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ANIMATION IN GEOGEBRA AND MATLAB APPLYING DIFFERENTIAL CALCULUS AND ANALYTICAL GEOMETRY

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Abstract: In this text, we will explore how the most fundamental concept of differential calculus can be applied to create animations that help visualize and better understand mathematical phenomena, focusing on two powerful tools: GeoGebra and MATLAB. The goal is to highlight the relationship between differential calculus and animations, and concrete examples will be mentioned of how these concepts can be applied in creating more realistic effects and movements in computer animations. The additional goal is to show that the aforementioned software is significant in the teaching and learning of mathematics, since it allows visualizing and experimenting with abstract concepts, facilitating its understanding and retention. In addition, they develop practical skills in the use of technological tools and programming, increasingly relevant in today's world. To illustrate the application of the derivative, we consider a problem, the solution of which also involves concepts of analytical geometry.

INTRODUCTION

In this talk, we will explore how differential calculus can be taken beyond traditional theoretical concepts to create dynamic animations that help visualize and better understand mathematical phenomena, focusing on two powerful tools: GeoGebra and MATLAB.

Before getting into the details of the application, it is important to understand what GeoGebra and MATLAB are, and their relevance in teaching and learning mathematics.

GeoGebra is a mathematics software that combines geometry, algebra and calculus in a single tool. It was developed with the aim of facilitating the learning of mathematics through a graphical and easy-to-use interface. GeoGebra is a powerful tool for teaching and learning mathematical concepts, from basic

geometry to advanced calculus. Additionally, it offers a wide range of educational resources, such as interactive textbooks, tutorials, and example problems. (Saldaña, 2017)

On the other hand, MATLAB is a programming language and development environment specialized in numerical calculation and data visualization, based on matrices. It is widely used in research and industry to solve complex problems in various fields, such as engineering, physics, biology, and finance. MATLAB offers a wide range of tools and functions for data analysis, simulation, and optimization. (MathWorks, 2024)

Both softwares have significant importance in teaching and learning mathematics, as they allow students to visualize and experiment with abstract concepts, facilitating their understanding and retention. Additionally, it allows students to develop practical skills in using technological tools and programming, which is increasingly relevant in today's world.

In the next section, we will see how GeoGebra and MATLAB can be used to create animations that illustrate differential calculus and its application in various contexts.

INTRODUCTION TO DIFFERENTIAL CALCULUS

One of the most significant applications of differential calculus lies in its geometric definition and interpretation, which establishes that the derivative of a function is the slope of the tangent line to the curve of the function at a specific point. This fundamental concept allows us to understand how a function changes at a given point and is essential for the analysis of the variation and behavior of functions in space.

Let us remember that the formal definition of the derivative is essential in differential calculus, since it provides a powerful tool to model and understand phenomena of change

and growth in various mathematical and scientific contexts.

Looking at Figure 2.1, let us remember that the slope of the secant line \overrightarrow{AB} is:

$$m = \frac{\Delta y}{h}$$

If h it is increasingly smaller, we can calculate the slope of the tangent line at A, using:

$$m = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

which is precisely the definition of derivative (Stewart, 2019)

“Definition The tangent line to the curve $y=f(x)$ at the point $P(a, f(a))$ is the line that passes through with a slope.

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

When the limit exists.”

