

## MATHEMATICAL MODEL OF LINEAR FLOW REGRESSION FOR A PROTOTYPE OF ARTIFICIAL WETLAND IN TEHUITZINGO PUEBLA MEXICO

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**Abstract:** Wastewater treatment is a crucial challenge for the preservation of the environment and public health. Deep wetlands have emerged as a sustainable and effective solution to purify wastewater naturally and biologically. The application of statistics plays an essential role in the evaluation, monitoring and optimization of these treatment systems. In this work, a deep wetland prototype adopted for wastewater treatment in the municipality of Tehuiztzingo, Puebla is modeled and analyzed; Statistics is applied to wastewater treatment and its importance in decision making by applying the Python programming language for its calculation.

## INTRODUCTION

Among the aspects that suffer the most are the availability of fresh water for human consumption due to natural phenomena caused by global warming, as well as the irresponsible use and poor management planning of water resources, which has led to overexploitation and contamination of water resources. bodies of water, aquifers and bodies of water such as dams or reservoirs.

At the same time, 64% of surface water bodies are contaminated to one degree or another, mainly by industrial wastewater discharges and drainage systems from cities and municipalities in the country. While it is true that 60% of the country's drainage water goes through some type of treatment, in many cases wastewater treatment plants (WWTP) do not operate as efficiently as they must. This creates a situation in which water from drains is discharged directly into streams, ravines, rivers and lakes with consequent pollution, which generates a negative impact on the environment, as well as on the populated areas near them.

This way, an artificial wetland imitates these ecosystems by using vegetation and

substrates to eliminate contaminants, bacteria, odors and fecal particles from the water, allowing it to be reused for other purposes or discharged into bodies of water as long as the treated water meets the requirements. Pollution requirements. standards required by Conagua. According to the last census carried out by the National Institute of Statistics and Geography (INEGI), the total population of the municipality is 12,672 inhabitants, with the administrative center of the municipality being the city with the largest population -6,644 inhabitants-. Just as the same census shows that 94.2% of this population has some type of drainage, either through wastewater collection in pipes or through septic tanks. Although the drainage coverage in the municipality is worthy of attention, the main problem of the municipality in terms of drainage is the lack of a functioning wastewater treatment plant since the currently existing plant is not operational due to lack of resources. economical to operate it. incorrect dimensions, as well as design solutions that negatively affected its long-term operation. This has caused contamination of the nearby environment due to the discharge of wastewater directly into the ravine, which also negatively affects the health of people living nearby.

Based on the above, the objective of this research work is to design an artificial wetland for the treatment of wastewater generated in the municipal center of Tehuitzingo, Puebla, thus improving the local environment, generating jobs and increasing the population's awareness of caring for water and, above all, helping to reuse this vital fluid, which is more important and difficult to obtain every day.

## **RESEARCH PROBLEM**

Access to drinking water is a fundamental service for people since it affects their quality of life and improves the health and well-being of the population. According to the UN and its sustainable development goals, 3 out of 10 people do not have access to drinking water services. These sewage systems are responsible for collecting and transporting wastewater generated in populated areas to treatment plants. At the national level, only 73% of the wastewater generated is treated, the rest is discharged into various bodies of surface water with the corresponding pollution that this entails.

On the other hand, the exploitation of aquifers and surface water bodies for the supply of drinking water, as well as the pollution caused by the discharge of untreated wastewater, have threatened the water supply in some regions of the country.

## **JUSTIFICATION**

This work proposes mathematical modeling through a linear regression of the flow of a prototype of a deep artificial wetland in the municipality of Tehuitzingo, Puebla. The Python programming language will be used to model the behavior of the flow. To complement this, tables and graphs will be made to help better visualize the data and results. This will be of great help to know the efficiency of the prototype design.

## **THEORETICAL FRAMEWORK**

According to CONAGUA, there are 2,786 wastewater treatment plants in operation in the country, with an installed capacity of 196,749.51 l/s, which treat 144,710 l/s, equivalent to 67.2% of the wastewater generated and collected. in the country. Therefore, investment in wastewater treatment systems is necessary, both conventional and alternative, such as artificial wetlands.

Wetlands are a type of transitional ecosystem between water and land, thus having characteristics of both an aquatic environment and a terrestrial environment, as well as unique characteristics. These have the function of capturing sediments and nutrients and preventing them from reaching other bodies of water, which is why they are vital for ecosystems.

