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OBTAINING THE GLOBAL OPTIMUM OF AN NP-HARD PROBLEM: SCHOOL SCHEDULE THROUGH THE THREE-STAGE STRATEGY

José Israel Hernández-Vázquez
Department of Industrial Engineering, National Technology of Mexico / Technological Institute of León

José Omar Hernández-Vázquez
Department of Industrial Engineering, National Technology of Mexico / Technological Institute of León

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Abstract: The preparation of school schedules is a problem that occurs in many educational institutions around the world. From the mathematical approach, school schedules are considered NP-hard, since the computational time in searching for the solution can increase exponentially by increasing the number of variables, or by the complexity of the restrictions. Different strategies are reported in the literature to solve this problem; however, these do not guarantee finding the best solution or global optimum of the problem. This document establishes a validation of the three-stage assignment strategy that has been used in the solution of school schedules, whose results are characterized by obtaining good solutions in short times through the exact branching and bounding technique. Validation consists of demonstrating that the strategy reaches the global optimum in a school schedule problem.
Keywords: NP-Hard, School schedule, Staged strategy, Global optimal.

INTRODUCTION

The preparation of activity schedules is a problem that occurs in a wide variety of areas, particularly in educational institutions. Its difficulty lies in the large number of options that can be generated and choosing the best one, or one that meets all the needs of the institution, is a task that consumes a large amount of time, even in problems with a few subjects, classrooms. and teachers.

The development of a school schedule from a mathematical point of view is classified as an NP-hard problem due to the large number of combinations present, causing the computational time in searching for a solution to increase drastically (Bardadym, 1996; Even, Itai & Shamir, 1976).

Different strategies have been reported in the literature to solve school schedules, highlighting different methods that guarantee a good solution in short times, or the use of exact techniques in the search for the global optimum of the problem.

The first works correspond to assignment models (Appleby, Blake & Newman, 1961; Csima & Gotlieb, 1964) and the graph coloring technique (De Werra, 1985; De Werra, Asratian & Durand, 2002; Welsh & Powell, 1967).

On another side, various authors have chosen to use Mathematical Programming in the search for an optimal solution, highlighting PE Integer Programming (Lawrie,1969), PEB Binary Integer Programming (Arratia-Martinez, Maya-Padron & Avila-Torres, 2021; Bakir & Aksop, 2008; b; Palma & Bornhardt, 2020; Sánchez-Partida, Martínez-flores, Cabrera-Rios & Olivares-Benitez, 2017; Son & Ngan, 2021; Tripathy, 1984) and finally Mixed Integer Programming (PEM) & Mason, 2016; Lindahl, Sørensen & Stidsen,2018; Rappos, Thiémard, Robert & Hêche, 2022; Sørensen & Dahms, 2014; Tassopoulos, Iliopoulou & Beligiannis, 2020).

In recent years, other strategies have been used that have allowed good quality solutions to be achieved in less time to large problems; however, these do not guarantee the global optimum of the problem. The use of local search methods stands out (Demirović & Musliu, 2017; Goh, Kendall & Sabar, 2017; Rezaeipanah, Matoori & Ahmadi, 2021; Saviniec, Santos & Costa, 2017, 2018; Song, Liu, Tang, Peng & Chen, 2018), metaheuristic tabu search techniques (Goh et al., 2017; Lü & Hao, 2010; Saviniec et al., 2018) and genetic algorithms (Arias-Osorio & Mora-Esquivel, 2020; Beligiannis, Moschopoulos & Likothanassis, 2009; Feng, Lee & Moon, 2017; Junn, Obit & Alfred, 2018; Khonggamnerd & Innet, 2009; Lin, Chin, Tsui & Wong, 2016; Niknamian, 2021; Raghavjee & Pillay, 2010; Rezaeipanah et al., 2021; Yigit, 2007), solutions based on minimal disturbance (Barták, Müller

& Rudová, 2003; Lindahl, Stidsen & Sørensen, 2019; Phillips, Walker, Ehrgott & Ryan 2017), in addition to hyper-heuristics (Ahmed, Özcan & Kheiri, 2015; Junn, Obit, Alfred & Bolongkikit, 2019; Kheiri, Özcan & Parkes, 2016) among others (Cruz-Rosales et al., 2022; Esmaeilbeigi, Mak-Hau, Yearwood, & Nguyen, 2022; Mirghaderi, Alimohammadlo & Fotovvati, 2023; Wouda, Aslan & Vis, 2023).

