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CONSIDERING THE DIRAC DELTA FUNCTION DDF AS AN ANALOGY OF THE BEHAVIOR OF THE COLCAP INDEX 2008-2019

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Abstract: Among his works Paul Adrian Maurice Dirac (*1902 †1984), is also recognized for his support with the so-called Dirac Delta function. The present document intends to articulate the usefulness of this function and analyze of the behavior of the shares in the stock market. One may assume that in a daily share price, there may be some *impulse events*, which will describe specific behavior in discrete periods in a normal or traditional trading market, and the mathematical development of its function, may constitute a tool for the analysis of these motions.
Keywords*:* DDF, Impulse events, COLCAP index, Heaviside.

INTRODUCTION

Among his works Paul Adrian Maurice Dirac (*1902 †1984), is also recognized for his support with the so- called Dirac Delta function DDF. The present document intends to articulate the usefulness of this function and analyze of the behavior of the shares in the stock market. One may assume that in a daily share price, there may be some *impulse events*, which will describe specific behavior in discrete periods in a normal or traditional trading market, and the mathematical development of its function, may constitute a tool for the analysis of thesemotions.

As stated by Tonidandel and Araujo (2015):

> Recognized internationally following Paul's work A.M. Dirac (created at the young age of 25 years), one can get an idea of the concept of *impulse* from the old example of the mechanics in which a forceconcentrated in a short period of time happens at once, for example when a soccer player *kicks* a ball,representing an *impulsive force* applied, to cause a finite change of the linear momentum (of the ball) in an infinitesimal time (p. 3306)[2].

This notion of impulse, arises in a field and in a historical moment, in which the questions related to the subatomic roots of matter, contradict the traditional logic of physical thought, with great thinkers like Lord Kelvin, Maxwell himself and Stokes and those had made an emphasis on the development of physics from a purely *mechanistic* position.

In a sense, the scientific debate focused on a struggle between matter/movement, against the further development of *quantum mechanics*, which, to some extent, respected the development of other ideas such as light, heat and electricity and that unleashed the *quantum generation*, which was closer to the constitution of a*mathematical machinery* than to an orthodox fidelity of the mechanistic program. In this context, the quantumtheory needed completeness in its approaches, but also a program. Dirac and others knew that quantum theory could not be represented only by common numbers, but that the theory needed complex numbers, which were capable of realizing a new reality shaken by a group of young people under thirty years old. Among others things, the experience of the development of *quantum generation* was the creation of a mathematical arsenal, which included the use of new techniques for the analysis of the physic *micro-phenomenon*. Two excellent works about the development of physics in the first three decades of the past century are the works of Rydnik (1969), Kumar (2011) and Cox & Forshaw (2011). However, Dirac's contributions are not summarized in his contribution to Quantum Mechanics, but to the field of function analysis (if the DDF can be called a function).

In this sense, the DDF can describe these discrete and impulsive behaviors. Unitary functions and especially single scales, tend to be a *group* of atypical functions that describe certain behaviors, generally of physical variables across time. One of them is the so-called unitary scalar function U^1SF,

2 Own translation.

or in some cases also called Heaviside's step function (HSF). This function is denoted as *H(x)*, *u(x)*, or in some cases by θ(x). First, we will analyze some unit scalar functions; later we will explore some characteristics of the delta function to make an analogy with the behavior of the shares in the stock market. Finally, the COLCAP index will be analyzed annually, to make some final considerations in its daily and annual development.

UNITARIAN SCALAR FUNCTIONS

French mathematician Jean D´Alembert (*1717 †1783) believed that the concept of function should be conditional upon the common methods of Algebra and Calculus. However, the great Swiss mathematician Leonhard Euler (*1707 †1783), in turn, had a seemingly simpler view, believing that a function could be defined easily if it were possible. You can draw the curve of $f(t) \times t$, just as you would with a pen by sliding it over a piece of paper.

Frege[3] (1904), - in honor to Boltzmann and with the purpose to define the concept of function -, considers that in recent times, within the very definition of function the word variable has predominated. This, however, is in need of clarification, taking into account that any variation occurs over time, therefore, the analysis must be related to the temporal occurrence, since it is the variables that are submitted for consideration. However, what the philosopher perceives is that the analysis does not correlate with time and even in trigonometry, time disappears from any basis of analysis.

Dirac proposes a function that has generally been associated with unit and/or scalar functions, one being Unit Step Function. USF has also been studied by Abramowitz & Stegun (1972) and can be expressed graphically as follows:

3 Gottlob Frege (*1848 †1925).

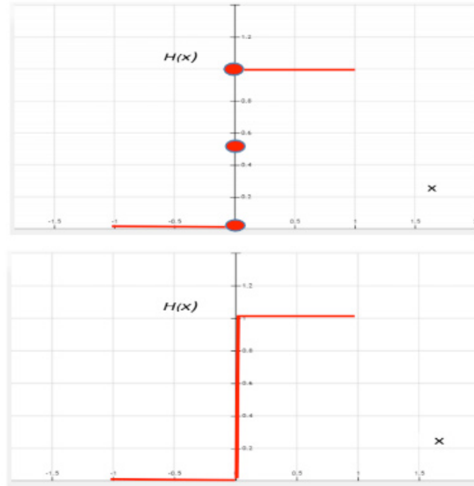

Fig. 1. The Unit Step Function an expression graphic. Observe as the function is expressed as signal in (0,0), (0, 0.5) and (0,1). In the second part as a continuous function.

Source: Own construction.

The function goes through -1 to 0 and emits three signals in *(0, 0)* in *(0, 0.5)*, *(0, 1)* and extends to *(1, 1)*. In the second case, there are no opposite signals but the function is continuous. [2]In this sense, the HSF is defined as a constant piecewise function. This function is represented by:

$$H(x) \begin{cases} 0 & X < 0 \\ ½ & X = 0 \\ 1 & X > 0 \end{cases} \quad (1)$$

If this is defined as a generalized function θ *(x)*, it can be expressed as:

$$\int \theta(x)\varphi'(x)dx = -\varphi(o) \quad (2)$$

Kanwall (1998) has defined that for φ´(x), the derivative is a smooth function that tends to decay veryquickly. It is usually used as a notation as follows:

$$Hc(x) \equiv H(x-c) \quad (3)$$

This function is also associated with the boxcar, sign function and ramp function. It can be either be defined with the function *boxcar* **BC:**

$$M(x) = H\left(x + \frac{1}{2}\right) - H\left(x - \frac{1}{2}\right) \quad (4)$$

Or it can be defined with the *sign function* **SF:**

$$H(x) = \frac{1}{2}[1 + sgn(x)] \quad (5)$$

Oppenheim & Wilsky (1983), have studied the relation between the mathematical expression and the signals, or signal function. First, the authors explain the importance of the difference between discrete and continuous measures with relation to time. The discrete-time signal is expressed by *x(n),* and with the use of brackets [.] (independent variable). In addition, an interesting example about the behavior of the shares is the COLCAP index, measured daily, where the independent variable (COLCAP Index), is a discrete-time signal used in Colombia for the analysis of the share´s market. Following, we see the COLCAP index in a short period of time:

Returning to the function, the derivative of the function is given by:

$$\frac{d}{dx} H(x) = \delta(x) \quad (6)$$

This involves the delta function, which is represented by δ(x).The function has also been commonlyassociated with the so-called *ramp function* **RP:**

$$R(x) = xH(x) \quad (7)$$

The derivative is expressed:

$$\frac{d}{dx} R(x) = H(x) \quad (7a)$$

Both functions can be combined in the following function:

$$R(x) = H(x) * H(x) \quad (7b)$$

The expression *H(x)** implies a *convolutive* behavior of both functions; the mathematical operator allows the functions to be transformed into another that represents *magnitude* of the superposition of the two initial functions, the second translated and inverted (Hirschman, 1955). Bracewell (2000), regarding the entities of thefunction considers:

$$H(x) * f(x) = \int_{-\infty}^{x} f(x')dx' \quad (8)$$

If we assume that x is time:

$$H(t) * f(t) = \int_{-\infty}^{\infty} H(u)H(t-u)du \quad (8a)$$

$$= H(0) \int_{0}^{\infty} H(t-u)du \quad (8b)$$

$$= H(0)H(t) \int_{0}^{\infty} H(t-u)du \quad (8c)$$

If it develops we have:

$$= H(0)H(t) \int_{0}^{t} H(t-u)du \quad (8d)$$

And it results in:

$$= tH(t) \quad (8e)$$

Thus, once the convolution of these functions is resolved, it can be observed in the function if a linear function *ax+b* is considered:

$$H(ax+b) = H[(x + \tfrac{b}{a})]H(a) + H[-x - \tfrac{b}{a})]H(-a) \quad (9)$$

$$\begin{cases} H[x + \frac{a}{b}] & a > 0 \\ H[-x - \frac{a}{b}] & a < 0 \end{cases} \quad (10)$$

The slope of the function on *a*, and the negative slope on a must be discounted from the negative values of *x*, as long as *a*, is less than 0. The USF, in terms of the limits can be defined by:

$$H(x) = \lim[\tfrac{1}{2} + \tfrac{1}{M} * tan^{-1}(\tfrac{x}{t})] \quad (11)$$

So:

$$= \frac{1}{\sqrt{M}} \lim_{t\_0} \int_{-x}^{\infty} t^{-1} e^{-u^2/t^2} du \quad (11a)$$

$$= \frac{1}{2} \lim_{t\_0} \operatorname{erfc}\left(\frac{-x}{t}\right) \quad (11b)$$

$$= \frac{1}{M} \lim_{t \to 0} \int_{-x}^{\infty} t^{-1} sinc(\frac{u}{t}) du \quad (11c)$$

So that:

$$= \frac{1}{2} + \frac{1}{M} \lim_{t \to 0} sin\ (\frac{M}{t}) \quad (11d)$$

$$= \lim_{t \to 0} \frac{1}{2} e^{x/t} \quad \text{for } x \leq 0 \quad (11e)$$

So that:

$$= \lim_{t \to 0} \frac{1}{2} e^{-x/t} \quad \text{for } x \geq 0 \quad (11f)$$

$$= \lim_{t \to 0} \frac{1}{1+e^{-x/t}} \quad (11g)$$

$$= \lim_{t \to 0} 1 + e^{-e^{-x/t}} \quad (11h)$$

$$= \frac{1}{2} \lim_{t \to 0} [1 + \tanh\ (\frac{x}{t})] \quad (11i)$$

And finally

$$= \lim_{t \to 0} \int_{-x}^{\infty} t^{-1} \wedge [\frac{x - 1/2t}{t}]\, dx \quad (11j)$$

The ERFC function (complementary error function), states that if *(x)* is the integral sine, *sinec (x)* is the cardinal sine function and ^(x) corresponds to the triangular function. As is known, any *monotonic* function that presents horizontal asymptotes that are unequal and constant is a Unitary Scalar Function. If we obtain the *Fourier* transform of this function, we obtain:

$$\mathcal{F}[1 + (x)] = \int_{-x}^{\infty} e^{-2\pi ikx} H(X) dx \quad (12)$$

Below, we have the graphs derived from the *Fourier* transformation:

So that:

$$= \frac{1}{2}[\delta(k) - \frac{i}{\pi k}] \quad (13)$$

Where δ (x), is the Dirac´s Delta Function. Next we will deepen the Dirac Delta Function.

ABOUT DE DDF

George & Imaz (1995), state that DDF can have two approaches, one related to mathematics which considers a distribution and another that considers it an unusual function that is mainly used in Physics and Engineering. Robinson (1966) has called it a non-standard function and its influence on signal theory is very important (Oppenheim, 1983). The function by Khuri (2004), searched the importance of the relation with the density interms of non-central moments. In the article written by Chi & Tam (1999), the authors proposed an efficient method to obtain a distribution of a function with one or more random variables applying the DDF in its generalized presentation. The applications of the DDF, can be show in the voltage in circuits with pressing momentary; dampen harmonic oscillator; at point of charge moving in some volume. Some application have relation with the analysis of the density of charge; integration between solutes and solvents and transmission of a pin-hole on a dark screen (Mathew & Walker, 1980).

The DDF function is according to Gasiorowicz (1974) a very particular type of function; it must disappear when $x \neq y$, and must be infinite when $x\text{-}y = 0$, since the integration range is infinitesimally small. In this way, it is not a function in the usual mathematical sense, but rather a generalized function or a distribution. It does not have a meaning in itself, but it can always be define in the form:

$$\int dx f(x) \delta(x - a) \quad (14)$$

A function *f (t)* can be understood simply as a rule that associates a value of t a value of *f* to each value of *t*;that is, it is a mapping from t to f symbolized by $f\,(t)\colon t \to f\,(t)$. Therefore, to claim that the DDF is not a functionsimply because it cannot be hand-drawn is insufficient. In fact,

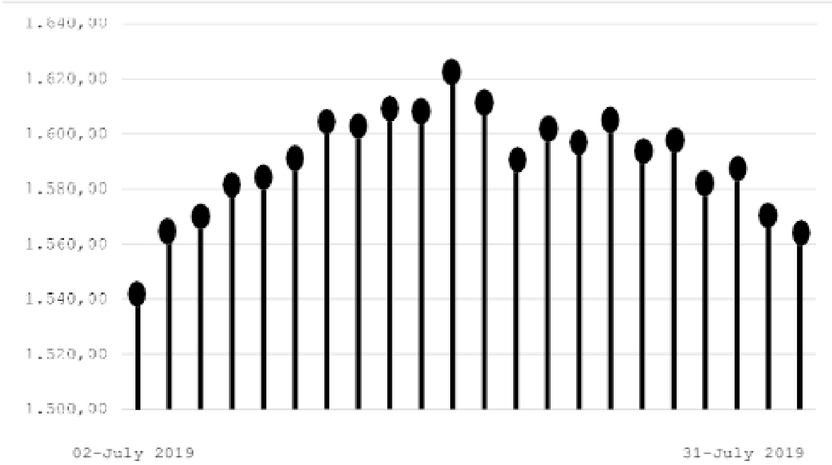

Fig 2. The behavior of the COLCAP Index between 02-July-19 and 31-July 2019. Expressedas a *sign*.

Source: Own construction.

