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MIXED AND NATURAL CONVECTION HEAT TRANSFER IN A TWO-DIMENSIONAL SQUARE CAVITY

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Abstract: In this work the numerical solution of three cases of flow in a two-dimensional square cavity filled with air is presented. Case 1 corresponds to the problem of mixed flow with the upper wall at a constant speed and the other three walls fixed, the temperature of the moving wall is constant and greater than the lower wall, the other two vertical walls remain thermally insulated; Case 2 is similar to Case 1, but considering the four walls are fixed and the bottom wall is the hot one, and in Case 3 all 4 walls are fixed, one vertical wall is kept at a constant higher temperature than the opposite vertical wall , and the other two horizontal walls are thermally insulated. The solution of the equations of conservation of mass, momentum and energy is done with the free software OpenFOAM. The results are compared with those reported in the bibliography, obtaining satisfactory results.
Keywords: Mixed, natural, openFOAM convection.

INTRODUCTION

In nature, at home or in industry, energy transfer is always present. In this sense, we can mention the multiple applications in which solar radiation is used, for example, water heating for domestic use, photovoltaic cells, etc. In electrical and electronic equipment, the increase in temperature can sometimes be counterproductive. The movement of air naturally outside or inside a building can be used to distribute the temperature and obtain thermal comfort conditions. Because the phenomenon of energy transfer in the form of heat is involved in multiple situations, it is concluded that its study and understanding is of the utmost importance. Currently there are many works related to this subject in theoretical and experimental form, but numerical simulation is becoming more interesting every day, in the sense that it has become a practical tool when designing a certain system or expanding experimental designs to other ranges or situations. Natural convection is one of the most studied heat transfer phenomena [1-5], for example, in most systems that use solar energy, convection is always present, such is the case of flat solar collectors, photovoltaic collectors, solar chimneys, Trombe walls, to name a few applications. It is worth mentioning that these applications can be modeled considering rectangular geometries.

De Vahl Davis and Jones [6] in 1983 presented results for the problem of natural convection in a square cavity in laminar flow regime for Rayleigh number up to $1x10^6$, the two horizontal walls were considered adiabatic (q=0) and the others two isothermal verticals (TC and TF), in this comparison exercise, results were reported, considered benchmarks, for maximum and minimum velocity components, as well as mean values of Nusselt number on the hot wall (TC). The work of De Vahl Davis and Jones is widely cited, among the most recent citations in 2020 are the works of Rani H. P., et al., [7] and Ferialdi H., et al., [8]; in these works the authors use the results of De Vahl Davis and Jones to verify their numerical solutions.

It is common to find engineering applications where heating occurs at the bottom surface, eg flat solar collectors, solar distillation, stove ovens, to name a few. Natural convection with heating from below is known as Rayleigh-Bénard convection, since they were the first to study this phenomenon in the 1900s [9 and 10]. Ouertatani, et al., 2008 [11] presented a numerical study of natural convection in a square cavity heated on the lower wall and cooled on the upper wall, the two vertical walls were considered adiabatic; the authors used the finite volume method for the discretization of the conservation equations, and the multigrid technique to accelerate the convergence of the iterative solution process of the discrete equation systems. The authors present results for the Rayleigh number interval from $1x10^3$ to $1x10^6$, corresponding to a laminar flow regime, they report benchmark results for the maximum velocity components and average Nusselt number on the horizontal walls. In their work Himrane, et al., [12] numerically analyze the heating of a room on its lower surface with a periodic sinusoidal variation of the heating temperature, the conservation equations are solved with the lattice-Boltzmann method, the authors use the benchmark results [11], to verify their solution methodology. Bouraoui and Nejma [13] were other authors who used the benchmark results [11] to validate their convective model of olive waste drying to obtain natural fertilizer.

Iwatsu, et al., [14] in 1993 presented a numerical study considering mixed convection in a square cavity filled with air, the authors considered a study for Reynolds number of 100, 400 and 1000 and Grashof number of 100, 10000 and 1,000,000, the upper wall of the cavity was considered to be at constant velocity and at a uniform temperature higher than the temperature of the lower wall, the other two horizontal walls were considered adiabatic; the authors present results in graphs for temperature and streamlines and values of the average Nusselt number of the upper wall. Sharif [15] in 2007 extended the study carried out by Iwatsu, et al., when carrying out a study considering the effect of the inclination of the cavity with angles of 0, 10, 20 and 30º, the author used the FLUENT software for the solution. of the conservation equations, to verify his solution model, the author compares the results of the average Nusselt number reported by Iwatsu, obtaining percentage differences of less than 6%.

