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EVALUATION OF THE MECHANICAL BEHAVIOR OF THE HIP JOINT AFFECTED BY BONE DECALCIFICATION USING FRACTAL METHODS

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Abstract: The hip joints of patients with osteopenia and osteoporosis show a complex mechanical behavior resulting from the decalcification of bone tissue, which modifies its internal structure and properties. The joints are subjected to stress because together with the musculature that surrounds them, they support the weight of the body in static and dynamic postures with fatigue. Under load conditions, the biomechanical failure of the joints is the product of the forces that act when they exceed their capacity for balance while they wear out. The mechanical behavior of the joints is important because they belong to the trunk and relate the coxal bone to the left or right femur during work activities. Then, through fractal methods, the results of bone densitometry were evaluated. By self-similar traces in Box-Dimension and in Mass-Dimension, the fractal dimension of the joints was determined to evaluate the irregularities and wear of the same. Likewise, it describes the variation of bone mass in the time that passes from the normal condition to osteoporosis.
Keywords: Fractal dimension, fractal behavior, wear, hip joints, mechanical behavior and bone tissues.

INTRODUCTION

In the study of the mechanical behavior of hip joints, it is important to take into consideration the fundamentals of sciences related to fractal geometry, such as fracture mechanics and materials mechanics. Therefore, the mechanics of materials deals with the study of the mechanical behavior of materials under load conditions. Fracture mechanics deals with the analysis of cracks or other imperfections (pores, inclusions, microcracks), their effect as stress risers. And it is known as fracture toughness as the limit load capacity that a structure can support before failure [1].

Among the processes that lead to failures is the accumulation of fatigue due to the loads to which a structure or the skeletal system of the human body is exposed [2], [3]. The damage is sometimes seen concentrating in regions of systems in which there is relative movement between two or more elements, forming tribological systems that naturally work with high precision and efficiency, providing the human body with an articulated structure [4], [5], [6]; In addition, they are affected by diseases such as osteoarthritis, osteoporosis and rheumatoid arthritis that affect the way in which friction phenomena are carried out and the efforts that it causes in these areas. lubrication and its effect on the mobility of the system as a whole; and wear, the speed with which it occurs, the resistance offered by the components to it and the consequent damage it causes in these systems [6].

In bone tissue, fatigue damage, in the form of microcracks, has been related to phenomena such as adaptation to altered stress environments. This implies that the damage that occurs during normal physiological loading is severe enough to cause failure if not repaired. Therefore, the contribution of fatigue to osteoporotic fracture is complex [4], [5]. By stimulating bone remodeling, fatigue promotes visible microarchitectural deterioration of cortical and trabecular tissue1 which has its most significant effect on hip joints (Figure 1) [7].

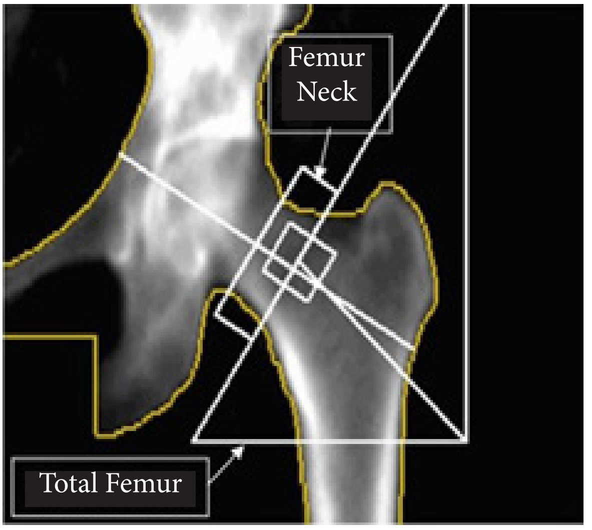

Figure 1. Analysis of bone mass in the left femur. Dual-energy X-ray absorptiometry image of the left hip. Source: Lorente Ramos et al., 2012.

Experimental data support the hypothesis that an important factor in the study of the relationship between the fracture and the geometric structure of the propagation surface is the fractal nature of the fractures, that is, self-similar characteristics of the crack surfaces at different scales. of observation [8], an analysis that provides the necessary parameters for the prediction of a future propagation and therefore, the ways to avoid it or, failing that, to be prepared from the moment a person begins to present decalcification of their bone tissues until reaching the risk of severe osteoporosis [16].

