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STUDY OF THE RESPONSE TO THE HARMONICS OF A MASS SPRING SYSTEM: CASE: QUASI RESONANCE (RESOLUTION)

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Abstract: Quasi-resonance in dynamic systems, of the Mechanical Oscillator type, occurs when the frequency of the system under study is "very close" to the frequency of the excitation signal. In these cases the response is in the form of an amplitude modulated (AM) signal made up of a carrier and a message. It is important to know both the frequency and the period of these signals in order to take precautions when mechanical equipment or constructions, such as bridges or buildings, are exposed to this type of events and thus avoid damage whose consequences in both human and economic lives become serious. catastrophic. In this article we present the procedure to obtain the value of these parameters.
Keywords: Quasi-resonance; Mechanical Oscillator; Modulated amplitude; Carrier; Message;

BACKGROUND

a) In Flores, A. J. A. & Rodríguez, F. A. (2018 Page 94) we began the study of the frequency response of a mechanical oscillator emphasizing the resonance phase; Due to the type of excitation signal, this behavior was caused by the third harmonic (n = 3). The solution obtained together with the graph is shown in Figure 1.

$$x(t) := \left[\sum_{n=1}^{2}\left[B(n)\cdot\frac{n\cdot\sin(3t) - 3\cdot\sin(n\cdot t)}{6\left(n^2-9\right)}\right] + \left(\frac{\sin(3\cdot t)}{54} - \frac{t\cdot\cos(3\cdot t)}{18}\right) + \sum_{n=4}^{10}\left[B(n)\cdot\frac{n\cdot\sin(3t) - 3\cdot\sin(n\cdot t)}{6\left(n^2-9\right)}\right]\right]\cdot\Phi(t)$$

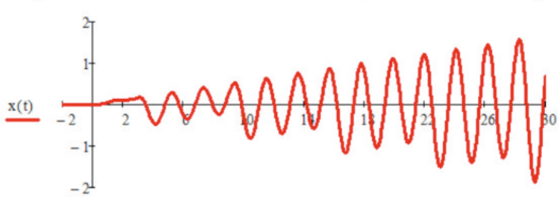

Figure 1.- Response function and graph that shows the response of a system in a state of resonance. Resonance is caused by the third harmonic.

b) In Flores, A. J. A. & Rodríguez, F. A. (2021) we continue with the study of this class of systems, but now the emphasis is on the phase that we define as "quasi-resonance"; Due to the type of excitation signal, this behavior was caused by the first harmonic (n = 1). We point out the fact that the previous behavior, resonance phase, is impossible to occur in practice. The solution obtained together with the graph is shown in figure 2.

$$x(t) := -\frac{22\cdot\sin(\pi\cdot t) - 2\cdot\pi\cdot\sqrt{11}\cdot\sin\left(\sqrt{11}\cdot t\right)}{11\cdot\pi^3 - 121\cdot\pi}$$

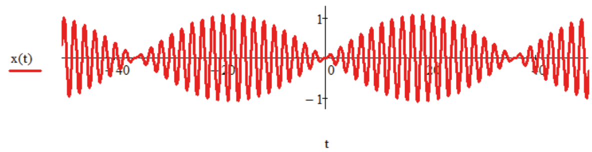

Figure 2.- Function and graph that shows the response of a system in a "quasi-resonance" state.

c) In this installment we make a detailed study of the response in a state of quasi-resonance, answering a series of questions such as: Why is this response given? . . . What are the associated frequencies? etc

d) Let us remember that we carry out the analysis based on a problem of the type:

Determine the function x(t) that allows us to know the position at all times of the mass m = 1 that is attached to a spring with "k = a", if starting from rest and from the point of equilibrium, a excitation signal given by the function:

$$f(t) := \begin{cases} (2-t) & \text{if } 0 \le t \le 2 \\ f(t-T) & \text{if } t > 2 \end{cases}$$

Periodic function with period T = 2 and angular frequency: $\omega_o = \pi$.

Note: The units are appropriately sized according to the system in which they work, be it "cgs or mks"".

And that the problem is designed for educational purposes, it is friendly in more ways than one.

In the resolution we use the strategy indicated in Flores A. J. A. (2021) in light of the scenario presented by the Fourth Industrial Revolution (Industry 4.0: Science, Technology, Engineering & Mathematics STEM).