From all of the above, it can be concluded that the study and understanding of the phenomenon of heat transfer by natural and mixed convection in rectangular cavities is a prior process when seeking to obtain simulations of more complex cases, such as geometries in non-rectangular coordinates. or in combination with other mechanisms of heat transfer.

DESCRIPTION OF THE METHOD

DEFINITION OF THE PROBLEM TO BE SOLVED

Figure 1 shows the physical and geometric characteristics of the problem to be solved, it consists of a square cavity of length H, filled with air initially at rest. The boundary conditions (C.F) are numbered clockwise, starting at the left vertical wall, the boundary conditions for all three cases are no-slip boundary condition for velocity, i.e., the velocity of the fluid. is the same as the velocity of the adjacent wall and for the temperature the boundary condition corresponds as presented in Table 1. The adiabatic boundary condition corresponds to q=*0*, and a wall heated uniformly at constant temperature T_C y T_F if it corresponds to a cold wall.

	Case 1[14]			Case 2 [11]			Case 3 [6]		
Boundary condition	***u***	***v***	***T***	***u***	***v***	***T***	***u***	***v***	***T***
C. F. 1	*0*	*0*	*q=0*	*0*	*0*	*q=0*	*0*	*0*	T_C
C. F. 2	*U*	*0*	T_C	*0*	*0*	T_F	*0*	*0*	*q=0*
C. F. 3	*0*	*0*	*q=0*	*0*	*0*	*q=0*	*0*	*0*	T_F
C. F. 4	*0*	0	T_F	0	0	T_C	0	0	q=0

Table 1. Boundary conditions.

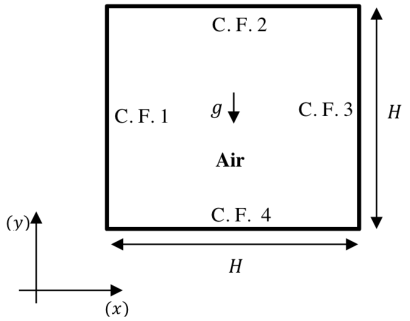

Figure 1. Square cavity filled with air.

GOVERNING EQUATIONS

It is considered that the air will move in a laminar flow regime in a steady state, viscous dissipation is neglected and the Boussinesq approximation will be valid in the density change in the buoyant force, the two-dimensional conservation equations for mass, momentum and energy are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_o) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

Where u and v are the velocity components in the x and y directions respectively, p is the air pressure inside the cavity and T its absolute temperature. g is the acceleration due to gravity. At a reference temperature T_ o=300 K the properties of the air correspond according to Incropera [16]: p = 1.1614 kg/m^3, v = 15.89x10^{-6} m^2/s, Pr = 0.71 and β = 0.333 (1/K)

For cases of natural convection, the characteristic dimensionless number is the Grashof number, defined as:

$$Gr=\frac{g\beta(T-T_o)H^3}{\nu^2} \quad (5)$$

Another characteristic dimensionless number is the Rayleigh number, defined as $Ra = GrPr$.

In mixed convection phenomena the characteristic dimensionless number is the Richardson number defined as:

$$Ri=\frac{Gr}{Re^2} \quad (6)$$

where the Reynolds number is defined as $Re=\frac{UH}{\nu}$, U is the velocity of the sliding wall (case 1), H is the length of the cavity, and ν is the kinematic viscosity of the air.

SOLUTION

The governing equations are solved with the free software openFOAM [17]. The solution starts by creating a discrete domain with Nx and Ny control volumes, as shown in figure 2, this is done with the blockMesh library. This computational domain is called solution space mesh, in each control volume the governing equations are solved. The control volumes are smaller in the vicinity of the walls, for the purpose of a correct solution of the hydrodynamic and thermal boundary layer.

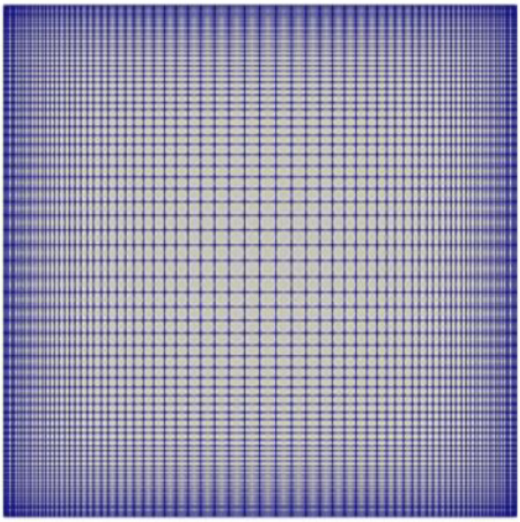

Figure 2. Non-uniform mesh generated with *blockMesh.*

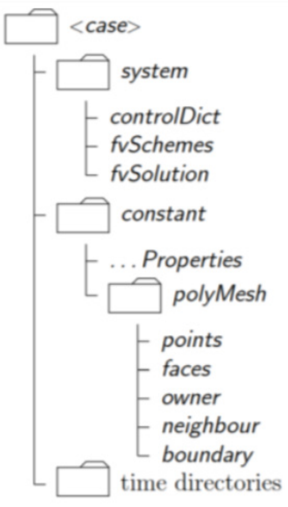

Figure 3. Necessary subfolders for the solution of a case with OpenFOAM.