Therefore, the purpose of this research is to study and evaluate the mechanical behavior of hip joints that present pathological conditions of bone decalcification using fractal geometry methods, since current medical imaging methods are tools that allow obtaining results. qualitative and quantitative measures of tissue density, but they are not predictive nor do they describe or generate the complex form of damage to estimate its magnitude. In addition, traditional methods are based on exposing patients to radiological effects every time the pathological condition is monitored, and even more so, these methods do not allow the precise amount of calcium to be supplied that can strengthen or homogenize the affected tissues from the point of view of the disease. of study of mathematics or from the foundations of Euclidean Geometry [9].

What is complex is that the hip joint is a type of ball-and-socket synovial joint that connects the pelvic girdle to the lower extremity. At this joint, the head of the femur articulates with the acetabulum of the coxal bone. Therefore, the mechanical behavior of this joint is very complex because it is a multiaxial joint that allows a wide range of movements: flexion, extension, abduction, adduction, external rotation, internal rotation, and circumduction. Likewise, this joint sacrifices mobility for stability and weight support [5]. [6]. The total weight of the upper part of the body is transmitted through this joint to the lower limbs during standing. This joint is the most stable in the human body. Consequently, with the fractal dimension of the wear surfaces of the hip joints, the degree of irregularity and fragmentation of the bone tissues will be quantified, and it will also describe the variation of bone mass through the time that a patient passes from the normal condition. diagnosed with decalcification until acquiring osteopenia or osteoporosis.

MATERIALS AND METHODS

MATERIALS

The materials used in this research are the images and numerical data obtained in the study by bone densitometry in female patients in the country. Similarly, to carry out this study, the GE Lunar Prodigy Advance DXA system manufactured by GE Medical Systems LUNAR [13] was used.

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METHODOLOGY FOR THE STUDY

Methodology for the analysis of medical images by bone densitometry (DXA)

Initially, a population of 40 female patients between 42 and 75 years of age, affected by bone decalcification, was defined. Then, the bone densitometry test (DXA) was carried out in the Diagnostic centers and medical centers in the period from August 2011 to September 2021. This consisted of the quantitative analysis of the bone tissue of each patient in the joint. hip, and from there they obtained the bone mineral content (BMC), bone mineral density (BMD) and the T-score and Z-score and the medical image of the analyzed area.

The T-score and Z-score express the severity of the disease in each patient, the T-score is the value of the standard deviation of bone mineral density with respect to a healthy 30-year-old person of the same sex, while the Z-score is the standard deviation of a person's bone density with that of an average person of the same age and sex [10], [11], [12].

Methods of fractal mechanics for the evaluation of mechanical behavior of the coxofemoral joints of patients with bone tissue decalcifications

The methodology used to evaluate the affected surfaces of the hip joints due to erosion due to bone decalcification consisted of:

First, the medical images obtained by densitometry were processed by saving them in image formats.

Second, the images were evaluated through the BENOIT software. In one case, the surface displayed in each of the images was analyzed using the method of self-similar traces by Box Dimension to obtain the type of fractal governed by an equation model that describes the damage caused by the disease and a fractal dimension that quantifies the degree of irregularity. In the other case, the method of self-similar Traces by Mass Dimension was used and in a similar way the type of fractal and the fractal dimension were determined [9].

Third, the graph of the affected part was obtained, in the case of both methods.

Fourth, the fractal found was interpreted and the fractal dimension was determined [9], [13], [14]. In the case of the analysis with the BOX DIMENSION method, the fractal dimension (Db) is defined as the exponent of the proportionality N(ε) ∞ 1, where N(ε) is the number of structures with self-similarity of linear size ε required to cover the structure. In this case, consider that the space is divided into a grid of boxes with size ε, and the number of boxes that contain the fractal is determined in the form Db = l´ím, ε →0 (1)

On the other hand, in the case of the MASS DIMENSION method, the fractal dimension (Dm) is defined from the proportionality m(r) ∞ rDm, where m(r) = M(r)/M is the mass contained in a certain circle of radius r, M(r) is the set of points contained in the circle and M is the total number of points in the set.

RESULTS AND DISCUSSION

RESULTS OF THE FRACTAL ANALYSIS OF THE COXOFEMORAL JOINTS OF PATIENTS WITH BONE TISSUE DECALCIFICATIONS

Table 1 shows the results of the fractal analysis obtained by means of the BENOIT software.