PROBLEM RESOLUTION

In the study we point out that a system operates in a state of "quasi resonance" when the frequency of the excitation signal has a value "very close" to the natural frequency of the system and it is an effect that must be avoided, since the response is an "amplitude modulated" signal, which for the effect of mechanical vibrations implies an oscillation whose amplitude increases within certain limits, not always within the safety range of the equipment.

In the aforementioned document we point out that the amplitude modulation is generated by the first harmonic of the signal (n = 1) and we show it by doing the corresponding study, obtaining as a general solution function for the position x(t) the expression (1), where: "√a" is the natural frequency of the system.

$$x(t,a) := \frac{2}{\pi}\cdot\left[\frac{\sqrt{a}\cdot\sin(\pi\cdot t) - \pi\cdot\sin(\sqrt{a}\cdot t)}{\sqrt{a}\cdot(a-\pi^2)}\right] \quad 1)$$

We presented the solution for the case in which "a = 11" and obtained the answer shown in figure 3 (p. 2015 of [2]) .

$$x(t,a) := \frac{2}{\pi}\cdot\left[\frac{\sqrt{11}\cdot\sin(\pi\cdot t) - \pi\cdot\sin(\sqrt{11}\cdot t)}{\sqrt{11}\cdot(11-\pi^2)}\right]$$

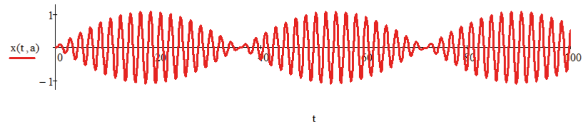

Figure 3: Graph of the response of the system in quasi-resonance: The oscillation increases its amplitude, causing the risk of destruction of the system.

NOTES

In the Amplitude Modulated (AM) technique in communications, the central frequency is called the carrier and the "envelope" the message, so to be consistent with this nomenclature we will call the internal signal "carrier" and the envelope "message." and we will identify them with the subscript "p" for carrier and "m" for message; At the outset we ask two questions:

a).- What is the frequency of the carrier and what determines it?

b).- What is the frequency of the message and what determines it?

GUESS

The quasi-resonant behavior of the mechanical system, as we already pointed out, originates when the frequency of the excitation signal is "very close" to the natural frequency of the system, regardless of whether it is above or below; In this problem the natural frequency of the system is "√11" and the frequency of the excitation signal, for the first harmonic, is "π"; the difference between them is: – π = 0.175 which is "very small" (Flores, A. J. A. & Rodríguez, F. A. (2021 page 2014). We propose the following conjecture that we obtained through observation and analysis of the graph of the solution signal:

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GUESS

The frequencies of the carrier and the message are determined by the times that there is a difference between the periods of the excitation signal and the natural frequency of the system, in each of them.

EXPLANATION OF THE CONJECTURE

We obtained this conjecture through the Scientific Method used in solving problems in science. We observe the event under study, look for regularities, parameterize them, subject them to validation and repeat the process over and over again until we find the "best guess"; Validation is in mathematics what experimentation is in physics.

Imagine Edison carbonizing his filament and inserting it into the light bulb again and again until he was successful, or Coulomb observing how the elder balls were attracted or repelled again and again depending on the magnitude of the charge he managed to add to them, or Archimedes sitting down and standing up. standing and watching how the water level moved in the bathtub. What determined that Edison's filament illuminated without burning? What determined whether the elder balls attracted or repelled each other in Coulomb's case? What determined whether the water level in the bathtub rose or fell depending on Archimedes' position?

CONSTRUCTION OF THE CONJECTURE:

For the analysis we identify two components in the system response:

a).- One determined by the first harmonic of the excitation signal; let's call it: $f_1(t)$, y that is: (2).

$$f1(t) := \frac{2}{\pi} \cdot \frac{\sqrt{11} \cdot \sin(\pi \cdot t)}{\sqrt{11} \cdot (11 - \pi^2)} \qquad 2)$$

b).- And another determined by the natural frequency of the system; let's call it: $f_2(t)$, that is (3)

$$f2(t) := \frac{-2}{\pi} \cdot \frac{\pi \cdot \sin(\sqrt{11} \cdot t)}{\sqrt{11} \cdot (11 - \pi^2)} \qquad (3)$$

The corresponding frequencies are:

$\omega_1 = \pi$ and $\omega_2 = \sqrt{11}$

The corresponding periods are:

$T_1 = 2$ and $T_2 = \frac{2\pi}{\sqrt{11}}$

In this part we resort to the resolution method used in the problems studied in the Higher Secondary Level on the least common multiple (lcm) of two quantities, of the type:

PROBLEM

There are two bells "A" and "B" that ring simultaneously at 7:00 A.M.; If bell "A" rings every 25 minutes and bell "B" rings every 40 minutes: After how long will they ring again?