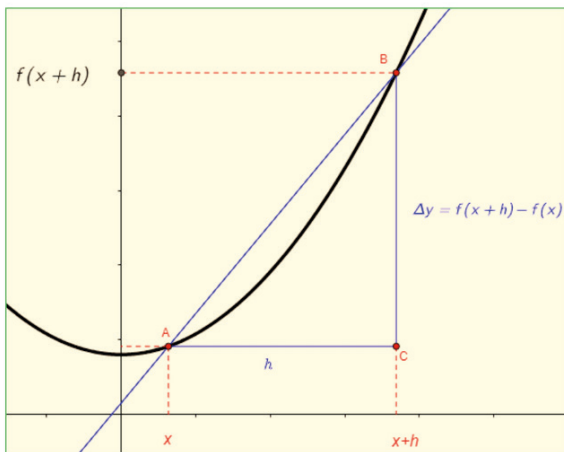


Figure 1: Line secant to the curve f

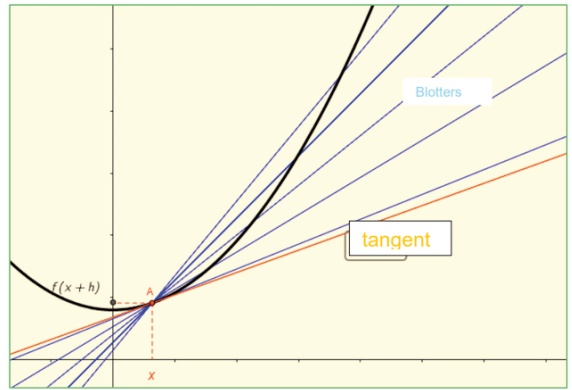


Figure 2: Line tangent to the curve f

The detailed definition opens the doors to one of the first applications given for the derivative, which is to calculate the tangent line to a curve at a given point $P(x_0, y_0)$. We can work on this type of problem in GeoGebra in the following way:

Suppose that it is necessary to calculate the tangent line to the curve $f(x) = \sqrt[3]{-x^3 + 3}$ at the point whose coordinate in x is -1 .

Using the view mode in GeoGebra: “Navigation Bar”, we observe the steps to follow to obtain the requested calculation, access it: <https://www.geogebra.org/classic/yungmk9f>

APPLICATIONS OF DIFFERENTIAL CALCULUS IN ANIMATIONS

To show the application of the derivative of a function, we will focus on a specific example. Suppose the following problem:

Create an animation in which a circle of radius $r = 1$ moves tangentially along the path of a sine curve given by the function:

$$f(x) = \frac{1}{2} \text{sen} \left(\frac{4}{3} x \right) + 2()$$

Solution:

Since this animation requires that the circle be tangent to the function, we will use precisely the derivative to achieve this purpose.

The following figure shows the outline of the problem posed: the circle with center W , is tangent to f in Q , and must be moved tangentially by f . The line $ec1$ is the line tangent to f at Q and $ec2$ is perpendicular to $ec1$.

The following figure shows the outline of the problem posed: the circle with center B , is tangent to f in A , and must be moved tangentially by f . The line $ec1$ is the line tangent to f at A and $ec2$ is perpendicular to $ec1$.

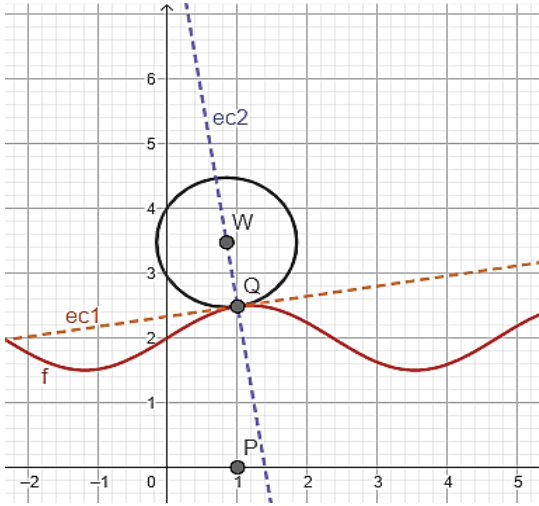


Figure 3: Circumference tangent to f in A

SOLUTION MADE IN GEOGEBRA

To begin the construction, we create in GeoGebra a slider and the function f given in (1), as well as its derivative and a point P associated with a slider Δ . The point P will serve to control the displacement of the circumference. All the following figures will depend, in their construction, on the point P .