Patient	TOBMD	BOX DIMEN	SD Db	MASS DIMEN	SD Dm
P10EA	0.833	1.869	0.004	2.495	0.161
P24AY	0.964	1.884	0.006	1.949	0.012
P25AMY	1.363	1.839	0.011	2.611	0.208
P27AH	0.901	1.889	0.088	1.898	0.008
P28AO	0.983	1.874	0.005	1.886	0.004
P29AL	0.913	1.875	0.011	1.974	0.049
P32ADR	1.036	1.866	0.005	1.899	0.018
P33AK	1.151	1.857	0.004	2.552	0.331
P38AAJ	1.099	1.848	0.007	2.081	0.143
P39AR	0.902	1.875	0.005	1.874	0.003
P40AS	1.102	1.862	0.005	3.06283	0.844
P41ALS	1.04	1.845	0.009	1.91272	0.011

TOBMD: Mineral density in the global region of the hip. SD Db,: standard deviation of Fractal Dimension by the Box Dimension technique and SD Dm; deviation Fractal Dimension by Mass Dimension technique. Source: self made.

Table 1. Values of the fractal dimension and bone mineral density of the total region (TOBMD) of the coxofemoral joints of patients with normal condition of bone decalcification.

The fractal dimension of the coxofemoral joints was determined by both techniques; BOX DIMENSION and MASS DIMENSION for all patients and whose values are found in Table 1. These measurements represent the quantification of the surface roughness in the global region of the hip joints of women who are presenting lack of care in their tissues. bony. It is observed that the cumulative damage of the pathological conditions behaves like a fractal.

From the fractal analysis, it can be deduced that the values of the fractal dimension obtained by Box dimension are smaller with respect to those obtained by MASS dimension. In a generic sense, with the first technique, the fractal dimension is a number that serves to quantify the degree of irregularity and fragmentation of the bones of the patients evaluated, while with the second technique, values of the dimension of the joints were obtained in terms of the lost mass, physical variable that characterizes the wear caused by decalcification

in the bone [15], [16]. Consequently, the determined fractal dimensions are proportional to the bone mineral density values of the global joint region for purposes of estimating wear and tear caused by disease [17], [18].

In the case of the fractal analysis of the first patient P10EA, Figure 2 shows the medical image obtained from the analysis of bone densitometry in the hip joint.

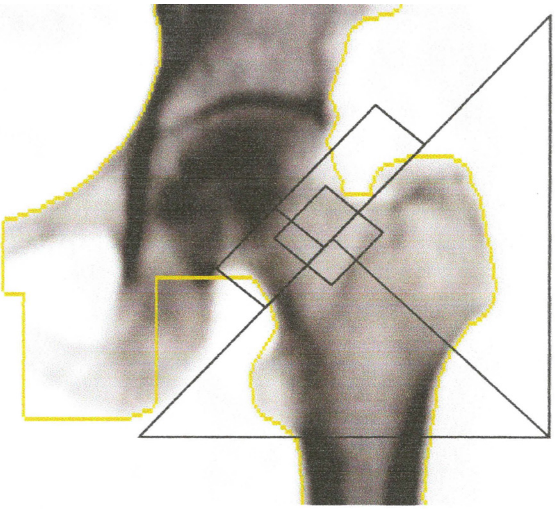

Figure 2. Medical image obtained from DXA analysis of the hip joint of patient P10EA. Normal condition of bone decalcification. Source: Courtesy GRUMED. DR.

La Figure 3, se observa la imagen procesada por el método box dimension para determinar la determinar la dimensión fractal del fractal encontrado.

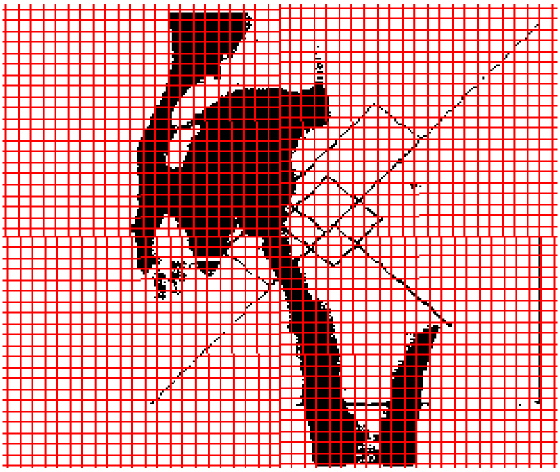

Figure 3. Processed image of the joint of patient P10EA using the box dimension method to determine the fractal model and its dimension. Normal condition of bone decalcification. Source: self made.