In another solution approach, authors have chosen to develop the school schedule in two stages, which has allowed the number of variables used in mathematical modeling to be greatly reduced, speeding up the search for the solution. (Birbas et al., 2009; Lindahl et al., 2018; Sørensen & Dahms, 2014; Yasari, Ranjbar, Jamili & Shaelaie, 2019).

Recently Hernández et al. (2020a, 2020b) developed a strategy in preparing university schedules, which consists of decomposing the problem into three stages, which significantly reduces the number of binary variables, limiting the solution space. This strategy allows the exact branching and bounding technique to generate the optimal solution for each stage in short times, however, it is not demonstrated whether it reaches the global optimum of the problem.

The main contribution of this document is the validation of the three-stage strategy proposed by Hernández et al. (2020a, 2020b) in obtaining the global optimum of the problem. To demonstrate whether it reaches the global optimum, it is considered to solve a school schedule through a single mathematical model with an exact technique, and then compare it with the solution achieved by the strategy in three stages in different instances.

THREE-STAGE ALLOCATION STRATEGY

The number of combinations of a mathematical model with binary variables is 2^n, where n is the number of variables. To limit the search space, the strategy evaluated in this study proposes solving the problem in stages, as other authors have done in the development of school schedules (Birbas et al., 2009; Lindahl et al., 2018; Sørensen & Dahms, 2014; Yasari et al., 2019), which has allowed the number of variables to be reduced. The study by Sørensen and Dahms (2014) shows the mathematical theoretical theorems that illustrate the advantages of working a binary problem in stages, substantiating the benefits of this strategy for its application in other cases of school schedules.

Unlike the studies mentioned in the previous paragraph, where a maximum of two stages were used, the strategy proposed by Hernández et al. (2020a, 2020b) considered the development of a university schedule in three of these, using binary variables from two indices. This strategy assigns each subject a time interval, a classroom and a teacher consecutively, through three mathematical models (see figure 1).

Table 1 shows a comparison between proposing a mathematical model to develop a university schedule, such as the studies presented in (Arratia-Martinez et al., 2021; Bakir & Aksop, 2008; Daskalaki et al., 2004; Palma & Bornhardt, 2020; Sánchez-Partida et al., 2017; Schimmelpfeng & Helber, 2007) which used PEB and exact techniques in the search for the solution, or the proposal developed by Hernández et al. (2020a, 2020b) of using three mathematical models, taking into account 126 subjects, 14 time intervals, 9 classrooms and 21 teachers. As can be seen, when the modeling is decomposed into three stages, the number of variables is considerably reduced by 98.34% and it is only necessary

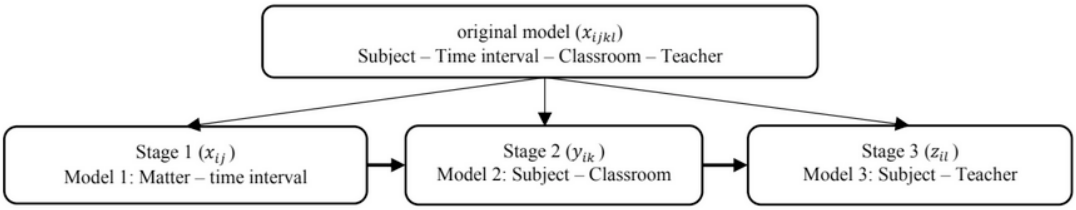

Figure 1: Decomposition of the original problem into three stages

Source: Hernández et al. (2020a)

1 Mathematical model		3 Mathematical models					
Model 1 x_{ijkl}		**Model 1 x_{ij}**		**Model 2 y_{ik}**		**Model 3 z_{il}**	
4-Index Binary Variable		Binary variable 2-indices		Binary variable 2-indices		Binary variable 2-indices	
i Subject	126	*i* Subject	126	*i* Subject	126	*i* Subject	126
j Time interval	14	*j* Time interval	14	*k* Classroom	9	*l* Teacher	21
k Classroom	9	Variables (*i* x *j*)	1,764	Variables (*i* x *k*)	1,134	Variables (*i* x *l*)	2,646
l Teacher	21						
Total binary variables (*i* x *j* x *k* x *l*)	**333,396**	Total binary variables	**5,544**				

Table 1: Comparison of the number of variables between one and three mathematical models

Source: Hernández et al. (2020a)

to use two indices in each variable, greatly simplifying the complexity of the problem.