In figure 3.1, it is observed that point Q is the point of tangency, the same one that is created in GeoGebra as $Q(x(P), f(x(P)))$. With these coordinates, it will move by (1) according to the value of the slider Δ , which will control the displacement. Based on the point, the slope of the tangent line Q , $m=f'(x(Q))$ is constructed $ec1$, as well as the slope of the line $ec2$ (perpendicular to $ec1$), $m_p = -\frac{1}{m}$. Since we have the slopes and the point through which the lines pass, it follows that:

$$ec1: y - f(x(Q)) = m(x - x(Q)) ()$$

$$ec2: y - f(x(Q)) = m_p(x - x(Q)) ()$$

Now, to obtain point W , that was showed in the figure 3.1, as the center of the tangential circle a f , we will use the formula:

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}} ()$$

which is used to calculate the distance from a point to the line (Wooton, Edwin, & Frank, 1985), where (x_1, y_1) are the coordinates of the point, and $Ax + Bx + C = 0$, is the equation of the line. Therefore, since the radius of the circle is $r = 1$, we know that the distance from to W , is precisely equal to $ec1$. We will denote the center of the circle $W(h, k)$.

To use equation (4), it is necessary to transform the into the general form from (2), in which, to facilitate the following operations we replace $f(x(Q))$, by y_Q and, $x(Q)$, by x_Q , then (2) becomes

$$y - y_Q = m(x - x_Q) ()$$

Then:

$$mx - y + (f_c y_Q - m x_c x_Q) = 0 ()$$

On the other hand, the equation of a line in its general format is

$$Ax + By + C = 0 ()$$

Then from (6) and (7), it follows that

$$A = m; B = -1; C = (y_Q - m x_Q)$$

Then in (4)

$$r = \frac{mh + (-1)k + (y_Q - mx_Q)}{\pm\sqrt{m^2 + (-1)^2}} ()$$

The sign of the denominator is determined by the position of the point W with respect to the line and the origin of coordinates $O(0,0)$, according to the following: considering that the line separates the Cartesian plane into 2 half-planes, if W and O are located in different