The mathematical model with logarithmic evolution that describes the fractal behavior of the affected area of the hip joint of the patient P10EA who is in a normal state of bone decalcification is shown in Figure 4.

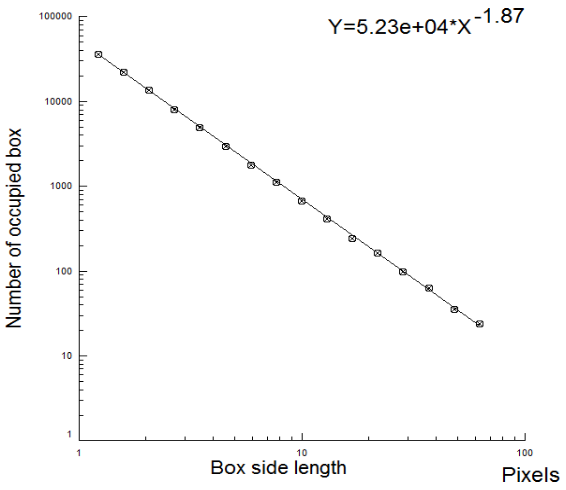

Figure 4. Fractal model resulting from the hip joint of patient P10EA in normal condition using the fractal Box dimension method to determine the type of fractal and its dimension. Source: self made.

The resulting fractal model of the hip joint of patient P10EA and its fractal dimension obtained by the Box Dimension method is that of equation 2.

$$Y = 5.23 \times ex{-1.8695} \tag{2}$$

The mathematical model of equation 2 is a logarithmic equation that describes the mechanical behavior of the hip joint of patient P10EA and serves to quantify the degree of irregularity and fragmentation of her bone tissues. This pattern is a wear indicator. Likewise, through the MASS Dimision method, the fractal model of equation 3 was obtained.

$$Y = 0.262 \times ex2.49515 \tag{3}$$

From the mathematical model of equation 3, the loss of bone mass can be studied, which has a standard deviation with respect to its fractal dimension of Db = 1.8695, its fractal dimension being Dm = 2.49515. This equation indicates that the loss of bone mass has an exponential behavior. This mechanical behavior of tissue decalcification in the hip joint is shown in Figure 5.

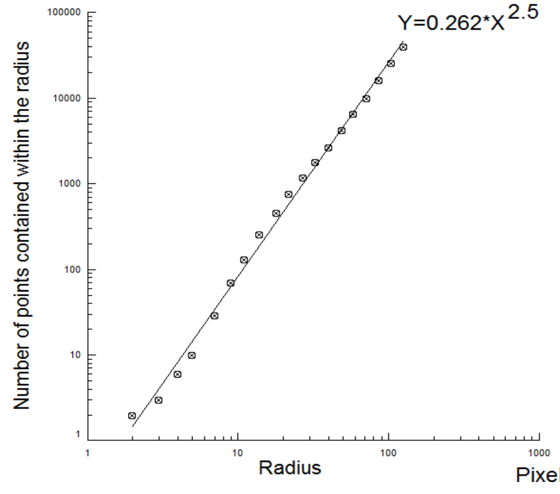

Figure 5. Fractal model resulting from the hip joint of patient P10EA in normal condition, using the fractal Mass dimension method to determine the type of fractal and its fractal dimension. Source: self made.

The graph that shows the behavior of the fractal model obtained by the Mass dimension method was obtained during the processing of the image in Figure 6.

Figure 6. Processed image of the joint of patient P10EA using the Mass dimension method to determine the fractal model and its dimension. Normal condition of bone decalcification. Source: self made.

The value of the fractal dimension Db is close to the mean of 1.8686 ± 0.014258, with a positive deviation of 0.000818 obtained through the Box dimension method for the patient's hip joint. However, the value of the fractal dimension of the affected area Dm is higher than the average of 2.1227 ± 0.32880, with a positive deviation of 0.37242 obtained with the Mass dimension method for the same joint. The higher value of both fractal dimensions in the affected area indicates that the patient has a relatively consistent bone tissue microarchitecture, which is corroborated by the fact that the patient has a bone mineral density in the femoral neck region of 0.823 g. /cm2, higher than the average value of 0.7924 g/cm2 ± 0.0615 with a positive deviation of 0.0306, that is, this explains the mechanical behavior of the normal condition of the bone tissue decalcification process. Likewise, the fractal study was applied to all the patients listed in Table 1.

