is then amplified by the magnetic core, leading to a magnetic flux density in Tesla. The flow of the magnetic flux density vector through the cross-sectional area of the core generates magnetic flux in Webers. The inductor stores energy within its magnetic field strength, making it fundamental for the operation of various devices such as transformers, circuit breakers, contactors, relays, generators, and electric motors.

In the following sections, we will present examples that will help understand the operation of an alternating current inductor, covering the fundamental quantities of electromagnetism and demonstrating the application of linear integral and integral through the surface in electrical engineering. Detailed step-by-step instructions will be provided during the examples. It is recommended that the reader review concepts of electricity, vector calculus, and electromagnetism to follow the sequence. We cite our sources at the end of this work as the main references for a deeper understanding of the subject. Ansys® Electromagnetics Suite, Release 2022 R2, presents a guide with the procedure for simulating problems using the finite element method. In Bastos' work (2019), the concepts of low-frequency electromagnetism are discussed. Krasnov, Kisseliov, and Makarenko (1981) present the fundamentals of vector analysis. Mendes et al. (2019) model the phenomenon of magnetic hysteresis. Bianchi (2005) discusses the concept and application of the finite element method, while Suarez (2007) addresses magnetoelasticity.

EXAMPLE 1

Consider an inductor formed by 314 turns of magnetic wire. The upper face of the magnetic core, located at $y = 0.043$ m as shown in figure 02, is subjected to a magnetic flux density given by:

$$\vec{B} = -\left(0.508 + 5.8 \times 10^{-5} sen\left(-\frac{\pi}{2} + 179.93x\right)\right)\vec{j}$$

Magnetic flux density in T, $-0.017 \leq x \leq 0.017$ m, $-0.018 \leq z \leq 0.018$ m and $t = 3$ ms. Calculate the magnetic flux through the surface.

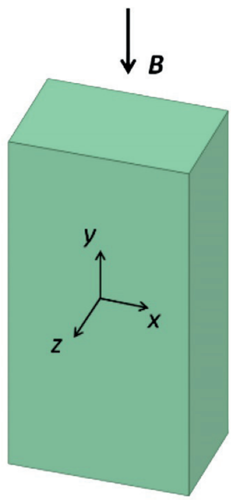

Figure 2 – Magnetic Core.

Source: Own authorship

Solution

The magnetic flux is given by:

$$\phi = \iint \vec{B} \cdot \vec{d}S$$

$$\phi = \iint \left(0, -0.508 - 5.8 \times 10^{-5} sen\left(-\frac{\pi}{2} + 179.93x\right), 0\right) \cdot (dydz, dxdz, dxdy)$$

For the upper face located at $y = 0.043$ m, and therefore $dy = 0$, we have:

$$\phi = \iint \left(0, -0.508 - 5.8 \times 10^{-5} sen\left(-\frac{\pi}{2} + 179.93x\right), 0\right) \cdot (0, dxdz, 0)$$

$$\phi = \iint \left[-0.508 - 5.8 \times 10^{-5} sen\left(-\frac{\pi}{2} + 179.93x\right)\right] dxdz$$

$$\phi = -0.508 \int_{-0.017}^{0.017} dx \int_{-0.018}^{0.018} dz - 5.8\times10^{-5} \int_{-0.017}^{0.017} sen\left(-\frac{\pi}{2} + 179.93x\right) dx \int_{-0.018}^{0.018} dz$$

$$\phi = -0.508(0.017+0.017)(0.018+0.018) - 5.8\times10^{-5} \int_{-0.017}^{0.017} sen\left(-\frac{\pi}{2} + 179.93x\right)\frac{179.93}{179.93} dx$$

$$(0.018+0.018)$$

$$\phi = -0.00062 + \frac{5.8\times10^{-5}}{179.93}\cos\left(-\frac{\pi}{2} + 179.93x\right)\Bigg|_{-0.017}^{0.017}(0.036)$$

$$\phi = -0.00062 + 1.16\times10^{-8}\left[\cos\left(-\frac{\pi}{2} + 3.0588\right) - \cos\left(-\frac{\pi}{2} - 3.0588\right)\right]$$

$$\phi = -0.00062\ Wb$$

The magnetic flux of 0.00062 Wb in the direction of the negative *y*-axis produces a flux linkage, for *t* = 3 ms, given by:

$\lambda = N\phi = 314 \times 0.00062$

$\lambda = 0.19\ Wb$

EXAMPLE 2

An inductor with a magnetic core was analyzed in the software Ansys® Electromagnetics Suite, Release 2022 R2, and we observed the behavior of the magnetic field strength as shown in figure 03. For this inductor, the magnetic field strength, in A/m, along the right side closed path can be approximated by:

$$\begin{cases} \vec{H} = -\left[-1.175\times10^{7}y^{4} - 1.147\times10^{2}y^{3} + 3.370\times10^{3}y^{2} - 1.442\times10^{-2}y + 20.831\right]\vec{j} & for \quad x = 0 \\ \vec{H} = \left[2.392\times10^{5}x^{3} - 1.085\times10^{4}x^{2} + 72.141x + 7.444\right]\vec{i} & for \quad y = -34.93\times10^{-3} \\ \vec{H} = \left[-1.272\times10^{7}y^{4} + 2.756\times10^{4}y^{3} + 5.377\times10^{3}y^{2} - 7.608y + 20.727\right]\vec{j} & for \quad x = 43.66\times10^{-3} \\ \vec{H} = -\left[3.587\times10^{3}x^{2} - 1.903\times10^{2}x + 8.866\right]\vec{i} & for \quad y = 34.93\times10^{-3} \end{cases}$$

Calculate the electrical current that generates this magnetic field strength for $0 \leq x \leq 0.044$ and $-0.018 \leq z \leq 0.018$.

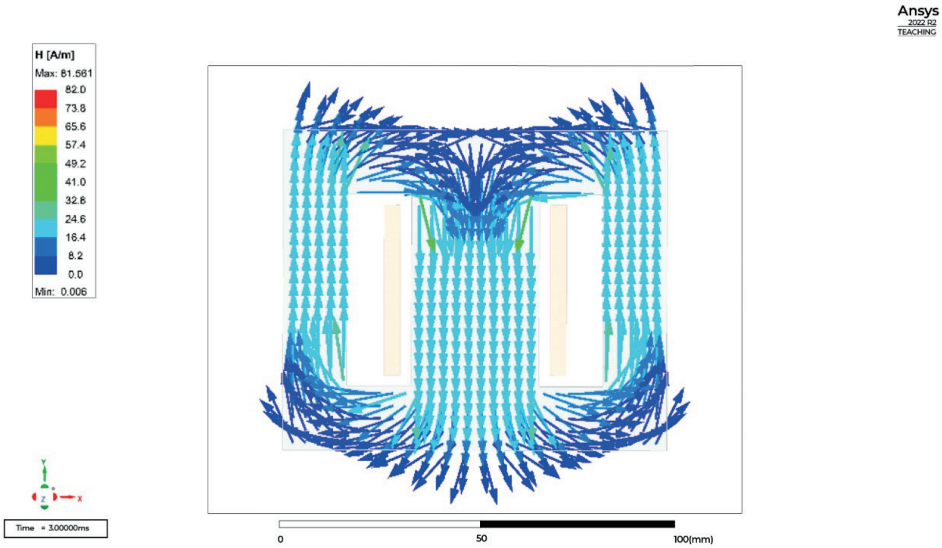

Figure 3 – The magnetic field strength of an E-I core inductor.

Source: Own authorship

Solution

Electric current is the circulation of the magnetic field strength along a closed loop. Thus,

$$I = \int_{l(s)} \vec{H} \bullet d\vec{l}$$

For the cartesian system, when $x = 0$, the change in x is zero, meaning $dx = 0$. Since the magnetic field has components in the x and y directions, we are dealing with a two-dimensional (2D) problem, where the change in z, dz, is also zero.