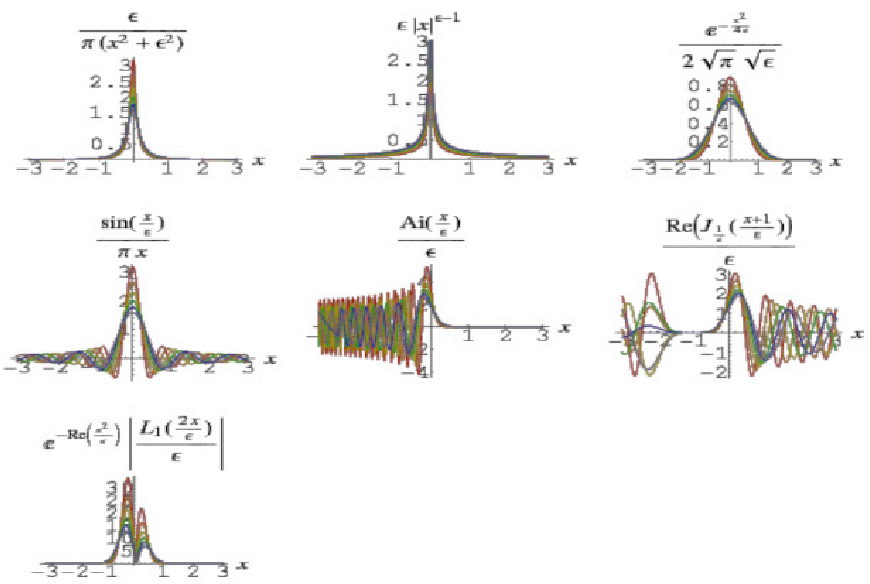

Fig 3. The *Fourier* transformer in a triangular function and the *erfc* function. **Source**: Bracewell (2000).

Bussines Time	Transaction time1	Transaction time2	Theta-time (θ)	Tau-time (τ)
Counts only whena given market isopen, chopping offnights and-week-ends. It is the most used.	Add one for each transaction.	Add the value ofeach transaction.	Add a measure of the seasonal volatility. Contract from or expand on the physical time.	Add some measureof the momentaryrecent volatility.Contract from or expand on thephysical time.

Table 1. Classification of the kinds of time in Economics. **Source:** Own construction based on Zumbach (1997).

it can be said that the DDF belongs to a more general class of functions, called functional or simply distributions. A functional α *(t)* can be understood as theprocess of associating to one arbitrary function β(t) and one number *N*α[β(t)]:

$$\int_{-\infty}^{\infty} \alpha(t)\beta(dt) = N\alpha[\beta(t)] \quad (14a)$$

The DDF is a generalized function that can be defined as the limit of a delta sequence class (Bracewell, 1999). From the normal point of view, the Delta function is associated with a Schwartz[4] space *(S)* or the spaceof all soft functions with compact support *D*, of the test functions. The delta action in *f*, is denoted as δ*[f]* or isalso usually expressed as < δ,*f* >. This function can be seen as a derivation of the unit scalar function:

$$\frac{d}{dx}[H(x)] = \delta(x) \quad (15)$$

The DDF has as its main attribute that the integral of infinity to less infinite is expressed as follow:

$$\int_{-x}^{\infty} f(x)\delta(x-a)dx = f(a) \quad (16)$$

Where it follows:

$$\int_{a+g}^{a-g} f(x)\delta(x+a)dx = f(a) \quad \text{for } \epsilon > 0. \ (16a)$$

Other identities included:

$$\delta(x+a) = 0 \quad (17)$$

For $x \neq a$, so that:

$$\delta(ax) = \frac{1}{|a|}\delta(x) \quad (17a)$$

$$\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x+a) + \delta(x-a)] \quad (17b)$$

In general, the DDF of a function x is given by:

$$\delta[g(x)] = \sum_i \frac{\delta(x-x_i)}{|g'(x_i)|'} \quad (17c)$$

Where corresponds to the set of the roots of g. Let's look at the following example:

$$\delta(x^2 + x - 2) = \delta[(x-1) + \delta(x+2)] \quad (17d)$$

Hence, $g'(x) = 2x+1$, so that $g'(x_1) = g'(1) = 3$ and $g'(x_2) = g'(-2) = -3$.

So,

$$\delta(x^2 + x - 2) = \frac{1}{3}\delta[(x-1) + \frac{1}{3}\delta(x+2)] \quad (17e)$$

The equation that expresses the derivative of this function is:

$$\int f(x)\delta^{(n)}x dx \equiv -\int \frac{\partial f}{\partial x}\delta^{(n-1)}(x)dx \quad (18)$$

If you leave $f(x) = xg(x)$, it results in:

$$\int xg(x)\delta'(x)dx = -\int \delta(x)\frac{\partial}{\partial x}[xg(x)] \quad (18a)$$

So that:

$$= -\int \delta(x)[g(x) + xg'(x)]dx \quad (18b)$$

So:

$$= -\int g(x)\delta(x)dx \quad (18c)$$

In this case the second term can be deleted to obtain:

$$\int xg'(x)\delta(x)dx = 0 \quad (19)$$

Wich involves:

$$x\delta'(x) = -\delta(x) \quad (19a)$$

If a parallel process is followed, it results in:

$$\int [x^n f(x)]\delta^{(n)}(x)dx = (-1)^n \int \frac{\partial^n[x^n f(x)]}{\partial x^n}\delta(x)dx \quad (20)$$

After Dirac, introduces the Bra and Ket vectors, in his Principles of Quantum Mechanics (1930), he refersto the DDF, as a function that satisfies the following conditions:

$$\int_{-x}^{\infty} \delta(x)dx = 1 \quad (21)$$

And it will present a different behavior for

$$\delta(x) = 0 \quad (21a)$$

If you want to have an image of the function, you must consider a function of the real variable *x* that is null and that is outside a domain of amplitude ε around the origin *x* =

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0 and that the inside of that domain is equal a

The exact form of the function internally does not matter as long as it does not have unnecessarily strong variations (for example if the function is of order ε^{-1}). Going to the limit for $\varepsilon \to 0$, the function will tend to be confused with $\delta(x)$.

$\delta(x)$ is not a function of x according to the traditional definition of function, since the definition would require having a defined value for each point in the domain. It is what I would like to define as an improper function, which highlights fundamental differences with ordinary functions. In this way, the DDF is a function that can allow within the mathematical analysis with the same comfort as that of the other functions, as long as non-logical consequences are generated.

THE ECONOMIC TIME

On the other hand, just as Frege (1907), introduced the notion of time in relation to the concept of function, there is a difference between *physical time, economic time* and *psychological time*. In principle, we can recognize that physical time is a space of analysis very different from that posed by other disciplines. Uttal (2008) has made an analysis of *physical* and *psychological* time, recognizing that the *physical phenomenon* has a certain criterion of affinity with the *psychological phenomenon*. For example, in physics, the instruments of observation of the minuscule have not allowed us to approach the internal reality of the atom and elementary particles much more direct manner. However, the mathematical arsenal of quantum mechanics provides a favorable and in-depth theoretical analysis of the reality of the particles. In this way, -the author insists-, there is a contradiction between behavioral currents, mainly of cognitive mentalism, which based on the questions: what happens in the mind?, and what happens with the behavior of elementary particles?, trying to build a fieldof similar analysis that provides an explanatory theoretical *corpus* but with a limitation, in contrast to theobserved reality.

But, to explain this space, that of the minuscule, different theorists have turned to analog models that try to account for the internal constitution of matter. Bohr[5] himself used a *heliocentric* model, with a paradigmatic idea, that gave light to the so-called quantum leap. Although, some locate the origin of quantum mechanics in the study of black body radiation, researched by Planck and which resulted in its famous constant, is not the concept of quantum leap, a fundamental pillar in such development? Why is it that the changes introduced by quantum theory are comparable to those that the *cognitive* revolution brings in the psychological field?.

Whatever the arguments for or against this comparison, it is very likely that the further development of physical science qualitatively demonstrates the hypotheses made by the physics of the twentieth century and the psychology that emerges as the basis of the associated ways of thinking the so-called cognitive mentalism.