Table 2 shows the results of the fractal analysis obtained in patients with osteopenia.

Patient	TOBMD	BOX DIMEN	SD Db	MASS DIMEN	SD Dm
P23AL	0.912	1.883	0.003	2.458	0.277
P26AC	0.792	1.866	0.007	1.946	0.017
P30AH	0.755	1.862	0.013	1.987	0.049
P31AH	0.687	1.871	0.009	1.933	0.004
P36AJ	0.714	1.881	0.002	2.010	0.267
P37AJ	0.673	1.862	0.008	2.001	0.030

TOBMD: Mineral density in the global region of the hip. SD Db: standard deviation of fractal dimension by the Box Dimension technique and SD Dm; fractal dimension deviation by the Mass Dimension technique. Source: self made.

Table 2. Values of the fractal dimension and bone mineral density of the total region (TOBMD) of hip joints of patients with osteopenia.

Like the data in Table 2, the medical image of the bone densitometry analysis of a patient is shown, which is shown in Figure 7.

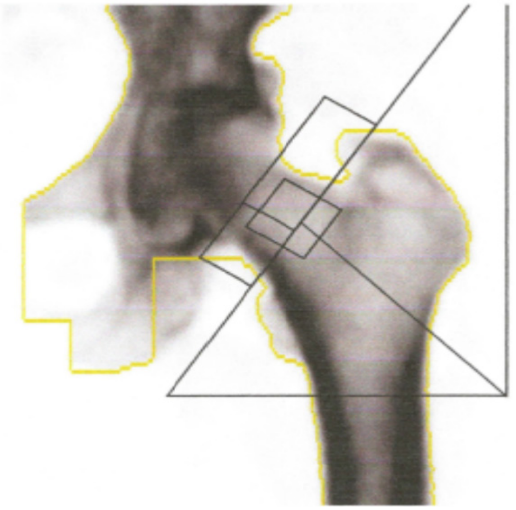

Figure 7. Medical image obtained from DXA analysis of the hip joint of patient P23AL with osteopenea. Source: Courtesy GRUMED. DR.

On the other hand, Figure 8 shows the image processed by the Box Dimension method to determine the fractal during the fractal analysis of the image of patient P23AL.

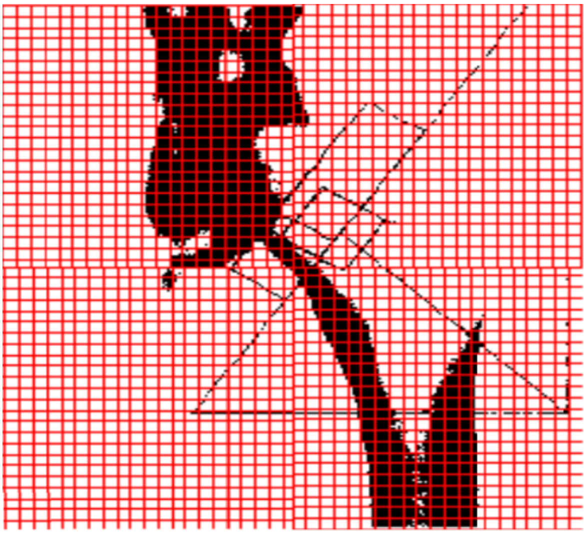

Figure 8. Processed image of the joint of patient P23AL using the box dimension method to determine the fractal model and its dimension. Source: self made.

Then, the mathematical model with logarithmic evolution that represents the fractal behavior of the medical results that indicate the osteopenia of the P23AL patient was obtained. In the case of the fractal dimension obtained by the Box Dimension method, this model is:

$$Y = 5.58 \times 10E04x{-1.8830} \qquad (4)$$

Equation 4 indicates that the loss of bone mass has an exponential behavior as the decalcification of the tissues occurs in the hip joint of the patient P23AL, in a condition of osteopenia, and is graphed in Figure 9.