$$I_1 = \int_{l(s)} \vec{H} \bullet d\vec{l} = \int_{l(s)} (0, H_y, 0) \bullet (dx, dy, dz)$$

$$I_1 = \int_{l(s)} (0, 1.175\times10^7 y^4 + 1.147\times10^2 y^3 - 3.370\times10^3 y^2 + 1.442\times10^{-2} y - 20.831, 0) \bullet (0, dy, 0)$$

$$I_1 = \int_{0.035}^{-0.035} (1.175\times10^7 y^4 + 1.147\times10^2 y^3 - 3.370\times10^3 y^2 + 1.442\times10^{-2} y - 20.831) dy$$

$$I_1 = \left[1.175\times10^7 \frac{y^5}{5} + 1.147\times10^2 \frac{y^4}{4} - 3.370\times10^3 \frac{y^3}{3} + 1.442\times10^{-2} \frac{y^2}{2} - 20.831 y \right]_{0.035}^{-0.035}$$

$$I_1 = 0.235\times10^7 (-0.035 - 0.035)^5 + 0.287\times10^2 (-0.035 - 0.035)^4 - 1.123\times10^3 (-0.035 - 0.035)^3 +$$

$$0.721\times10^{-2} (-0.035 - 0.035)^2 - 20.831 (-0.035 - 0.035)$$

$$I_1 = -2.105 \quad A$$

For y = -0.035, the change in y is zero, meaning dy = 0, and the change in z is also zero, dz = 0.

$$I_2 = \int_{l(s)} \vec{H} \bullet d\vec{l} = \int_{l(s)} (H_x, 0, 0) \bullet (dx, dy, dz)$$

$$I_2 = \int_{l(s)} (2.392\times10^5 x^3 - 1.085\times10^4 x^2 + 72.141x + 7.444, 0, 0) \bullet (dx, 0, 0)$$

$$I_2 = \int_{0}^{0.044} (2.392\times10^5 x^3 - 1.085\times10^4 x^2 + 72.141x + 7.444) dx$$

$$I_2 = \left[2.392\times10^5 \frac{x^4}{4} - 1.085\times10^4 \frac{x^3}{3} + 72.141 \frac{x^2}{2} + 7.444x \right]_{0}^{0.044}$$

$$I_2 = 0.598\times10^5 (0.044 - 0)^4 - 0.362\times10^4 (0.044 - 0)^3 + 36{,}071 (0.044 - 0)^2 + 7.444 (0.044 - 0)$$

$$I_2 = 0.313 \quad A$$

For x = 0.044, the change in x is zero, meaning dx = 0, and the change in z is also zero, dz = 0.

$$I_3 = \int_{l(s)} \vec{H} \bullet d\vec{l} = \int_{l(s)} (0, H_y, 0) \bullet (dx, dy, dz)$$

$$I_3 = \int_{l(s)} (0, -1.272\times10^7 y^4 + 2.756\times10^4 y^3 + 5.377\times10^3 y^2 - 7.608y + 20.727, 0) \bullet (0, dy, 0)$$

$$I_3 = \int_{-0.035}^{0.035} (-1.272\times10^7 y^4 + 2.756\times10^4 y^3 + 5.377\times10^3 y^2 - 7.608y + 20.727) dy$$

$$I_3 = \left[-1.272\times10^7 \frac{y^5}{5} + 2.756\times10^4 \frac{y^4}{4} + 5.377\times10^3 \frac{y^3}{3} - 7.608 \frac{y^2}{2} + 20.727y \right]_{-0.035}^{0.035}$$

$$I_3 = -0.254\times10^7 (0.035 + 0.035)^5 + 0.690\times10^4 (0.035 + 0.035)^4 +$$

$$1.792\times10^3(0.035+0.035)^3-3.804(0.035+0.035)^2+20.727(0.035+0.035)$$

$$I_3=-2.056 \quad A$$

For $y = 0.035$, the change in y is zero, meaning $dy = 0$, and the change in z is also zero, $dz = 0$.

$$I_4=\int_{l(s)}\vec{H}\bullet d\vec{l}=\int_{l(s)}(H_x,0,0)\bullet(dx,dy,dz)$$

$$I_4=\int_{l(s)}(-3.587\times10^3x^2+1.903\times10^2x-8.866,0,0)\bullet(dx,0,0)$$

$$I_4=\int_{0.044}^{0}(-3.587\times10^3x^2+1.903\times10^2x-8.866)dx$$

$$I_4=\left[-3.587\times10^3\frac{x^3}{3}+1.903\times10^2\frac{x^2}{2}-8.866x\right]_{0.044}^{0}$$

$$I_4=-1.196\times10^3(0-0.044)^3+0.952\times10^2(0-0.044)^2-8.866(0-0.044)$$

$$I_4=0.676 \quad A$$

The total electric current along the closed loop is:

$I = I_1 + I_2 + I_3 + I_4$

$I = -2.105 + 0.313 - 2.056 + 0.676$

$I = -3.172\ A$

Therefore, the electric current in the inductor is 3 A, flowing out of the plane.