In this context, the analysis of time comes from a Greek tradition that can be traced from *Anaximander* in the differentiation between *apeiron* (τὸ ἄπειρον), and *arche (*ἀρχή), that is, the unlimited and the infinite. Time is the indefinitely extensive and that in contrast to the conformation of the *cosmos*; it had neither a beginning nor an end. Other thinkers as *Melissus of Samos*, -follower of Parmenides-, denied the future and considered that what exists is *apeiron* has neither beginning nor end. That concept extends itself even to space - the unlimited applies to extension and duration. The delimitation of the unlimited, through the imposition of the number, which provides regularity, proportion and measure, gives a characteristic of geometric proportion

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to the extension, which is consolidated in the different forms that matter is acquired.

Time, on the other hand, is directly related to the movements of the stars and the celestial bodies that providea measure to it through the daily, nocturnal, monthly and annual cycles. Time ceases to be a succession to constitute a measurable element. In *Timaeus* (Τίμαιος), Plato argues that time is the result of a divine creation and in a later phase the being is located (an ideal model of the cosmos in the future), the becoming (which obeys the disorganized matter) and space, which therefore does not leave a mythical vision of a divine creation of time. Aristotle, on the other hand, introduces the idea of Before and After. The number of the movement with respect to the Before and After, is what for the *stagirite*, represents time, which can only exist if there are conscious beings capable of performing the action of enumeration.

We generally conceive the idea of a clock as something which serves to measure time, but the *Platonic- Aristotelian* idea is itself a type of clock, a criterion for the measurement of time. Time is then the movement or the measure of movement in its regularity and constancy. However, Aristotle considered that time was the sphere of everything, because everything happens in time, in the surrounding sphere (Guthrie, 1964).

Without a doubt the version of time from a physical posture, could be summed up in the idea of its symbioticunion with space. *Space-time* implies a different vision of the traditional perception of time in classical mechanics. A schematic posture no longer persists and the rupture of that traditional vision cannot be analyzed without recognizing the influence of physical thought of the early twentieth century. In relation to *economic time*, Zumbach (1997), performs an interesting analysis of the difference between *physical time* and *economic time*. Based on the classic definitions from dictionaries, it establishes some time scales for the use of financial phenomena. For the author, the scales are:

BEHAVIOR OF THE SHARES (HEAVISIDE AND DELTA)

For the behavior of a share or one set of shares, a situation such as the following occurs:

We assume that a stock has a constant market price over time and at one point, there is a significant increasethat is again sustained over a relatively short period. These increases may be due to external shocks or endogenous situations of listed companies (purchase of assets, incorporation of new technologies, innovation, trust, oligopolistic movements, increase in profits, etc). In this sense, the function that best describes the behavior of the share (whether common or preferred), is the Unit Scalar Function specially the HSF.

However, a share - unlike the case of a behavior similar to that of the Unit Scalar Function - may present a significant increase in its value, and over time, return to its initial state. Generally, the behavior of these shares can be associated with a unit *impulse function*, which, like the Scalar Function, is rare in its practical applications. In addition, similar behavior can be witnessed in short periods of variation in the price of the shares. Dirac's function can be overlapped to a stock impulse event, which can be interpreted as follows (Butkov, 1968; Moore, 1990). If the DDF is expressed as:

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases} \tag{22}$$

So that ∀a, b ∈ ℝ a<0<b:

$$\int_a^b \delta(x)dx = 1 \tag{23}$$

Be *f*: ℝ_ℝ a continuous function, we have:

$$\int_a^b \delta(x)f(x)dx = f(0), \forall a, b \in \mathbb{R}\ a < 0 < b \tag{24}$$

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If we consider an increase in the stock market price at a given time, which we will call $p1$ at a time tn andwe have a base value of the $p0$ share, we can find the following situation:

$$\delta(t) = \begin{cases} p_1 \rightarrow \infty & x = t_n \\ p_0 & x \neq t_n \end{cases} \quad (25)$$

Suppose now the integral between two periods' $t1$ and $t2$. Assume that this integral is equal to 1, asproposed in the development of the DDF function:

$$\int_{t1}^{t2} \delta(x)dx = 1 \quad (26)$$

If we evaluate the integral for a value *a* and *b* and find an *impulse event* in *c* that increases the price up to1/2 *(t)*, we will have to:

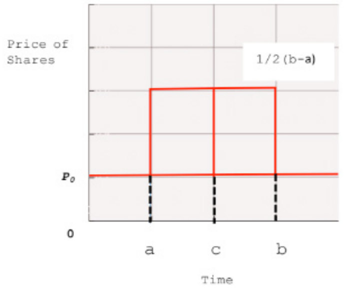

Fig 5. Integral for a value *a* and *b* and an *impulse event.*

Source: Own

CONSTRUCTION.

The extrapolation of these ideas implies changes with respect to the DDF, in relation to the case of the shares.It is based on a base price, which does not necessarily imply that it is located at $x = 0$. On the other hand, the impulse is given in discrete periods of time and the *impulse event* does not happen at $y = 0$. However, the analogy allows analyzing the behavior of the share and its market price in three instant moments. Hence:

$$d(b\text{-}a)t = \begin{cases} ½\,(b\text{-}a) & a < c < b \\ p_o & \text{in other values.} \end{cases} \quad (27)$$

If we assume that *b-a=2*, we will have to meet that:

$$\int_{t1}^{t2} \delta(t)dt = 2(b-a) * \frac{1}{2(b-a)} = 1 \quad (28)$$

So that:

$$= \int_a^b \frac{1}{2(b-a)} = \left(\frac{t}{2}\right)\left(b-a\right)\Big[_a^b \quad (29)$$

$$= \frac{(b-a)}{2(b-a)} + \frac{(b-a)}{2(b-a)} = \frac{(b-a)+(b-a)}{2(b-a)} = \mathbf{1} \quad (29a)$$

To derive the DDF, we need to define it in an interval *(a-β, a+β)*, which joins with a linear stroke:

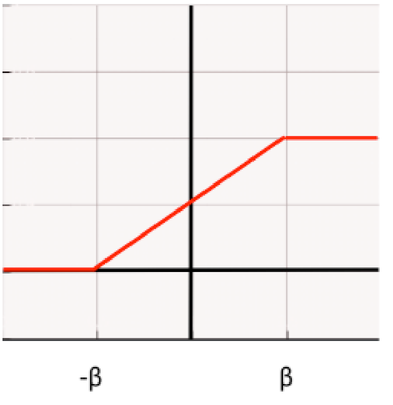

Fig 6. Derivate form the DDF with an interval *(a-β, a+β)*. **Source:** Own

CONSTRUCTION

The derivative of the function will present two jumps and it has a enormous utility in Physics and Engineering. We considered the δ(x) function with two linear traces. So the graphic expression is as follows:

Fig 7. Jumps in the derivate form the DDF.

Source: Own Construction

In the process of derivation δ′(x), we obtain:

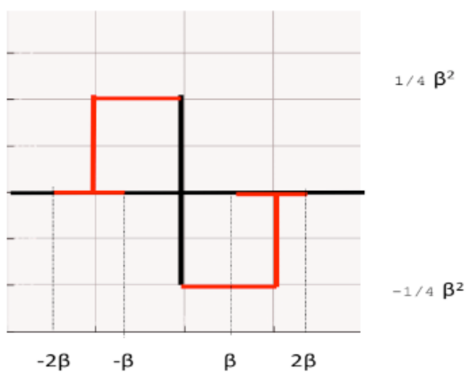

Fig 8. The derivate form of the DDF

Source: Own Construction

In this process, two well-defined areas are observed on the negative and positive side of the plane that havethe same area. In this way, it is shown that the function has a jump or jump in β and another in -*β* and accordingto the substitution rule, it has two linear strokes. For calculations related to the *nth* derivate see George & Imaz(1997).