In OpenFOAM the solution of a case (case) or problem is always necessary in three subfolders, as shown in figure 3; two subfolders *constant* y *time directories* they correspond to the pre-processing process, and in the system subfolder the parameters for the process of the numerical solution of the conservation equations are defined. in the same folder *time directories* the solution is stored for later post-processing. The suitable solver for the defined cases is the buoyantSimpleFoam, which allows the solution of natural, forced and mixed convection problems for compressible and incompressible flows.

RESULTS

NUSSELT NUMBER

The Nusselt number is an important dimensionless parameter in convective heat transfer, it is defined as:

$$Nu = \frac{q_{conv}}{q_{ref}}, \quad q_{conv} = -k\frac{dT}{d\eta} \quad y \; q_{ref} = k\,\frac{T_C - T_F}{H} \quad (7)$$

The average of the Nusselt number is obtained from:

$$\overline{Nu} = \frac{1}{H}\int_0^H Nu\, d\eta \quad (8)$$

where $\eta = x$ or y, depending on the case to solve.

CASE 1: MIXED CONVECTION IN A SQUARE CAVITY.

Table 2 compares the results of this work with those reported by Iwatsu, et al., [14] for the mean values of the Nusselt number on the upper wall, which is in motion. The maximum difference of 5.7% is found when Re= 400 and Gr=10^6.

CASE 2: CAVITY HEATED FROM THE BOTTOM.

For this case, the average values of the Nusselt number on the lower wall are compared. The openFOAM solution of this work has a maximum percentage difference of 6.6% for Ra=10^4, compared to the value reported by Ouertatani, et al., [11], as shown in Table 3.

Gr	10^2		10^4		10^6	
Re	Present work	Iwatsu [14]	Present work	Iwatsu [14]	Present work	Iwatsu [14]
100	2.01	1.94	1.37	1.34	1.00	1.02
400	4.03	3.84	3.79	3.84	1.15	1.22
1000	6.52	6.33	6.47	6.33	1.77	1.77

Table 2. Comparison of case 1 with the results of Iwatsu, et al., [14]. Average Nusselt values.

		Ra			
Values		**10^3**	**10^4**	**10^5**	**10^6**
Nu	Present work	1.011	2.015	3.901	6.283
	[11]	1.000	2.158	3.910	6.309

Table 3. Comparison of case 2 with the benchmark results [11]. Average Nusselt values.

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CASE 3: DIFFERENTIALLY HEATED CAVITY.

Table 4 presents the results obtained for the different Rayleigh numbers, for comparison purposes the benchmark values reported by De Vahl Davis et al., [6] are included, the maximum percentage difference is 5.5% for the x coordinate. where the vertical component of maximum velocity is presented with $Ra=10^6$.

Values		Ra 10^3	Ra 10^4	Ra 10^5	Ra 10^6
Nu	Present work	1.113	2.239	4.519	8.824
	[6]	1.118	2.243	4.519	8.800
u_{max}	Present work	3.655	16.240	35.209	65.234
	[6]	3.649	16.178	34.730	64.630
y	Present work	0.810	0.820	0.850	0.850
	[6]	0.813	0.823	0.855	0.850
v_{max}	Present work	3.689	19.626	68.606	219.865
	[6]	3.697	19.617	68.590	219.360
x	Present work	0.180	0.120	0.070	0.040
	[6]	0.178	0.119	0.066	0.0379

Table 4. Comparison of case 3 with the benchmark results [6].

CONCLUSIONS

The momentum mass and energy conservation equations were presented, in two dimensions and considering the Boussinesq approximation, the boundary conditions correspond to three cases reported in the literature, considered benchmarks, for purposes of verifying a correct solution of the equations. The cases correspond to a square cavity filled with air. The first case corresponds to a problem of mixed convection and two of natural convection. The equations are solved with the free software openFOAM, using the buoyantSimpleFoam library, which is useful for convective heat transfer problems with compressible and incompressible flow. For the three cases proposed, satisfactory results are obtained, for case 1 of mixed convection the maximum absolute percentage difference is 5.7%, in case 2 of natural convection heated by the lower wall and cooled on the upper wall the maximum difference is 5.5%, finally, for the case of heated and cooled natural convection in the vertical walls is 6.6%.

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