In our problem we have two functions: $f_1(t)$ y $f_2(t)$ that at t = 0 are found at x = 0; the first returns to x = 0 after 2 seconds and the second does so after seconds (Fig. 4): question How long will it take to coincide at x = 0 for the first time after the start? In the answer to this question, the concept of the LCM is present.

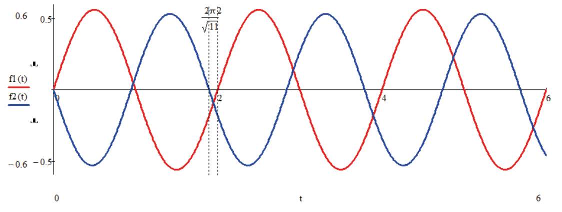

Figure 4: Graph of the functions that define the response of the system in quasi-resonance: The function f1(t) (in red) and f2(t) (in blue) coincide at the point (0, 0): The red one after 2 seconds it returns to x = 0 and the blue one does so after 2π/√11 seconds: What is the time t1 in which they coincide for the first time at the point x = 0; (t1, 0)?

Our conjecture is based on assuming that the small difference between the signals is presented as an additional force that during the positive cycles pushes the mass reaching a maximum and during the negative cycles it stops it, returning it to zero, thus determining the message of the response signal.

VALIDATION OF THE CONJECTURE

Continuing with what was stated above, in the system response we have two components:

$$f1(t) := \frac{2}{\pi} \frac{\sqrt{11}\cdot\sin(\pi\cdot t)}{\sqrt{11}\cdot(11-\pi^2)} \quad \text{and} \quad f2(t) := \frac{-2}{\pi}\cdot\frac{\pi\cdot\sin(\sqrt{11}\cdot t)}{\sqrt{11}\cdot(11-\pi^2)}$$

With their corresponding frequencies and periods:

Frequencies:

$$\omega_1 = \pi \quad \text{and} \quad \omega_2 = \sqrt{11}$$

Periods:

$$T_1 = 2 \quad \text{and} \quad T_2 = \frac{2\pi}{\sqrt{11}}$$

The difference between periods is:

$$T_1 - T_2 = 2 - \frac{2\pi}{\sqrt{11}} = \frac{2\cdot\sqrt{11} - 2\pi}{\sqrt{11}} \quad ;$$

$$\frac{2\cdot\sqrt{11} - 2\pi}{\sqrt{11}} = 0.10554835$$

We calculated the number of times that the difference of periods fits in the period of the natural frequency of the system; We call this magnitude “c”; Using the method of guesswork and the method of trial and error, we find that twice this magnitude “c” is the time in which the two signals are first spliced and determines the period of the mensaje [T_m] and therefore its frequency [f_m].

$$\frac{\frac{2\pi}{\sqrt{11}}}{\frac{2\cdot\sqrt{11}-2\pi}{\sqrt{11}}} \text{ simplify } \rightarrow \frac{\pi}{\sqrt{11}-\pi}$$

Approximate value”

$$\frac{\pi}{\sqrt{11}-\pi} = 17.948662$$

Double is the message period and the inverse is the frequency, that is, the natural frequency of the system determines the message frequency:

$$T_m = 2\cdot\left(\frac{\pi}{\sqrt{11}-\pi}\right) \qquad T_m = 35.8973239$$

$$f_m = \frac{\sqrt{11}-\pi}{2\pi} \qquad f_m = 0.0278572$$

We showed this result in figures 5 and 6.

$$x(t) := \frac{2}{\pi}\cdot\left[\frac{\sqrt{11}\cdot\sin(\pi\cdot t) - \pi\cdot\sin(\sqrt{11}\cdot t)}{\sqrt{11}\cdot(11-\pi^2)}\right]$$

$$c := \left(\frac{\pi}{\sqrt{11}-\pi}\right)$$

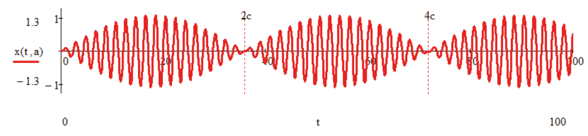

Figure 5: Graph of the system response in quasi-resonance: We quantify the period of the response envelope and therefore its frequency. At time ”2c” one cycle of the envelope is completed.