THE COLCAP INDEX

One of the most recognized indices in ColombiaistheCOLCAPindex,corresponding to the calculation and registration of twenty (20) of the most liquid shares that are traded on the Colombian BVC Stock Exchange. In the index, the market capitalization value determines the weighting to participate in it. The index started on January 15th, 2008 and is a base index of 1,000 points. In the event of the disappearance of shares registered in the COLCAP basket, a rebalancing process must be carried out, with a maximum per company of 20% of the total basket. If this limit is exceeded, the rebalancing procedure must be carried out and the surplus will be distributed among the other participants. Below we observe the behavior of the COLCAP index, between the period of January 2008 to August 2019.

Fig 9. The behavior of the COLCAP Index 2008-2019[6] .

Source: Own construction basedon BVC data.

The annex shows the behavior of the COLCAP index until mid-August of this year. The following equation expresses the calculation of the index:

$$I^k(t) = E\ \Sigma_{i=1}\ w_i^k\ \ Pi(t) \qquad (30)$$

Where:

$I^k(t)$= Index value for the period *(t).*

(t) = Day or instant at which the index is calculated.

k = Term of validity of the COP or weighting for share *i*, fixed for *k*.

E = factor that allows continuity to the index if there is rebalance of the basket or corporate adjustments.

n = number of shares in the index at the

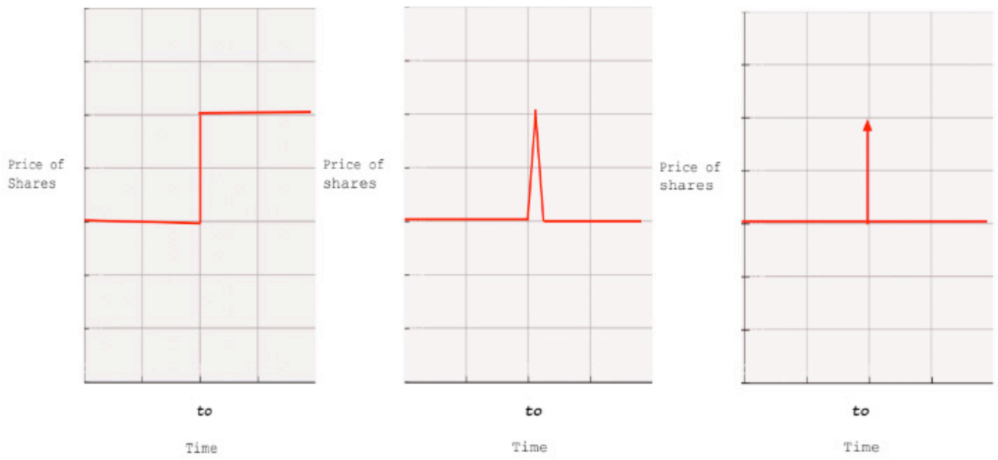

Fig 4. Probability of the behavior of the shares in the short time. Analogical as an *impulse event*.

Source: Own Construction.

Requirement.	Description.
Cash operation.	It refers to the fact that in the period of 90 calendar days prior to thedate of selection of the basket, there is at least one cash transaction
Registration of the share.	The share must be registered 30 calendar days before the effective date of the basket.
Active Condition of the share.	The status of the share must be *Active*, that is, that the share has presented an official quotation during the last 30 calendar days and / or that it presents offers in effect in the trading system.
Shares that generate dividends.	Registered shares must generate dividends
Colombian Global Market.	The group of foreign securities issued outside the country by nationalor foreign issuers will not be taken into account in the COLCAP selection process.
Annual re-composition of thebasket.	This review will include and exclude actions that meet or not meet thenecessary requirements respectively, the baskets will be valid for one year. In the re-composition process, the participation in the *index* of each selected stock for the following quarter is determined. COLCAPwill be recomposed after the market closes on the last business day ofOctober and will be effective between the first business day of November of the same year and the last business day of October of the following year.
Quarterly Weighting.	The index basket is recomposed annually. The weighting of the basketwill be recalculated quarterly with rebalancing. Each share within theindex basket varies daily according to the behavior of the prices of thesame in the secondary market. Thus, in order to avoid excessive participation of an issuer for extended periods, the BVC rebalances theindex basket quarterly. The rebalancing of COLCAP will take place on the last business day of the months of January, April and July of each year.

Table 2. Requirements of the COLCAP Index.

Source: Own construction based on BVC.

time *(t)*.

P = Current closing price of the share i in t.

W_i^k=Weight or weighting for share i, fixed during k.

For the COLCAP basket, the shares must meet the following requirements:

The different behaviors that shares can adopt are reflected in the behavior of the COLCAP index. Thus, when observing some specific years, the significant changes that can be witnessed result in the expression of *impulse events*, which for the specific case of the Colombian index we could group into:

i) Rebalancing: as explained previously, this change corresponds to a process of calibration of the intervening shares, which, as shown in the before table, allows the most important shares of the Colombian stock market to be located therein. *Impulse events* are, in this case, endogenous events.

ii) Changes in the price of underlying assets: this case refers to changes in the market price of the assets that are related to the companies in their sectoral location. In Colombia´s case, some shares have had strong shocks, generally due to the international prices of the products offered. These *impulse events* are generallyexogenous and tend to behave randomly in the short term, but aspects related to the economic cycle can also be witnessed (Hull, 2017).

iii) Irregularity of the *impulse events*: these may arise specifically from the fact that the shares are traded in a free competition market and that their market prices is subject to the daily movement in the stock quote wheel and that may be affected by the economic agents that can act through expectations, not always in a rational way. This is an intrinsic and endogenous aspect of the process itself, in the logic of the stock market. Analysis made from the observation of the random walk and the Brownian motion are relevant aspects of the relationship between Economics and Physics that have formally accounted for this aspect (Sinha *et. al,* 2001).

iv) Rational consumer behavior: in this case, the individual who buys shares in the stock market will, in principle, seek to satisfy the axiom of profit maximization and cost minimization. However, this behavior is only considered linked to the consumer and is not considered extensive to the corporations and listed companies (criticisms of this approach can be seen in Tversky & Kahneman, (1974, 1979, 1981). On the other hand, the influence that may have on the consumer, buying trends that generate momentum events should not be dismissed. *X*, can act irrationally, buying shares *a*, without a duly justified cause and this share will generate a chain reaction and in a sense a *wagon effect* in the stock market.

METHODOLOGY

First, we obtain the *standard deviation* per year form the data of COLCAP index. The next step, is analyze the situation of the behavior of the index in periods of reference, for example, we have four years with wide dispersion. However, the different functions studied, must show a similar behavior as with the DDF. It is possible that the behavior may be very similar, but it is important to recognize, that the analogy proposed is the result of the observation in different graphs. However, the demonstration is related to the impulse function or DDF. In the years observed, seven have a positive *slope* in the behavior of the index. We can perceived a relative

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improvement in the understanding of the importance of deepening of the stock market. Then, we analyze the specific moments of possible impulse events in the years of greatest dispersion.

RESULTS

In varying years, impulse moments affected by various processes are observed. However these processes happenin a limited time and measured discreetly, usually in daily periods. The behavior of the index shows significant variations in the years 2009, 2010, 2014 and 2015, while in the other years, there are less dispersed behaviors. Therefore, it can be deduced that in these years referenced in principle, there is a greater probability of occurrence of impulse events than in the other years. This postulate can generate debate, however, we will concentrate on these four years, to evaluate the influence of impulse events in the index and determine the existence or not of similar behaviors.

The next table shows the standard deviation:

Year	Standard Deviation
2008	75.36319
2009	187.4582
2010	182.3206
2011	71.36613
2012	60.91
2013	81.72
2014	91.26476
2015	92.02925
2016	68.59593
2017	56.1457
2018	65.88096
2019	68.23215

Table 3. Standard deviation COLCAP index 2008-2019.