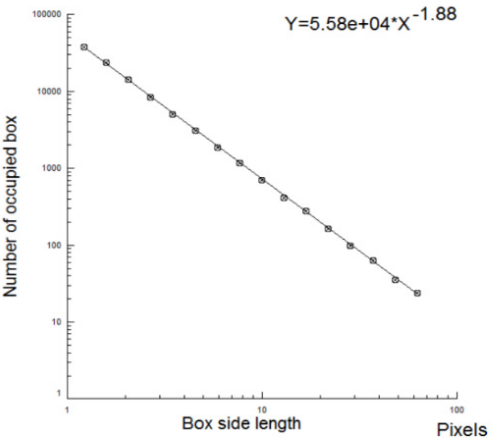

Figure 9. Fractal model resulting from the hip joint of the patient P23AL with osteopenia using the fractal Box dimension method to determine the type of fractal and its dimension.

In the case of the fractal dimension obtained by the Mass Dimension method from the densitometry results of the hip joint of the patient P23AL with ostopenia, the mathematical model is that of equation 5.

$$Y = 0.37 \times 10E04x2.4588 \qquad (5)$$

This fractal model represents the loss of bone mass, it has a fractal dimension of Dm = 2.4588, it is graphed in Figure 10.

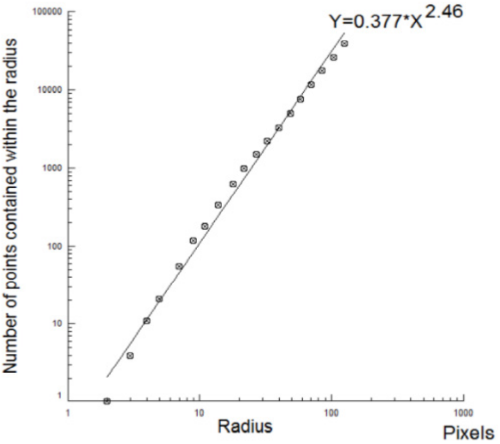

Figure 10. Resulting fractal model of the hip joint of patient P23AL using the fractal Mass dimension method to determine the type of fractal and its fractal dimension. Source: self made.

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The graph of Figure 10 that shows the behavior of the fractal model used the Mass dimension method was obtained during the processing of the image of Figure 11.

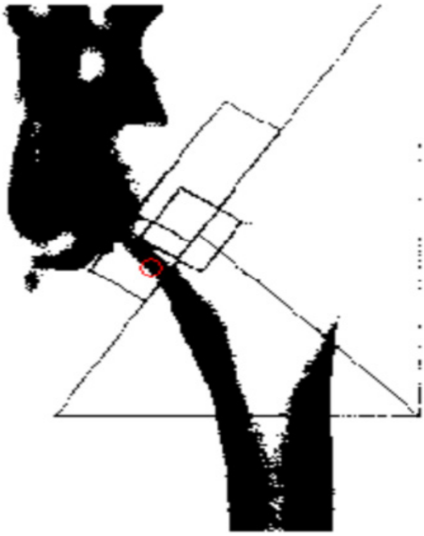

Figure 11. Processed image of the joint of patient P23AL with osteopenia using the Mass dimension method to determine the fractal model and its dimension. Source: self made.

The value of the fractal dimension Db is close to the mean of 1.8686 ± 0.014258, with a positive deviation of 0.0143. The value of the fractal dimension Dm is higher than the average of 2.1227 ± 0.32880, with a positive deviation of 0.3361. The higher value of both fractal dimensions indicates that the patient has a consistent microarchitecture of the bone tissue, which is correlated with the fact that the patient has a bone mineral density in the region of the neck of the femur of 0.845 g/cm2, higher than the average value of 0.7924 g/cm2 ±0.0615 for patients with osteopenia, as in this case, with a positive deviation of 0.0526. On the other hand, Table 3 shows the results of the fractal analysis obtained by means of the BENOIT software in patients with osteoporosis.

Patient	TOBMD	BOX DIMEN	SD Db	MASS DIMEN	SD Dm
P34AK	0.627	1.893	0.009	1.904	0.002
P35AK	0.64	1.873	0.008	2.049	0.119

TOBMD: Mineral density in the global region of the hip. SD Db,: standard deviation of Fractal Dimension by the Box Dimension technique and SD Dm; deviation Fractal Dimension by Mass Dimension technique. Source: self made.

Table 3. Values of the fractal dimension and bone mineral density of the total region (TOBMD) of the hip joints of patients with osteoporosis.