EXAMPLE 3

In the right side region of the winding shown in figure 04, the electric current density is given by:

$$\vec{J}=(0,0,14230.828) \quad A/m^2$$

For $0.020 \leq x \leq 0.025$ and $-0.023 \leq y \leq 0.023$ determine the electric current in the winding.

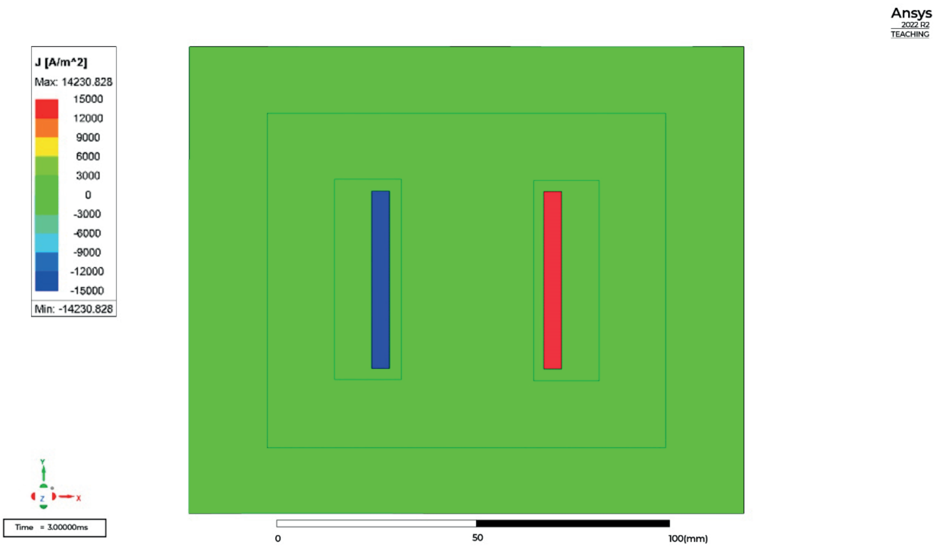

Figure 4 – The electric current density of an E-I core inductor.

Source: Own authorship.

Solution

$$I = \iint \vec{J} \bullet d\vec{S} = \iint (0,0,14230.828) \bullet (dydz, dxdz, dxdy)$$

$$I = \iint (0,0,14230.828) \bullet (0,0,dxdy) = \iint 14230.828 dxdy$$

$$I = 14230.828 \int_{0.020}^{0.025} dx \int_{-0.023}^{0.023} dy$$

$$I = 14230.828 \left[x \right]_{0.020}^{0.025} \left[y \right]_{-0.023}^{0.023}$$

$$I = 14230.828 \left[0.025 - 0.020 \right] \left[0.023 + 0.023 \right]$$

$$I = 3.273 \quad A$$

The electric current is 3.272 A, flowing out of the plane.

EXAMPLE 4

On the right-hand side of the winding shown in figure 05, the coil presents an electric current density, in A/m^2, given by:

$$\vec{J} = (0,0,J_z) \quad in \quad which$$

$$\begin{cases} J_z = -284620 + 14230828x & for \quad 0.020 \le x \le 0.021 \\ J_z = 14230.828 & for \quad 0.021 \le x \le 0.024 \end{cases}$$

Calculate the current in the coil for -0.0232 ≤ y ≤ 0.0232 and the divergence of the electric current density.