Source: Own constructionbased on BVC.

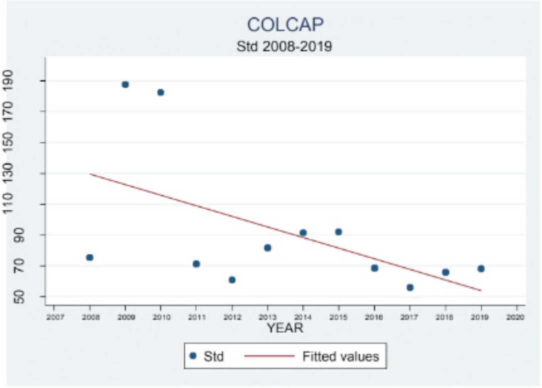
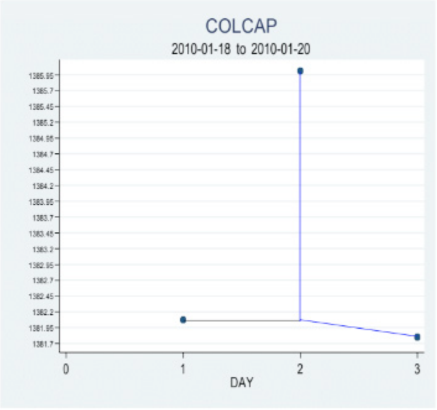


Fig 10. Standard deviation COLCAP index 2008-2019.

Source: Own construction based on BVC.

For the year 2009, an index behavior is found, which can be seen in the following figure in the months of August and in the month of December. In August, the days 27th, 28th and 31th are observed with an index value of 1210.09, 1214.38 and 1209.09 respectively. For the month of December the behavior of the actions of days 23th, 24th and 28th has been taken. The values are 1364.13, 1372.72 and 1365.06.

For the year 2010, we have as reference dates from January 18th to January 20th. The values for these days are 1382.07, 1386.01 and 1381.80 respectively. In this same year, another change is observed between February 1th and 3th, with the following values 1358.49, 1363.85 and 1359.43. Finally, there is a significant change between March 24th and 26th with the following values respectively.



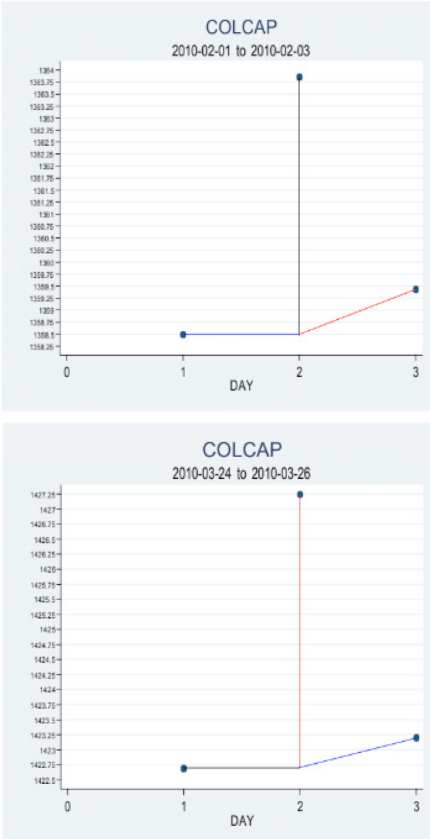


Fig 12. Behavior of COLCAP index 2010.

Source: Own construction based on BVC

For the year 2014, we have as reference dates from February 21th to February 25th. The values for these daysare 1520.60, 1535.11 and 1501.40 respectively. In this same year, another change is observed between April 15th and 21th, with the following values 1669.24, 1677.73 and 1668.85 respectively.

For the year 2015, we have as reference dates from February 13th to February 17th. The values for these days are 1397.81, 1398.97 and 1397.20 respectively. In this same year, another change is observed between February 20th to 24th, with the following values 1368.85, 1375.86 and 1365.36 respectively.

CONCLUSIONS

It is possible to analyze the behavior of the shares of different companies participating in the stock market, through different quantitative tools and methods that can be provided by other disciplines or sources, such as physics.Although, the relationship between Economics and Physics is the relationship between a daughter discipline of modernity and another born in Greek antiquity, it allows, however, a fruitful dialogue and the exchange of analyticalinstruments (Vera, 2018).

The measurement of a stock index such as COLCAP, in its discrete expression, can be analyzed from the analogy as a variable very similar to the variables studied in physics present in *impulse events*. An impulse event, analyzed as a discrete variable over time, allows the daily change of the share to be seen as an expression of this impulse. In most cases and from a vocation that we could call *Macro Time Series Analysis*, there is a description more consistent with the idea of a continuous variable, expressed by a certain function. But, unlike many physical variables, whose behavior, far from being random, move through certain patterns likewise to the actions in the stock market and, in fact, they reflect behaviors of excessive dispersion.

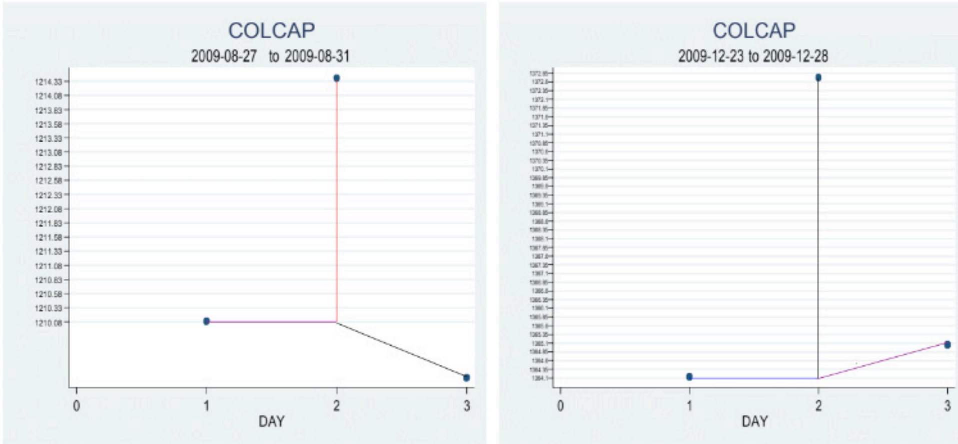


Fig 11. Behavior of COLCAP index 2009. **Source:** Own construction based on BVC.

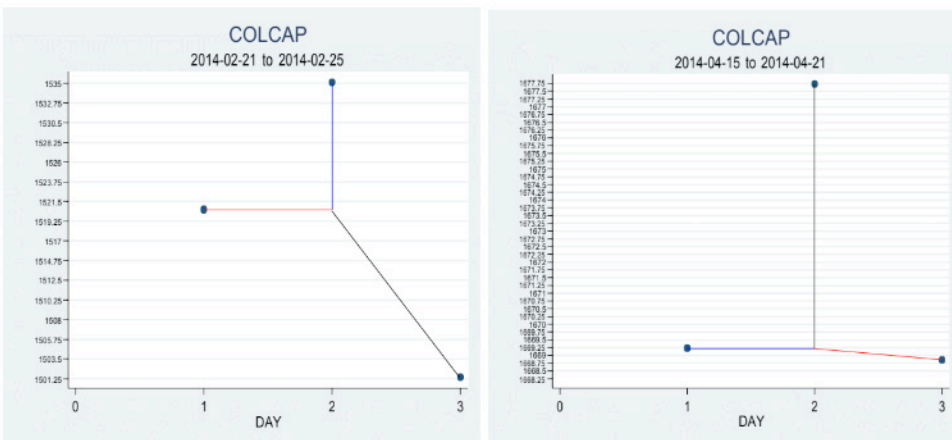


Fig 13. Behavior of COLCAP index 2014.