As mentioned previously, one of the benefits of wetlands is the absorption of pollutants, a fact that did not go unnoticed by man, since for centuries wetlands were used as a discharge area for human and animal waste. It was in the 1950s in Germany when studies began to be carried out on the feasibility of designing and building wetlands, in such a way that they would have better efficiency in the treatment of pollutants than natural wetlands. This technology had several advantages when compared to a conventional treatment plant, such as being cheaper to build and maintain, being simpler in maintenance and operation, having a more pleasant visual appearance, and serving as a habitat for plants and animals. These advantages meant that, for small cities and towns, which did not have the financial resources to build a conventional treatment system or to connect to a larger treatment plant, they could build an artificial wetland and thus be able to comply with the environmental regulations of wastewater treatment.

Another advantage is that wetlands can be designed depending on the volume to be treated, having wetlands that work in cities like wetlands that can serve a family home. Constructed wetlands can be classified into 2 types according to the flow with which water circulates in them, surface flow constructed wetlands (HAFS) and subsurface flow constructed wetlands (HAFSS). Being that, in the former, water flows in contact with the atmosphere, while, in the latter, water flows

under a substrate of gravel and sand.

The treatment of water within a natural wetland occurs through the interaction between the elements of the wetland and the physical, chemical and biological phenomena within the wetland, with the intervention of the wetland alone as the main source of energy. Chemical phenomena include processes such as decomposition by oxidation, which helps the destruction of viruses and pathogenic bacteria, as well as by precipitation and absorption in the substrate of heavy metals, nitrogen and phosphorus, the latter being particularly difficult to eliminate in flow wetlands. subsurface, so the use of specific substrates that enhance phosphorus retention has been proposed. Finally, the biological part of the wetland is represented by the vegetation and microorganisms present in the wetland substrate; these elements are capable of purifying nitrogen, phosphorus, heavy metals and organic matter. Finally, when designing an artificial wetland, certain considerations must be taken into account so that the system has the expected performance, such as the inlet flow, temperature, the amount of pollutants present and the type of vegetation used.

## METHODOLOGY

Review of artificial wetland design methodologies. This information is necessary to be able to have a design that meets the desired quality requirements of the water leaving the wetland. One of the methodologies that will be used is the method proposed by the National Water Commission (CONAGUA), which in turn is based on the methodology of the United States Environmental Protection Agency (EPA), as well as on the method proposed by Kadlec and Knight researchers. The methodology proposed by CONAGUA is based on an equation derived from a first-order kinetic model and treats wetlands as biological reactors whose performance can

approach those described in a plug flow reactor. However, in exchange, it gives us a very complete wetland design that includes length, width, depth of the medium, depth of water, slope, number of plants per m<sup>2</sup> and residence time.

To use these design methods, the prototype was built that has the following characteristics shown in Table 1:

Parameters	Value
Total area	0.1347 m <sup>2</sup>
Total width	36 cm
Total length	38 cm
Width of each cell	18 cm
Area of each cell	0.684 m <sup>2</sup>
Depth	25 cm
Inlet flow	7.15 l/d
Hydraulic residence time	45 hours

Table 1: Characteristics of the prototype of the artificial wetland prototype

Figure 1 and Figure 2 show a plan of the prototype, as well as a cross section of it.

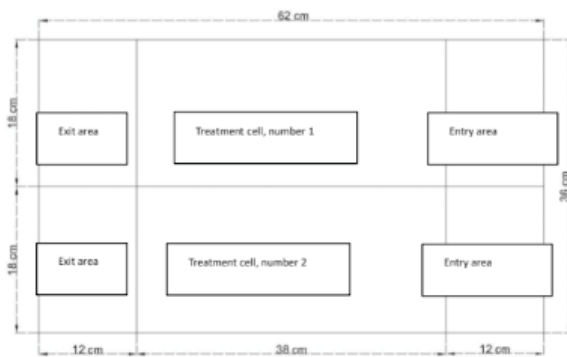


Figure 1: Top view of artificial wetland prototype

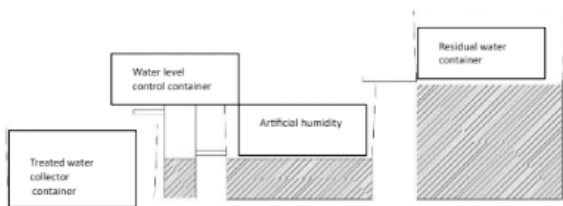


Figure 2 Side view of the artificial wetland prototype

Statistic analysis. All statistical analyzes were done with Google Colab using the python programming language. Applying the linear regression model, remembering that a linear regression model is a statistical analysis technique used to predict or explain the relationship between a dependent variable and one or more independent variables.