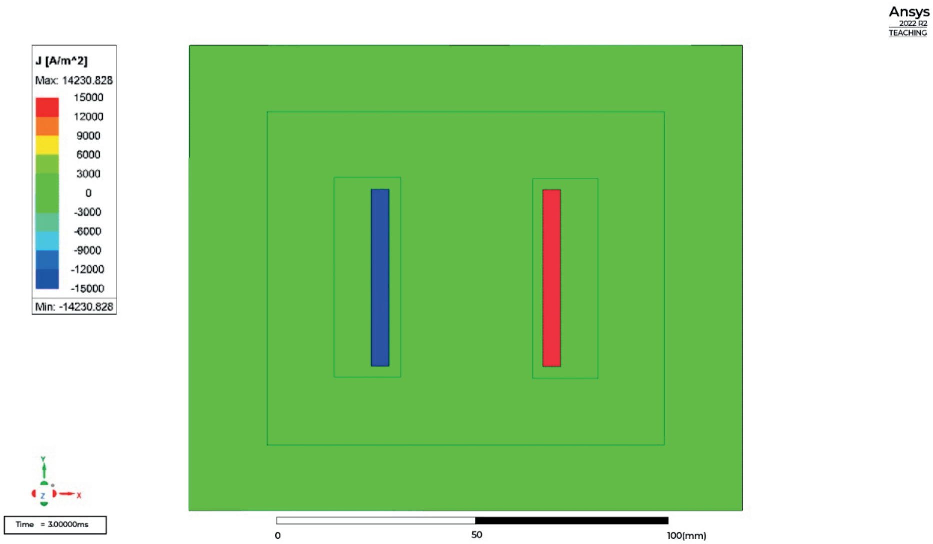

Figure 5 – The electric current density of an E-I core inductor.

Source: Own authorship.

Solution

For 0.020 ≤ x ≤ 0.021, we have:`

$$I_1 = \iint \vec{J} \bullet d\vec{S} = \iint (0, 0, -284620 + 14230828x) \bullet (dydz, dxdz, dxdy)$$

$$I_1 = \iint (0, 0, -284620 + 14230828x) \bullet (0, 0, dxdy)$$

$$\mathrm{I}_1 = \iint (-284620 + 14230828x) dxdy$$

$$\mathrm{I}_1 = -\iint 284620 dxdy + \iint 14230828 x dxdy$$

$$I_1 = -284620 \int_{0.020}^{0.021} dx \int_{-0.023}^{0.023} dy + 14230828 \int_{0.020}^{0.021} x dx \int_{-0.023}^{0.023} dy$$

$$I_1 = -284620 \left[x \right]_{0.020}^{0.021} \left[y \right]_{-0.023}^{0.023} + \frac{14230828}{2} \left[x^2 \right]_{0.020}^{0.021} \left[y \right]_{-0.023}^{0.023}$$

$$I_1 = -284620 \left[0.021 - 0.020 \right] \left[0.023 + 0.023 \right] + 7115414 \left[0.021^2 - 0.020^2 \right] \left[0.023 + 0.023 \right]$$

$$I_1 = 0.327 \quad A$$

For 0.021≤ x ≤ 0.024, we have:`

$$I_2 = \iint \vec{J} \bullet d\vec{S} = \iint (0,0,14230.828) \bullet (dydz, dxdz, dxdy)$$

$$I_2 = \iint (0,0,14230.828) \bullet (0,0,dxdy)$$

$$I_2 = \iint 14230.828 dxdy = 14230.828 \iint dxdy$$

$$I_2 = 14230.828 \int_{0.021}^{0.024} dx \int_{-0.023}^{0.023} dy$$

$$I_2 = 14230.828 \left[x\right]_{0.021}^{0.024} \left[y\right]_{-0.023}^{0.023}$$

$$I_2 = 14230.828 \left[0.024 - 0.021\right] \left[0.023 + 0.023\right]$$

$$I_2 = 1.964 \quad A$$

The total electric current in the coil is:

$I = I_1 + I_2 = 0.327 + 1.964 = 2.291$ *A*

The divergence of the electric current density is given by:

$$div\vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}$$

For $0.020 \le x \le 0.021$, we have:`

$$div\vec{J} = \frac{\partial}{\partial x} 0 + \frac{\partial}{\partial y} 0 + \frac{\partial}{\partial z} (-284620 + 14230828x)$$

$$div\vec{J} = 0$$

For $0.021 \le x \le 0.024$, we have:`

$$div\vec{J} = \frac{\partial}{\partial x} 0 + \frac{\partial}{\partial y} 0 + \frac{\partial}{\partial z} (14230.828)$$

$$div\vec{J} = 0$$

This inductor operates on alternating current with a frequency of 60 Hz, considered low frequency then the first Maxwell's equation is expressed as:

$$rot\vec{H} = \vec{J}$$

Applying the divergence operator, we have:

$$div\left(rot\vec{H}\right) = div\vec{J}$$

$$0 = div\vec{J}$$

EXAMPLE 5

For the central leg of the inductor shown in figure 06, the vector magnetic potential, in Wb/m, is given by:

$$\vec{A} = (0,0,A_z) \quad in \quad which \quad A_z = 0.009 \times 10^{-3} + 0.508x$$

$$for \quad -0.017 \le x \le 0.017 \quad and \quad -0.001 \le y \le 0.001$$

The vector magnetic potential presents this behavior near the horizontal line drawn on the central leg shown in figure 06. Determine the magnitude of the magnetic flux density in this range.

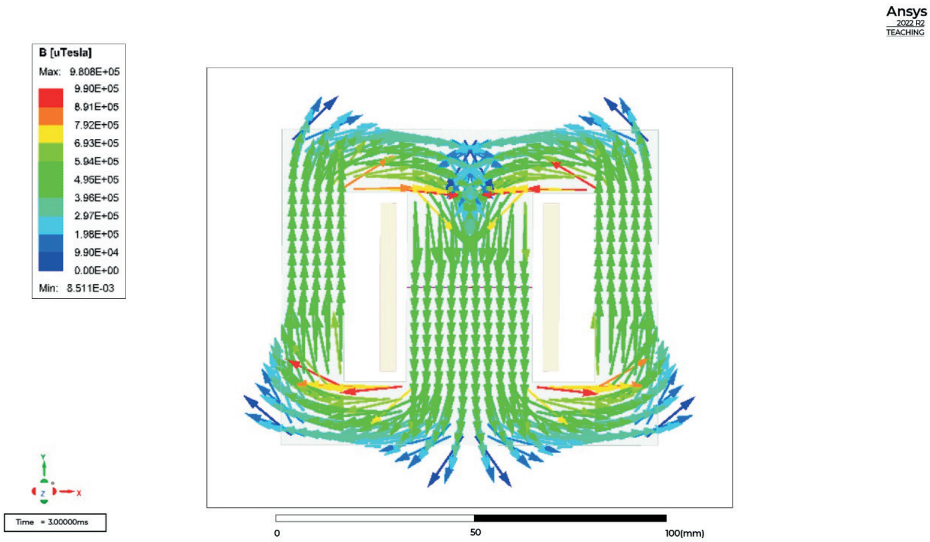

Figure 6 – Vector magnetic flux density of an E-I core inductor.

Source: Own authorship.

Solution

$$\vec{A} = (0, 0, A_z) = (0, 0, 0.009 \times 10^{-3} + 0.508x)$$

$$\vec{B} = rot\vec{A} = \left(\frac{\partial A_z}{\partial y}, -\frac{\partial A_z}{\partial x}, 0 \right) = (0, -0.508, 0) \quad |\vec{B}| = B = 0.508 \quad T$$

CONCLUDING REMARKS

In this chapter, we explored the calculation of fields in an inductor with a magnetic E-I core, with 314 turns and a depth of 34.93 mm. We used a two-dimensional model of the inductor in the Ansys® Electromagnetics Suite, Release 2022 R2, choosing the transient magnetic solution with Cartesian geometry. The inductor model was developed considering its different parts: a nonlinear anisotropic core, the copper coils, and the simulation region filled with vacuum. For excitation, we applied a sinusoidal voltage of 127 V at 60 Hz, and for boundary conditions, we established a zero vector magnetic potential at the edges of the simulation region.

The simulation of the inductor in ANSYS Maxwell allowed us to define fields with more realistic behaviors, from which we calculated the magnetic flux, flux linkage, electric current, divergence of the electric current density, and magnetic flux density.

The expressions of the fields used can adequately reproduce the behavior of the inductor, achieving the goals proposed for the study. This work has contributed significantly to a better understanding of the relationships between electromagnetic quantities, such as magnetic flux density, magnetic flux, magnetic field strength, electric current, electric current density, and vector magnetic potential.

DEDICATION

This work is dedicated to my beloved daughters: Lesly Daiana Barbosa Sobrado and Ketty Keisy Barbosa Sobrado Suarez.

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