As in the works (Arratia-Martinez et al., 2021; Daskalaki et al., 2004), the three-stage proposal uses subsets in each of the mathematical models, with the intention of separating subjects by group and interval of time, allowing modeling of certain constraints to be easier and reducing iteration between variables. In addition, coefficients are used in the objective function intentionally to speed up the search in binary variables.

VALIDATION OF THE ALLOCATION STRATEGY IN THREE STAGES

The validation of the three-stage strategy consists of demonstrating that it reaches the global optimum of the problem through the exact branching and bounding technique in a short time, compared to solving the problem in a single mathematical model. For which a school schedule problem was proposed in which it seeks to maximize the number of subjects to be assigned to a time interval, classroom and teacher considering the following restrictions:

- Each subject must not be assigned more than once.
- Per classroom in each time interval, only one subject can be assigned.
- Do not exceed the capacity of the classrooms (number of students).
- Only one subject can be assigned per time slot for each teacher.
- The maximum number of subjects per teacher must not be exceeded.

To validate the strategy, four mathematical models were designed. The first of them was established to solve the problem in a single step, allowing us to know the global optimum of the problem. Subsequently, the three mathematical models concerning each of the stages of the strategy appear, whose sequential resolution will generate the solution to the problem. The comparison of these solutions

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in different instances will determine if the staged strategy reaches the global optimum of the problem.

NOMENCLATURE USED IN MATHEMATICAL MODELS

SETS

I Subjects $I = \{Course_1, Course_2, \dots, Course_{|i|}\}$.
J Time intervals $J = \{T_1, T_2, \dots, T_{|J|}\}$.
K Classrooms $K = \{C_1, C_2, \dots, C_{|K|}\}$.
L Teachers $L = \{P_1, P_2, \dots, P_{|L|}\}$.

INDICES

i Subject $i \in$ I.
j Time Interval $j \in J$.
k Classroom $k \in K$.
l Teacher $l \in L$.

SUBSETS

$I_n \subset I$ Subjects assigned to time intervals n. $n \in \{T_1, T_2, \dots, T_{|J|}\}$.

PARAMETERS

a_{ijkl} = Number of students of subject i in time interval j in classroom k of teacher l.
a_{ik} = Number of students of subject i in classroom k.
Classrooms avaliable = Number of classrooms available.
Classrooms capacity$_k$ = Maximum capacity of the number of students in the classroom k.
Subjects per teatcher$_l$ = Maximum number of subjects per teacher *l*.

BINARY VARIABLE USED IN THE MATHEMATICAL MODEL 1

x_{ijkl}
1 Subject i is assigned in time interval j, classroom k and teacher l.
0 Subject i is not assigned in time interval j, classroom k and teacher l.

BINARY VARIABLE USED IN MATHEMATICAL MODEL 2 - STAGE I

x_{ij}
1 Subject i is assigned in time interval j.
0 Subject i is not assigned in time interval j.

BINARY VARIABLE USED IN MATHEMATICAL MODEL 3 - STAGE II

y_{ik}
1 Subject i is assigned to classroom k.
0 Subject i is not assigned to classroom k.

BINARY VARIABLE USED IN MATHEMATICAL MODEL 4 - STAGE III

z_{il}
1 Subject i is assigned to teacher l.
0 Subject i is not assigned to teacher l.

MATHEMATICAL MODEL 1: SUBJECT ASSIGNMENT-TIME INTERVAL-CLASSROOM-TEACHER

Mathematical model 1 was designed to find the solution to the problem in a single step. Consider the assignment of subjects to a time interval, classroom and teacher, taking into account all the constraints of the problem.

$$Z_{\text{Max}} = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} x_{ijkl} \tag{1}$$

Subject to:

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} x_{ijkl} \le 1 \quad \forall i \in I \tag{2}$$

$$\sum_{i\in I}\sum_{l\in l} x_{ijkl} \le 1 \quad \forall j \in J, \forall k \in K \tag{3}$$

$$\sum_{i\in I}\sum_{l\in l} a_{ijkl}x_{ijkl} \le Classroom\ capacity_k \ \forall j \in J, \forall k \in K \tag{4}$$

$$\sum_{i\in I}\sum_{k\in K} x_{ijkl} \le 1 \quad \forall j \in J, \forall l \in L \tag{5}$$

$$\sum_{j\in J}\sum_{j\in J}\sum_{k\in K} x_{ijkl} \le Subjects\ per\ teacher_l \ \forall l \in L \tag{6}$$

$$x_{ijkl} \in \{0,1\} \tag{7}$$

The Objective Function (1) seeks to maximize the number of subjects to be assigned in a time interval, classroom and teacher. The restrictions provide that:

- (2) Each subject must not be assigned more than once.
- (3) Only one subject can be assigned per classroom in a time interval.
- (4) It obliges not to exceed the capacity of the classrooms.
- (5) Only one subject can be assigned per time slot for each teacher.
- (6) The maximum number of subjects per teacher must not be exceeded.

MATHEMATICAL MODELING OF THE THREE-STAGE ALLOCATION STRATEGY

MATHEMATICAL MODEL 2 - STAGE I: SUBJECT ASSIGNMENT-TIME INTERVAL

Mathematical model 2 was established to generate the solution of stage I, which consists of the assignment of subjects with the time interval, taking into account the restrictions related to it.

$$Z_{\text{Max}} = \sum_{i \in I} \sum_{j \in J} x_{ij} \tag{8}$$

Subject to:

$$\sum_{j \in J} x_{ij} \leq 1 \qquad \forall i \in I \tag{9}$$

$$\sum_{i=1}^{I} x_{ij} \leq \textit{Classrooms available} \qquad \forall j \in J \tag{10}$$

$$x_{ij} \in \{0,1\} \tag{11}$$

The objective function (8) seeks to maximize the number of subjects to be assigned to a time interval. The restrictions consider that:

- (9) Each subject must be assigned in no more than one time slot.
- (10) The subjects assigned per time interval must not exceed the number of available classrooms.

MATHEMATICAL MODEL 3- STAGE II: SUBJECT-CLASSROOM ASSIGNMENT

Mathematical model 3 was developed to obtain the solution of stage II, which consists of assigning the subject to the classroom, taking into account the restrictions of the problem.

$$Z_{\text{Max}} = \sum_{i \in I_n} \sum_{k \in K} y_{ik} \tag{12}$$

Subject to:

$$\sum_{k \in K} y_{ik} \leq 1 \qquad \forall i \in I_n \tag{13}$$

$$\sum_{i \in I_n} a_{ik} y_{ik} \leq \textit{Classroom capacity}_k \quad \forall k \in K \tag{14}$$

$$y_{ik} \in \{0,1\} \tag{15}$$

The objective function (12) maximizes the number of subjects assigned to classrooms. For their part, the restrictions establish that:

- (13) Each subject must be assigned in no more than one classroom.
- (14) Groups of students larger than their capacity must not be assigned to each classroom.

MATHEMATICAL MODEL 4 – STAGE III: SUBJECT ASSIGNMENT-TEACHER

Mathematical model 4 was developed to generate the solution of stage III, which consists of assigning the subject with the teacher, taking into account the restrictions related to it.

$$Z_{\text{Max}} = \sum_{i \in I_n} \sum_{l \in L} Z_{il} \tag{16}$$

Subject to:

$$\sum_{l \in L} Z_{il} \leq 1 \qquad \forall i \in I_n \tag{17}$$

$$\sum_{i \in I_n} Z_{il} \leq 1 \quad \forall l \in L \tag{18}$$

$$\sum_{i=1} Z_{il} \leq Subjects\ per\ teacher_l \quad \forall l \in L \tag{19}$$

$$Z_{il} \in \{0,1\} \tag{20}$$

The objective function (16) seeks to maximize the number of subjects to be assigned to teachers. The restrictions determine that:

- (17) Each subject must be assigned to no more than one teacher.
- (18) In each time interval, no more than one subject must be assigned per teacher.
- (19) The maximum number of subjects per teacher must not be exceeded.

EXPERIMENTS

To validate the strategy in three stages in obtaining the global optimum, five instances were solved, considering different sizes of the problem in terms of the number of subjects, time intervals, classrooms and teachers as shown in table 2.