In the context of a flow, a linear regression model could be an excellent option, used to predict the flow of a river (dependent variable) as a function of independent variables, such as rainfall, temperature, geography, etc. The linear regression model seeks to establish a linear relationship between variables, which means that it is assumed that a change in an independent variable translates into a constant change in the dependent variable. The model can be expressed as follows:

$$\text{Flow rate} = \beta_0 + \beta_1 * \text{Variable1} + \beta_2 * \text{Variable2} + \dots + \epsilon$$

where:

Flow rate is the dependent variable that we want to predict.

$\beta_0$  is the intercept (the flow value when all independent variables are zero).

$\beta_1, \beta_2, \dots$  are the coefficients that represent the relationship between the independent variables and the flow.

$\epsilon$  is the error term, which captures the unexplained variations in the model.

The objective is to find the best values of  $\beta_0, \beta_1, \beta_2, \dots$  that minimize the total error and best fit the observed data. This is done using optimization and statistical methods.

In summary, a linear regression flow model seeks to predict the flow of a river as a function of independent variables through a linear relationship, which can be useful in water management, flood forecasting, and other applications related to water resources.

To collect data, water level measurements were taken inside the residual water tank of the prototype, in such a way that a volume in

liters was obtained that, when measured over a certain time, a flow rate in lt/d was calculated. This same thing was repeated in the wetland outlet reservoir, thus having a measurement of the flow at the entrance and exit of the wetland.

Regarding the contaminant removal data, these were obtained through physicochemical analyzes carried out in the laboratory.

## MODEL

The following regression model is used for the result of physicochemical analysis of our wetland through Google Colab.

First physicochemical analysis with linear regression with their respective residuals on each line. (Fig. 3, Fig. 4, Fig. 5)

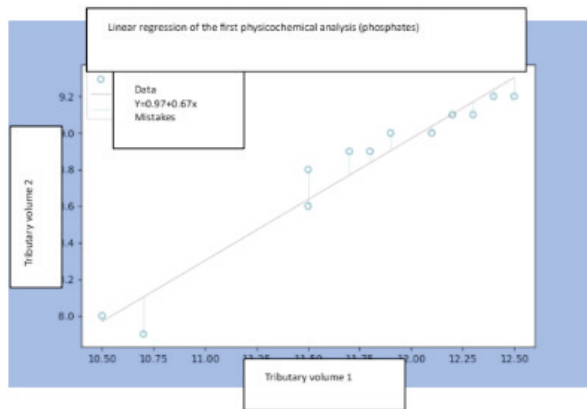


Figure 3: Phosphates

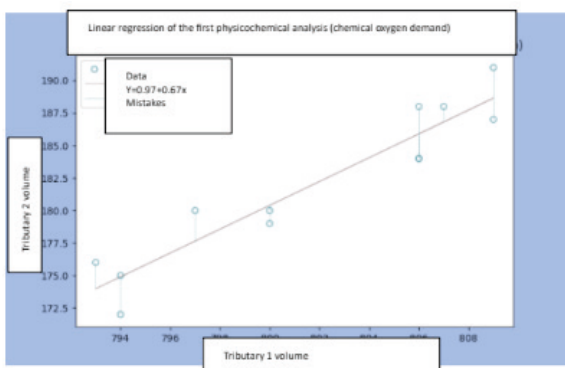


Figure 4: DQO

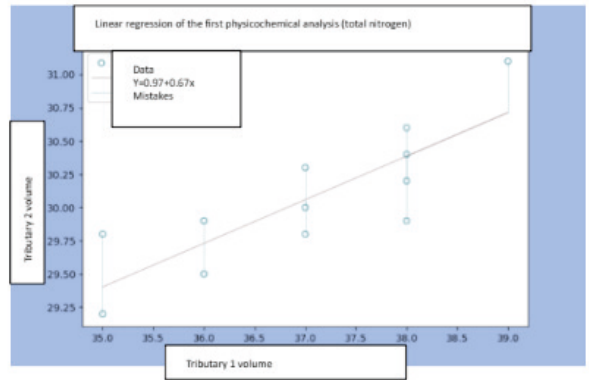


Figure 5: Total nitrogen

The following tables present the parameter values used in our initial analysis, along with the statistical analyzes and the associated analysis of variance (ANOVA) table that was calculated. (Table 1, Table 2, Table 3)

Volume 1 in waste water tank	Volume 2 in waste water tank	Effluent Volume	Removal percentage
10.5	8.0	2.5	0.76
10.7	7.9	2.8	0.74
11.5	8.6	2.9	0.75
11.5	8.8	2.7	0.77
11.7	8.9	2.8	0.76
11.8	8.9	2.9	0.75
11.9	9.0	2.9	0.76
12.1	9.0	3.1	0.74
12.2	9.1	3.1	0.75
12.3	9.1	3.2	0.74
12.4	9.2	3.2	0.74
12.5	9.2	3.3	0.74