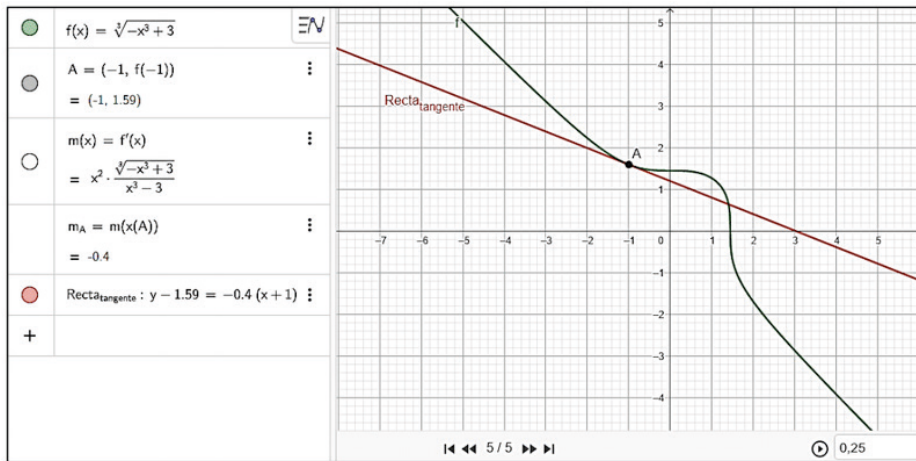


Figure 3: Line tangent to f at A

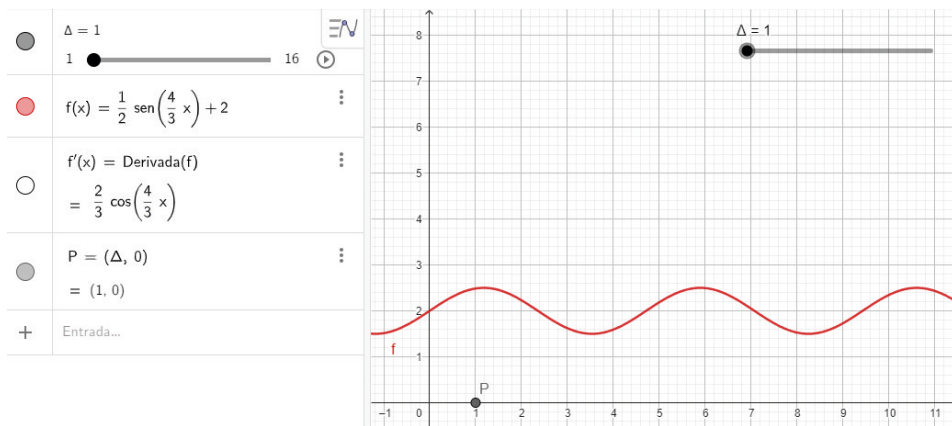


Figure 4: Constructions of: slider, function, derivative and point P

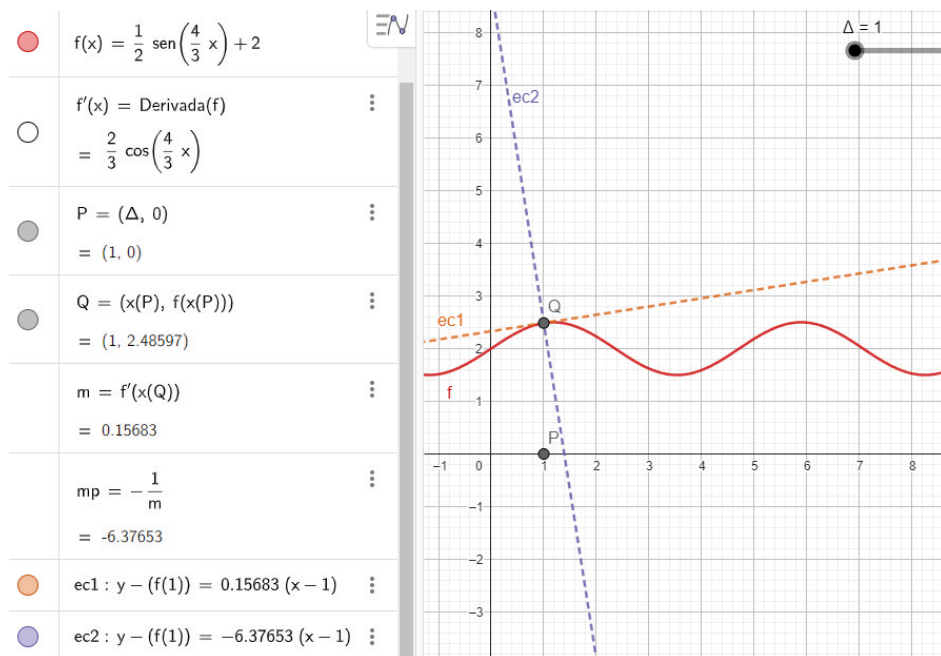


Figure 5: Constructions of: point A , lines ec1 and ec2

half-planes, The sign of the denominator in (8) will be positive, however, if W and O are located in the same semi plane, the sign of the denominator in (8) will then be negative (Wooton, Edwin, & Frank, 1985).

Therefore, in (8) with $r = 1$ we have to:

$$-\sqrt{m^2 + 1} = mh - k + (y_Q - mx_Q)$$

Then:

$$mh - k = -\sqrt{m^2 + 1} - y_Q + mx_Q \quad (9)$$

Now, let's consider equation (3)

$$y - y_Q = m_p(x - x_Q)$$

This line also passes through $W(h, k)$, that is

$$k - y_Q = m_p(h - x_Q)$$

Then:

$$m_p h - k = m_p x_Q - y_Q \quad (10)$$

Subtracting (9) from (10), it follows that

$$mh - m_p h = -\sqrt{m^2 + 1} + mx_Q - m_p x_Q$$

$$h = \frac{-\sqrt{m^2 + 1} + (m - m_p)x_Q}{m - m_p} \quad (11)$$

then replacing (11) in (10), we have

$$m_p \left(\frac{-\sqrt{m^2 + 1} + mx_Q - m_p x_Q}{m - m_p} \right) - k = m_p x_Q - y_Q$$

$$k = y_Q - m_p x_Q + m_p \left(\frac{-\sqrt{m^2 + 1} + mx_Q - m_p x_Q}{m - m_p} \right)$$

$$k = \frac{m y_Q - m_p y_Q - m_p m x_Q + m_p^2 x_Q - m_p \sqrt{m^2 + 1} + m_p m x_Q - m_p^2 x_Q}{m - m_p}$$

$$k = \frac{(m - m_p)y_Q - m_p \sqrt{m^2 + 1}}{m - m_p}$$

Therefore, the point W is:

$$W \left(\frac{(m - m_p)x_Q - \sqrt{m^2 + 1}}{m - m_p}; \frac{(m - m_p)y_Q - m_p \sqrt{m^2 + 1}}{m - m_p} \right)$$

With this point, we can create the circumference that depends on the coordinates of (x_q, y_q) , which, as seen in Figure 3.4, is the point that will move along the curve when moving the slider, in turn moving the circumference and creating the animation.