Figure 12 shows the medical image obtained from the bone densitometry analysis of the patient with osteoporosis condition.

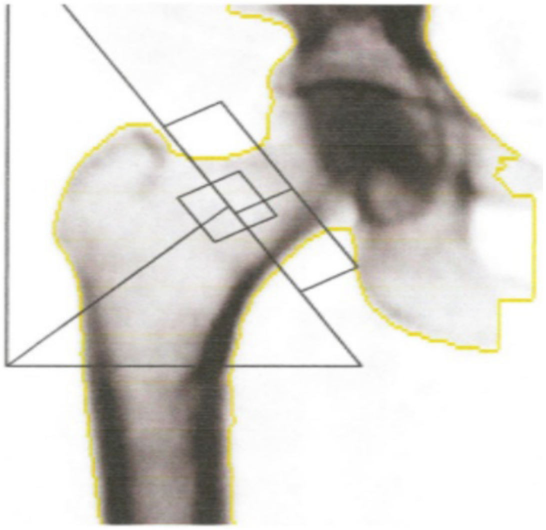

Figure 12. Medical image obtained from DXA analysis of the hip joint of patient P35AK with osteoporosis. Source: Courtesy GRUMED. DR.

Then, the image in Figure 12 was processed by the Box Dimension method, the results of which are shown in Figure 13.

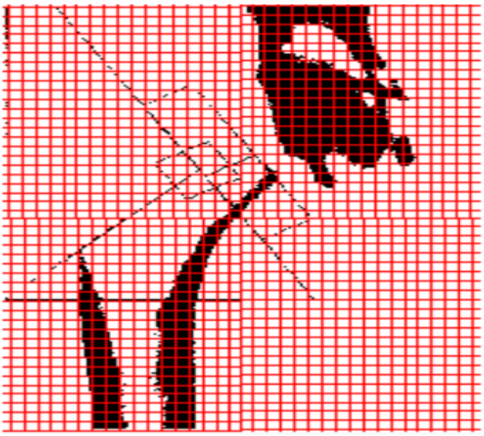

Figure 13. Processed image of the joint of the patient P23AL with osteoporosis using the box dimension method to determine the fractal model and its dimension. Source: self made.

The fractal dimension obtained by the Box Dimension method, whose mathematical model with logarithmic evolution of the method of the osteoporosis disease of patient P35AK is equation 6.

$$Y = 6.24 \times 10E04x{-1.8725} \quad (6)$$

This equation indicates that the loss of bone mass has an exponential behavior as tissue decalcification occurs in the hip joint, as shown in Figure 12.

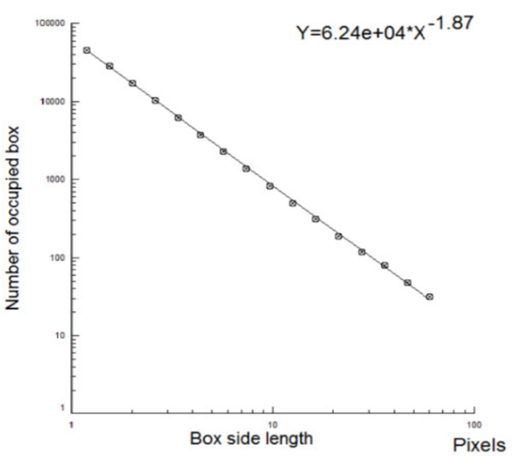

Figure 14. Fractal model resulting from the hip joint of the osteoporosis patient P35AK, using the fractal Box dimension method to determine the type of fractal and its dimension. Source: self made.

Likewise, for the P35AK patient with osteoporosis, the fractal dimension obtained by the Mass Dimension method and the mathematical model that represents the fractal behavior of the disease is that of equation 7.

$$Y = 2.2 \times 10E04x2.04945 \quad (7)$$

This fractal model that describes the behavior of the osteoporosis of the patient P35AK shows a logarithmic evolution of disease and is shown in Figure 15.