Table 2: Phosphates (First analysis)

Tributary 1 Volume	Tributary 2 Volume	Effluent Volume	Removal percentage
794	175	619	0.22
809	187	622	0.23
806	184	622	0.23
809	191	618	0.24
806	184	622	0.23
807	188	619	0.23
800	180	620	0.22
793	176	617	0.22
794	172	622	0.22
797	180	617	0.23
800	179	621	0.22
800	188	618	0.23

Table 3: COD (First analysis)

Tributary Volume 1	Tributary Volume 2	Volumen Efluyente	Removal percentage
695	479	619	0.69
681	473	622	0.69
695	477	622	0.69
672	452	618	0.67
671	453	622	0.68
696	473	619	0.68
677	442	620	0.65
676	448	617	0.66
675	451	622	0.67
675	444	617	0.66
685	455	621	0.66
686	451	618	0.66

Table 4: Total nitrogen (first analysis)

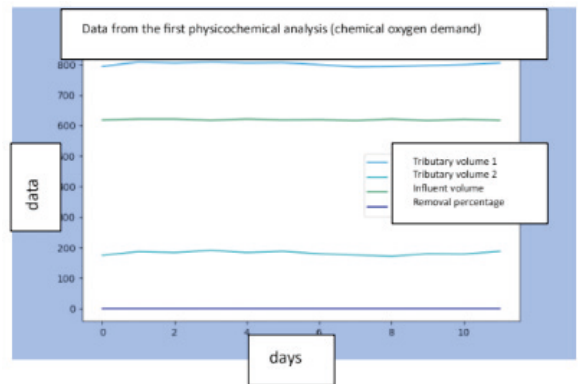


Figure 7: DQO

A relationship graph was constructed with respect to the days observed, taking into account influent volume 1, influent volume 2, effluent volume and removal percentage. (Fig. 6, Fig. 7, Fig. 8) This procedure was carried out for the second and third analyses. In the second part of our database, a linear regression analysis was applied to relate the volume of tributary 1 to tributary 2. In addition, the data were compared based on the days of observation, including those in which rains in the location. Subsequently, a statistical analysis was performed and the corresponding ANOVA table was generated. In the QR code below are the lines of code used to obtain our results and graphs. (Fig. 9)

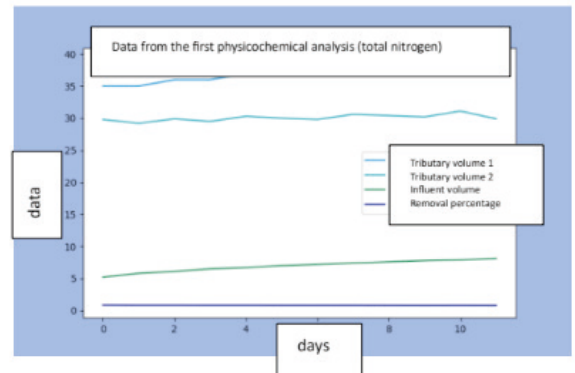


Figure 8: Total nitrogen



Figure 9: QR code

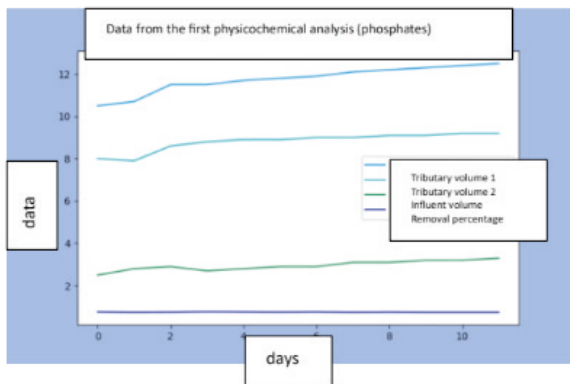


Figure 6: Phosphates

## CONCLUSIONS

It can be seen that the removal of contaminants remained constant throughout the time in which the observations were carried out. The pollutants where the best removal percentages were obtained were both total nitrogen and phosphates, something that goes hand in hand with what has been reported in other scale wetlands, where it is

mentioned that these are the pollutants with the best removal rates. Regarding the chemical oxygen demand, it was observed that it did not have a good removal, this is because the mechanism to remove this contaminant is found in the contact of the residual water with the roots of the vegetation, so it is theorized that These were not yet fully developed during data collection.

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