To facilitate viewing the animation, two

buttons are included in the construction that when pressed start or end the animation (Figure 3.5). The final product is available at: <https://www.geogebra.org/classic/uxuyehdu>

SOLUTION MADE IN MATLAB

To solve the problem in MATLAB, we will use the same analytical solution developed in GeoGebra and as a graphic reference, Figure 3.1. To begin the algorithm, we create the vector X with values in the interval and the function given in (1), plus its derivative denoted as . From the latter, we obtain the derivative of the perpendicular line in Q and name it . The lines of code are:

```
X= linspace (0,16);
Y=(1/2)*sin(4*X/3)+2;
m=(2/3)*cos(4*X/3); %derivative of Y
mp =-1./m;
```

Then, we need to create a loop with the for statement, which allows us to go through each value of the vector Below are the lines of code included in the loop:

```
for k=1: length(X)
axis ([0 16 -10 10]);
f=@(x) Y(k)+m(k)*(x-X(k)); % tangent line to f
g=@(x) Y(k)+ mp (k)*(x-X(k)); %perpendicular line
%Center C(h,K) and circumference radius:
h=(- sqrt (m(k)^2+1)+(m(k)- mp (k))*X(k))/(m(k)- mp (k));
K=g(h);
r=1;
%Circumference:
x= linspace (hr,h+r);
y1= K+sqrt (abs (r^2-(xh). ^2));
y2=K- sqrt (abs (r^2-(xh). ^2));
fplot (@(x)(1/2)*sin(4*x/3)+2)
hold on
fplot (f, ':')
fplot (g, ':')
plot (x,y1, 'r', x,y2, 'r')
axis ([0 16 -10 10]);
pause(0.05)
clf
end
```