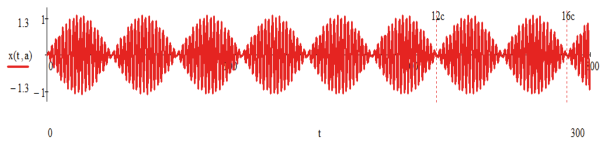

Figure 6: Graph of the system response in quasi-resonance: We extend the time range and verify that the conjecture is indeed satisfied. Every “2c” the envelope is repeated. Of course it is empirical evidence, but: When, if not on these occasions, are ICTs of great help and support in the resolution and study of certain types of problems?

Now we calculate the number of times that the difference of periods fits in the period of the first harmonic; again making guesses and through trial and error we find that this value determines the number of cycles that fit in the message.

$$\frac{2}{\frac{2\cdot\sqrt{11}-2\pi}{\sqrt{11}}} \text{ simplify } \rightarrow \frac{\sqrt{11}}{\sqrt{11}-\pi} \quad ;$$

$$\frac{\sqrt{11}}{\sqrt{11}-\pi} = 18.948662$$

This is the number of times the period difference fits in one message cycle and is equal to the number of carrier cycles that fit there.

Now, to calculate the frequency and period of the carrier we consider the following: If in a time period "A" there are "B" cycles of a signal, then the A/B ratio provides us with the duration of each cycle, that is That is, your period, for example, if 5 cycles fit into 20 seconds, each cycle lasts 4 seconds and that is your period.

Therefore, if the message period is:

$$T_m = 2\cdot\left(\frac{\pi}{\sqrt{11}-\pi}\right)$$

And the number of times that the period difference fits in the message is:

$$\frac{\sqrt{11}}{\sqrt{11}-\pi}$$

The ratios of these two magnitudes provide us with the period and frequency of the carrier which, as we can see, is equal to the natural frequency of the system:

$$\frac{\frac{2\pi}{\sqrt{11}-\pi}}{\frac{\sqrt{11}}{\sqrt{11}-\pi}} = \frac{2\pi}{\sqrt{11}} \qquad T_p = \frac{2\pi}{\sqrt{11}} = 1.8944517$$

The inverse is the carrier frequency.

$$f_p = \frac{\sqrt{11}}{2\pi} = 0.5278572$$

We show this result in figure 7.

$$x(t,a) := \frac{2}{\pi}\cdot\left[\frac{\sqrt{11}\cdot\sin(\pi\cdot t)-\pi\cdot\sin(\sqrt{11}\cdot t)}{\sqrt{11}\cdot(11-\pi^2)}\right]$$

$$\frac{\sqrt{11}}{\sqrt{11}-\pi} = 18.948662$$

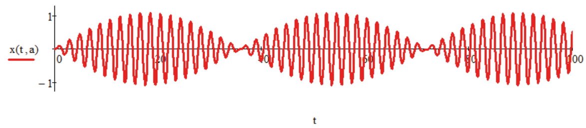

Figure 7: Graph of the system response in quasi-resonance: If we look at the graph and count the number of cycles within a cycle of the message, effectively the number is around 18.

NOTE

We solved this same problem for different values of "a" (with √a natural frequency of the system), above and below the first harmonic "π" and the results are consistent with the conjecture.

The stage would be a set of structures such as buildings, towers, bridges, etc. that are subjected to an external force, the product of, for example, an earthquake; Each one of them will experience a different effect depending on their natural frequency. It is the same force characterized by its frequency acting on systems with different natural frequencies so the responses will be different: This one remained unscathed; that one suffered minor damage but the one beyond it collapsed, figure 8.

Figure 8: Photo showing the differentiated damage left by an earthquake in the surrounding area depending on the type of construction. The Nuevo León building was completely destroyed and the Veracruz building was not damaged. Different natural frequencies or deficiencies in their construction?

FINAL COMMENTS

Proponemos que los múltiples casos de destrucción de sistemas como [Chasechocolate (2010); Phisisc Girl (2018) **y** Arredondo, M. (2015)] que se tienen registrados, por supuesto efecto de la resonancia, son más bien efecto de la cuasiresonancia en los que la vibración, desplazamiento de los cuerpos, llega a un punto en el que el cuerpo/sistema se destruye.

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