Source: Own construction based on BVC

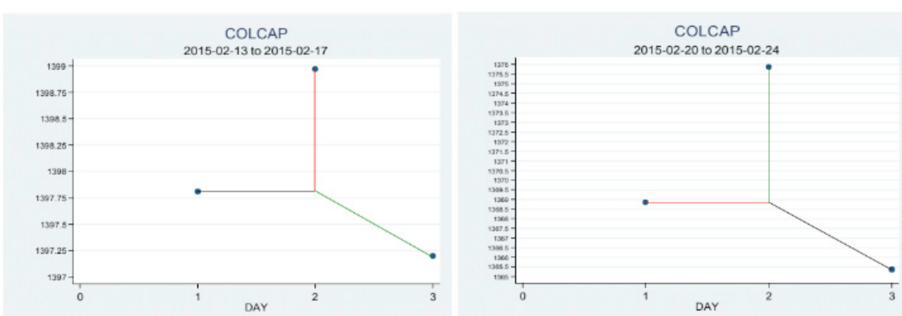


Fig 14. Behavior of COLCAP index 2015.

Source: Own construction based on BVC

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ANNEX 1

Year 2008.

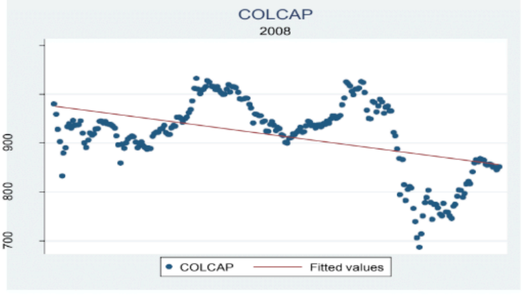
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	236	915.8525	75.36319	686.65	1032.08

. regress COLCAP DAY

Source	SS	df	MS
Model	283505.471	1	283505.471
Residual	1051202.96	234	4492.3203
Total	1334708.43	235	5679.6103

Number of obs = 236
F(1, 234) = 63.11
Prob > F = 0.0000
R-squared = 0.2124
Adj R-squared = 0.2090
Root MSE = 67.025

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
DAY	-.5087533	.0640416	-7.94	0.000	-.634925	-.382581
_cons	976.1397	8.753687	111.51	0.000	958.8936	993.385



Year 2009

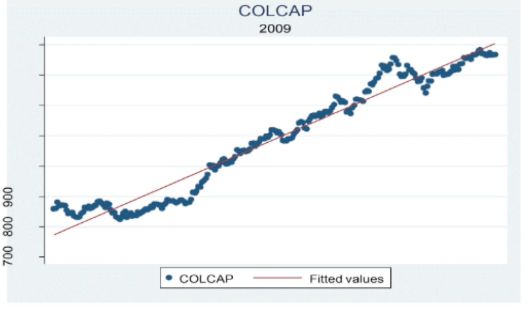
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	242	1089.132	187.4582	825.17	1382.63

. regress COLCAP DAY

Source	SS	df	MS
Model	8083496.56	1	8083496.5
Residual	385382.528	240	1605.7605
Total	8468879.09	241	35140.5771

Number of obs = 242
F(1, 240) = 5034.06
Prob > F = 0.0000
R-squared = 0.9545
Adj R-squared = 0.9543
Root MSE = 40.072

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
DAY	2.6162	.0368733	70.95	0.000	2.543563	2.68883
_cons	771.2637	5.167852	149.24	0.000	761.0835	781.443



Year 2010

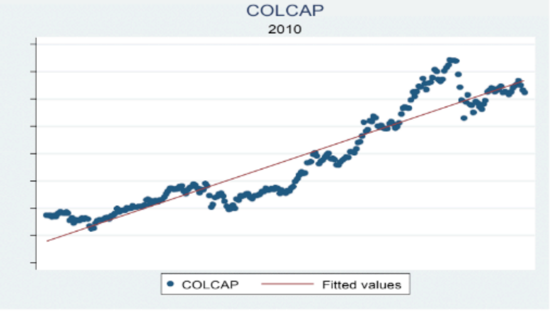
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	245	1572.887	182.3206	1326.08	1942.37

. regress COLCAP DAY

Source	SS	df	MS
Model	7124992.49	1	7124992.4
Residual	985758.7	243	4056.6201
Total	8110751.19	244	33240.783

Number of obs = 245
F(1, 243) = 1756.39
Prob > F = 0.0000
R-squared = 0.8785
Adj R-squared = 0.8780
Root MSE = 63.692

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
DAY	2.411223	.0575344	41.91	0.000	2.297893	2.52455
_cons	1276.307	8.163191	156.35	0.000	1260.227	1292.38



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Year 2011

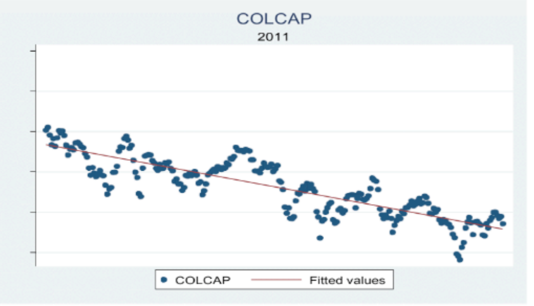
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	246	1663.034	71.39613	1482.59	1810.27

. regress COLCAP DAY

Source	SS	df	MS
Model	903127.909	1	903127.909
Residual	345736.855	244	1416.95433
Total	1248864.76	245	5097.4072

Number of obs	= 246
F(1, 244)	= 637.37
Prob > F	= 0.0000
R-squared	= 0.7232
Adj R-squared	= 0.7220
Root MSE	= 37.642

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	-.8532301	.0337963	-25.25	0.000	-.9197999	-.7866604
_cons	1768.408	4.814662	367.30	0.000	1758.924	1777.892



Year 2012

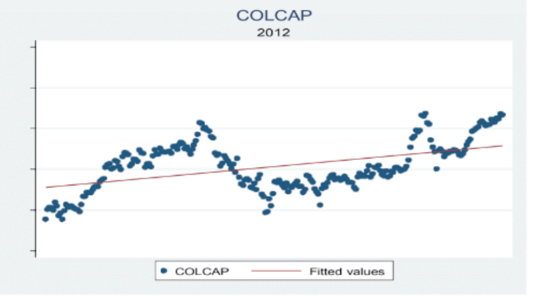
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	244	1706.615	60.9152	1577.71	1835.82

. regress COLCAP DAY

Source	SS	df	MS
Model	212426.844	1	212426.844
Residual	689264.003	242	2848.19836
Total	901690.848	243	3710.66192

Number of obs	= 244
F(1, 242)	= 74.58
Prob > F	= 0.0000
R-squared	= 0.2356
Adj R-squared	= 0.2324
Root MSE	= 53.369

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	.4189037	.0485059	8.64	0.000	.3233561	.5144514
_cons	1655.299	6.854196	241.50	0.000	1641.798	1668.801



Year 2013

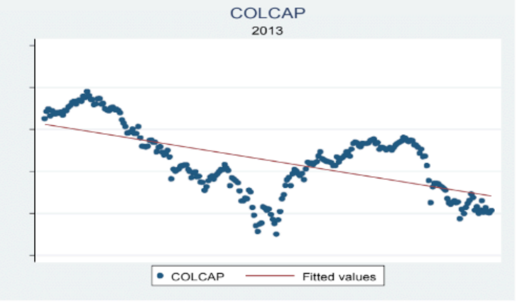
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	244	1727	81.72673	1550.7	1889.3