Instance	Subjects	Timeslots	Classrooms	Teachers
1	4	2	2	2
2	8	4	2	2
3	32	8	4	6
4	40	8	5	7
5	48	8	6	8

Table 2: Instances considered in the experimentation

Source: self made

A computer with an AMD Ryzen 3 processor, with 8 GB of RAM, using the Windows 10 Home system was used. The data matrices were programmed in Excel, which were linked to the Lingo 17 software where the mathematical models were captured and solved through the exact branching and bounding technique.

RESULTS

Table 3 shows the comparison between using a mathematical model and using the three-stage assignment strategy, considering the number of binary variables, restrictions, non-zeros, the optimal value and the search times for the solution in each one of the instances. The optimal value of model 1 determines the objective that must be achieved in each of the models 2-4 corresponding to the staged strategy, to conclude that it found the global optimum of the problem.

In all instances, the global optimum was obtained with the three-stage strategy, but with a significant decrease in the number of variables and the solution time, with respect to using a single mathematical model, as detailed in Table 4.

The three-stage strategy allows a reduction in the number of variables of up to 94%, significantly limiting the solution space, simplifying the time spent searching for the solution by up to 99% in the last instance.

Instance		1 Model	Three stages	Reduction
1	Binary variables	32	24	**25%**
	Time	<1 second	<1 second	**0%**
2	Binary variables	128	64	**50%**
	Time	<1 second	<1 second	**0%**
3	Binary variables	6,144	576	**91%**
	Time	15 second	<2 second	**87%**
4	Binary variables	11,200	800	**93%**
	Time	23 second	<2 second	**91%**
5	Binary variables	18,432	1,056	**94%**
	Time	8 h	2 second	**+99%**

Table 4: Reduction in the number of variables and solution time through the three-stage strategy

Source: self made

Instance		Model 1	Three stages			
			Stage I Model 2	Stage II Model 3	Stage III Model 4	Total
1	Binary variables	**32**	8	8	8	**24**
	Restrictions	**15**	7	9	11	**27**
	Non-zero	**160**	24	24	32	**80**
	Optimal value	**4**	4	4	4	
	Time	**<1 second**	<1 second	<1 second	<1 second	**<1 second**
2	Binary variables	**128**	32	16	16	**64**
	Restrictions	**27**	13	17	19	**49**
	Non-zeros	**640**	96	48	64	**208**
	Optimal value	**8**	8	8	8	
	Time	**<1 second**	<1 second	<1 second	<1second	**<1 second**
3	Binary variables	**6,144**	256	128	192	**576**
	Restrictions	**103**	41	65	87	**193**
	Non-zero	**30,752**	768	384	768	**1,920**
	Optimal value	**32**	32	32	32	
	Time	**15 second**	<1 second	<1 second	<1 second	**<2 second**
4	Binary variables	**11,200**	320	200	280	**800**
	Restrictions	**128**	49	81	104	**234**
	Non-zeros	**56,040**	960	600	1120	**2,680**
	Optimal value	**40**	40	40	40	
	Time	**23 second**	<1 second	<1 second	<1 second	**<2 second**
5	Binary variables	**18,432**	384	288	384	**1,056**
	Restrictions	**153**	57	97	121	**275**
	Non-zeros	**92,208**	1,152	864	1,536	**3,552**
	Optimal value	-----------	48	48	48	
	Time	**8 hours**	1 second	<1 second	<1 second	**2 second**

Table 3: Comparison between a mathematical model and the three stages

Source: self made

CONCLUSIONS

This article presented the validation of the three-stage allocation strategy proposed by Hernández et al. (2020a, 2020b), in obtaining the global optimum of an NP-hard school schedule problem. The strategy proposes the decomposition of the problem into three mathematical models, greatly simplifying its complexity, which allows obtaining solutions in short times compared to solving the problem in a single step.

The validation of the strategy consisted of demonstrating that it reaches the global optimum of the problem through the exact technique of branching and bounding in short times, compared to solving the problem in a single mathematical model.

Five instances were resolved. The three-stage assignment strategy managed to find the global optimum of the problem with a significant reduction in the number of variables of up to 94%, reducing the search time for the solution by up to 99% compared to solving the problem in a mathematical model only.

For future research, it will be attractive to demonstrate whether the three-stage strategy manages to find the global optimum of university schedules, where there are restrictions of greater complexity than those presented in this study. Furthermore, this strategy could be used in the solution of other NP-hard problems, with the intention of finding the global optimum.

INTEREST CONFLICT

The authors declare that there is no conflict of interest with respect to the research, authorship, and/or publication of this article.

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