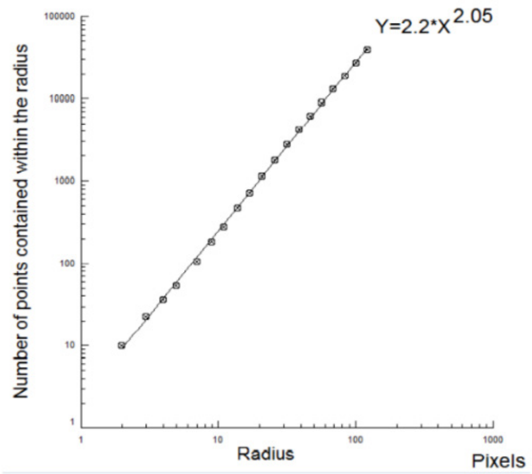

Figure 15. Resulting fractal model of the hip joint of patient P35AK using the fractal Mass Dimension method to determine the type of fractal and its fractal dimension.

The graph (Figure 15) that shows the behavior of the fractal model used by the Mass dimension method in the analysis of osteoporosis presented by patient P35AK, was obtained during the processing of the image of Figure 16.

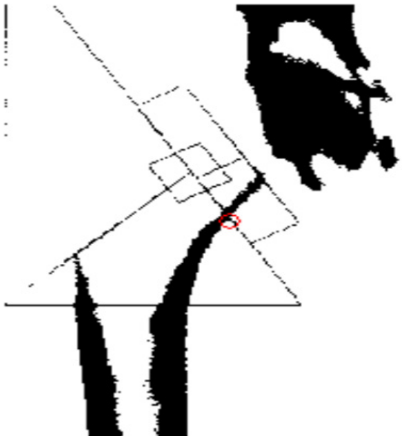

Figure 16. Processed image of the joint of patient P35AK osteoporosis, using the Mass dimension method to determine the fractal model and its dimension.

The lower value of the fractal dimension Dm indicates that the patient has a deteriorated microarchitecture of the bone tissue, which does not correlate with the fact that the patient has a bone mineral density in the region of the neck of the femur of 0.612 g/cm2, lower than the average value of 0.6155 g/cm2 ±0.6155 for patients with osteoporosis condition, with a negative deviation of –0.0035.

As seen in Table 3, for the BOX DIMENSION method, the fractal dimension values are lower for healthy people, while for the MASS DIMENSION method, the fractal dimension increases proportionally to the bone mineral density value.

It is important to have good quality medical images for the study; since lower calcium content results in less X-ray attenuation in bone, so photographic film becomes darker and the brightness of the digital image is reduced [19].

If you take as a reference the value of the fractal dimension of a bone from a healthy person, in this case of the patient P40AS (TOBMD=1.102 g/cm2), whose dimension Db = 1.86195 is relatively low and Dm = 3.06283 is very high, because its microarchitecture density is very high.

With the fractal geometry methods of Box Dimension and Mass Dimension, the fractal dimension that characterizes the roughness that occurs in the hip joints as a result of the decalcification of bone tissues was determined, in such a way that when the effect of loss of bone mass as a fractal this explains the pathological conditions of osteopenia and osteoporosis [10], [16], [19].

CONCLUSIONS

Fractal geometry was used as a useful tool for image analysis and bone densitometry results. The Box Dimension and Mass Dimension methods were applied in the evaluation of medical images to determine the fractal dimension and the fractal coefficient. In this work it can be seen that the estimation of the fractal dimension can be determined by means of digital image correlation procedures, based on small-scale computed tomography images, in such a way that cortical bone tomographies show large differences in the density distribution. These changes can be adequately represented by the fractal dimension.

The Mass Dimension method shows a more significant correlation, with a Pearson coefficient > 0.50 in almost all cases of the patient mass values compared to the Box Dimension method, which is also a good technique for analysis and discussion of results. Therefore, bone microstructure at different scales can be represented by a self-similar statistical ensemble in three dimensions (X – Y gray scale) that encodes the fractal dimension. At that time, the fractal dimension represents the geometric properties of this self-similar set; therefore, the irregularities in the density distribution and the main geometric characteristics of the bone microstructure can be adequately represented by this parameter.

Estimates can be made based on the results of the fractals and fractal dimension obtained from the fractal geometry methods used in this research; since the loss of bone mass and the mechanical behavior of the coxofemoral joints of female patients affected by diseases with bone decalcification were evaluated, and from here, the values of the fractal dimension were used to evaluate the mechanical behavior of these hip joints. hips in the pathological conditions of normal, osteopenia and osteoporosis. Finding that decalcification over time has a fractal behavior, which describes the degree of progress in its sequence of deterioration.

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