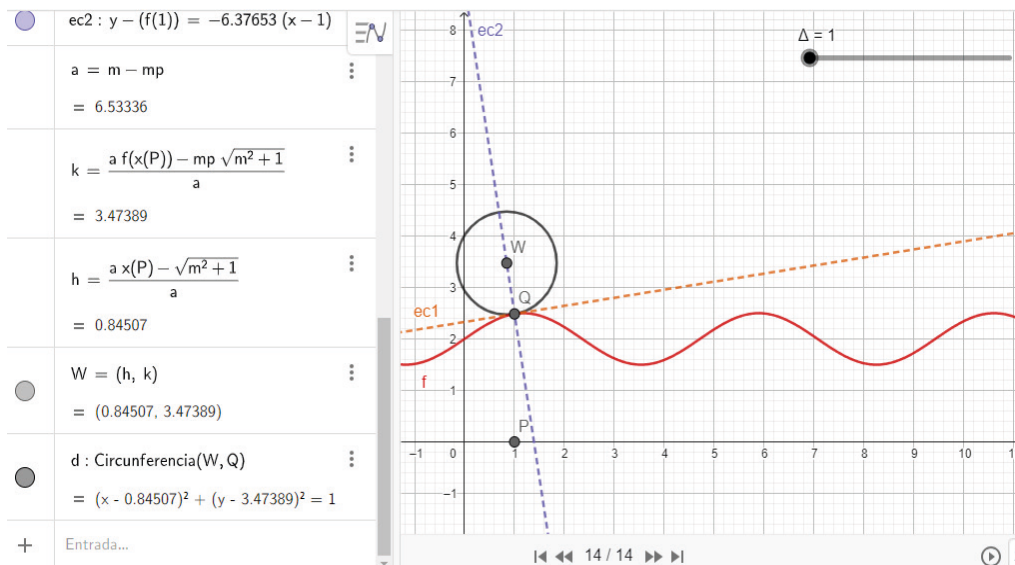



Figure 6: Construction of the circumference

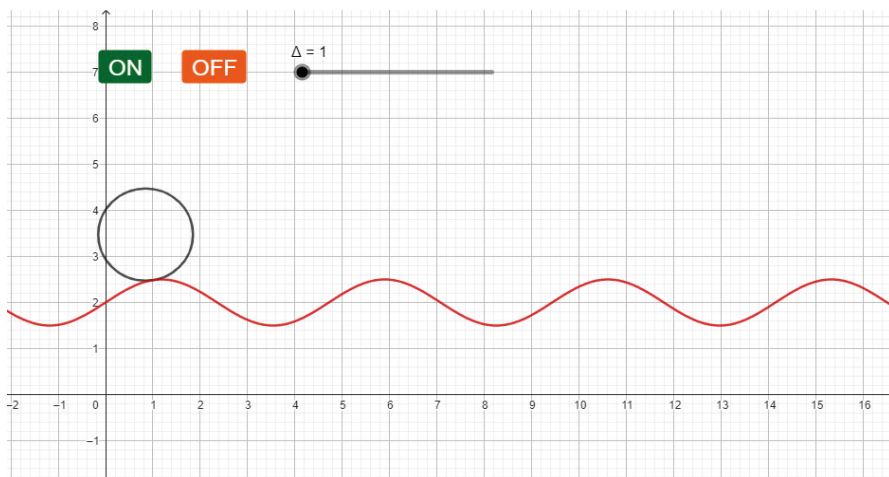


Figure 7: Animation made in GeoGebra

An important code during the execution of the loop is the “pause” command, since since it does not exist, the animation could not be displayed due to the high speed that MATLAB executes the programmed codes. The description of the mentioned command is detailed below:

“pause(n) pauses execution for n seconds before continuing. For this call to take effect you must enable pause.”(MathWorks, 2024)

clf” command is used, which deletes the graph created in each cycle and allows a new graph to be drawn, thus creating the animation effect.

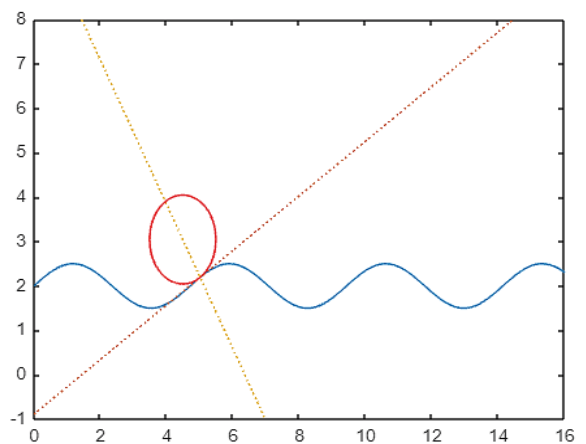


Figure 8: Graph paused during animation in MATLAB

The complete script can be found at: https://github.com/Jacobiano84/MATLAB_JEI/blob/main/Animacion.m

CONCLUSIONS

The definition of the derivative of a function has a fundamental geometric interpretation that allows a wide variety of practical applications.

The geometric interpretation of the derivative, as the slope of the tangent line to a curve, not only facilitates the understanding of the local behavior of functions, but is also a powerful tool for solving real problems in multiple disciplines.

GeoGebra offers a variety of interactive tools that allow you to visualize and manipulate functions and their derivatives in real time.

The animations and dynamic visualizations that GeoGebra allows you to create are

valuable for effectively demonstrating the principles of differential calculus and its various applications.

MATLAB, thanks to its programming algorithm structure, allows you to understand in detail the calculation process necessary to generate animations based on the definition of the derivative.

MATLAB's ability to handle complex algorithms and generate dynamic visualizations allows users to not only perform accurate calculations, but also effectively interpret and communicate the results using animations. This makes MATLAB an ideal platform for exploring and teaching the principles of differential calculus.

The integration of GeoGebra and MATLAB, with the concept of the derivative, not only simplifies the teaching and learning of this topic, but also opens new possibilities for its practical application.

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