. regress COLCAP DAY

Source	SS	df	MS
Model	594725.195	1	594725.195
Residual	1028334.41	242	4249.31573
Total	1623059.6	243	6679.25762

Number of obs	= 244
F(1, 242)	= 139.96
Prob > F	= 0.0000
R-squared	= 0.3664
Adj R-squared	= 0.3638
Root MSE	= 65.187

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	-.7009188	.0592474	-11.83	0.000	-.8176251	-.5842124
_cons	1812.863	8.372032	216.54	0.000	1796.371	1829.354



Year 2014

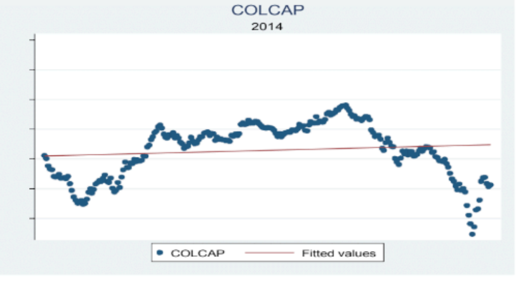
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	244	1628.697	91.26476	1348.38	1780.25

. regress DAY COLCAP

Source	SS	df	MS
Model	17898.834	1	17898.834
Residual	1192646.17	242	4928.28994
Total	1210545	243	4981.66667

Number of obs	= 244
F(1, 242)	= 3.63
Prob > F	= 0.0579
R-squared	= 0.0148
Adj R-squared	= 0.0107
Root MSE	= 70.202

DAY	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
COLCAP	.0940386	.0493449	1.91	0.058	-.0031616	.1912388
_cons	-30.66037	80.49338	-0.38	0.704	-189.2174	127.8967



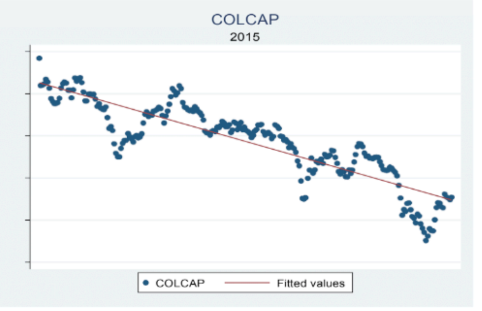
Year 2015

Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	242	1286.869	92.02925	1051.25	1483.89

. regress COLCAP DAY

Source	SS	df	MS		
Model	1576167.04	1	1576167.04	Number of obs =	242
Residual	464954.227	240	1937.30928	F(1, 240) =	813.59
Total	2041121.27	241	8469.38287	Prob > F =	0.0000
				R-squared =	0.7722
				Adj R-squared =	0.7713
				Root MSE =	44.015

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	-1.15524	.0405015	-28.52	0.000	-1.235024	-1.075457
_cons	1427.23	5.67635	251.43	0.000	1416.049	1438.412



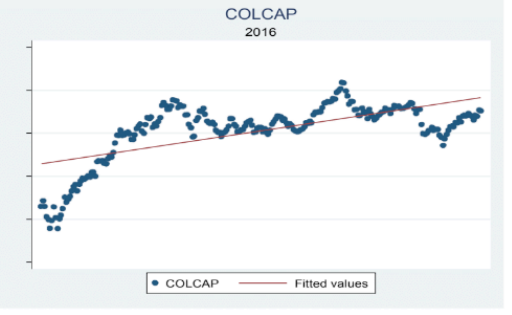
Year 2016

Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	245	1305.949	68.59593	1078.69	1417.57

. regress COLCAP DAY

Source	SS	df	MS		
Model	487957.81	1	487957.81	Number of obs =	245
Residual	660160.163	243	2716.7084	F(1, 243) =	179.61
Total	1148117.97	244	4705.4015	Prob > F =	0.0000
				R-squared =	0.4250
				Adj R-squared =	0.4226
				Root MSE =	52.122

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	.63101	.0470833	13.40	0.000	.5382666	.723753
_cons	1228.335	6.680351	183.87	0.000	1215.176	1241.49



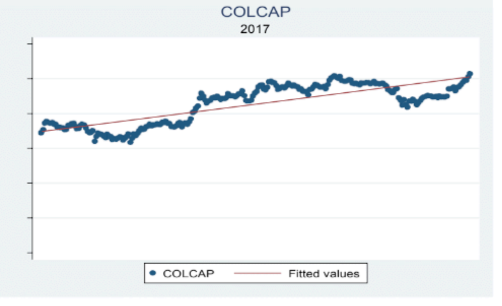
Year 2017

Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	242	1426.846	56.1457	1317.98	1513.65

. regress COLCAP DAY

Source	SS	df	MS		
Model	500904.323	1	500904.32	Number of obs =	242
Residual	258809.435	240	1078.3726	F(1, 240) =	464.50
Total	759713.759	241	3152.3392	Prob > F =	0.0000
				R-squared =	0.6593
				Adj R-squared =	0.6579
				Root MSE =	32.839

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	.6512514	.0302173	21.55	0.000	.5917264	.7107764
_cons	1347.719	4.235007	318.23	0.000	1339.376	1356.061



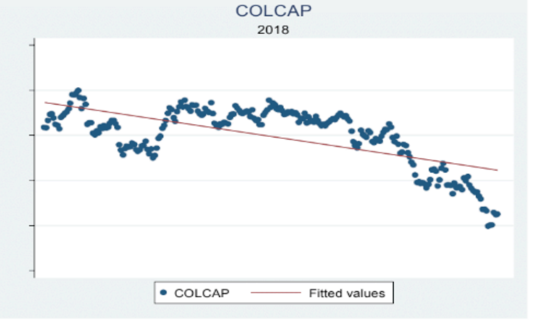
Year 2018

Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	243	1498.023	65.88096	1298.37	1598.4

. regress COLCAP DAY

Source	SS	df	MS		
Model	460327.485	1	460327.485	Number of obs =	243
Residual	590025.31	241	2448.2378	F(1, 241) =	188.02
Total	1050352.8	242	4340.30081	Prob > F =	0.0000
				R-squared =	0.4383
				Adj R-squared =	0.4359
				Root MSE =	49.48

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	-.6204665	.0452493	-13.71	0.000	-.7096011	-.531331
_cons	1573.72	6.367889	247.13	0.000	1561.176	1586.26



Year 2019

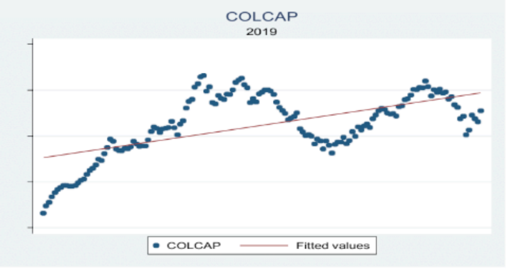
Variable	Obs	Mean	Std. Dev.	Min	Max
COLCAP	151	1523.816	68.23215	1332.8	1631.3

. regress COLCAP DAY

Source	SS	df	MS
Model	253770.013	1	253770.013
Residual	444573.856	149	2983.71715
Total	698343.868	150	4655.62579

Number of obs =	151
F(1, 149) =	85.05
Prob > F =	0.0000
R-squared =	0.3634
Adj R-squared =	0.3591
Root MSE =	54.623

COLCAP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
DAY	.9404915	.1019797	9.22	0.000	.7389784	1.142005
_cons	1452.338	8.934722	162.55	0.000	1434.683	1469.993



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