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FLIPPED CLASSROOM IN THE APPLICATION OF DIDACTIC SEQUENCE FOR THE IMPROVEMENT OF MATHEMATICAL COMPETENCES REFERRED TO MODELING AND REPRESENTATION OF MATHEMATICAL OBJECTS

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Abstract: Forming competent human beings, mathematically speaking, requires experience in the application of knowledge and skills when facing real situations, a task that is evaluated every year in schools. The results obtained in Colombia in the internal and external tests show shortcomings when giving meaning to school knowledge - daily environment. Showing the effectiveness of didactic sequences mediated by the inverted classroom model, through situations close to the quadratic function that would improve the level of competence referred to the modeling and representation of mathematical objects of high school students, was the objective. Qualitative research, case study method. Ten Activities were designed, mediated by WhatsApp and Geogebra through the flipped classroom model. Data were analyzed under the study a priori - a posteriori, revealing that the students do not know the meaning of function, classes and characteristics. Omission of the quadratic model in the representation of real phenomena. Mathematical skills in modeling and representing objects show low performance. It is concluded that the design of didactic sequences allows the student to see linearity between Activities, giving meaning to school knowledge in their environment. Student awareness is required when participating in the flipped classroom.
Keywords: Mathematical competence, quadratic function, inverted classroom.

INTRODUCTION

Mathematics is part of the curriculum, being a fundamental area. Being mathematically competent refers to abilities that the student demonstrates when using mathematical objects solving situations outside the school context (OECD, 2008). In Colombia, the results obtained in national and international standardized tests show that this competence is an unfulfilled task, since according to the Program for International Student Assessment PISA report, the gap that is progressing negatively in Latin American countries is worrying (Paul, 2019).

At the national level, the participating institution during the 2019-2021 cycle has been characterized with an average of 2.4, ranking in the second level of performance according to the Colombian Institute for Educational Evaluation ICFES, data that reflects that mathematical competencies - modeling of real problem situations - are not part of the school tasks developed in the classroom. This reflects the privilege of procedural Activities that do not generate reflective analysis of the studied mathematical object, leaving aside contextualized situations that allow the student to recognize semiotic representation systems of objects giving meaning to it in non-school situations, fundamental processes in conceptual work.

Designing scenarios that lead the student to new learning is an arduous task for the teacher; interpreting, reasoning, arguing are competencies that the student must strengthen in each exposed experience. Creating didactic situations is the responsibility of the teacher, since he must be skilled in choosing mathematical situations in media that lead the student to the created sequence (Brousseau, 2007). The mandatory confinement generated by the Covid 19 Health Emergency must have been a springboard for the adaptation of teaching methodologies that would strengthen the student's interest in learning in a different way, those that, regardless of economic, social, or technological factors, would achieve the objective of the school.

Advancing in the level of mathematical skills of tenth and eleventh grade students led to the use of the flipped classroom model to demonstrate the effectiveness of didactic sequences around the concept of quadratic function, to improve skills in modeling and representation of mathematical objects. Responding to the Question how to improve the level of mathematical competence of modeling and representation of mathematical objects in students close to concluding their school stage when studying the quadratic function, led to the design of a sequence of Activities that used the Flipped Classroom giving the student the leading role in their own learning. The use of technological tools, the WhatsApp application and Geogebra allowed generating conditioned and flexible self-learning for the context of the participating institution. In addition to inducing students to the task of being scientists by solving real situations using mathematics, understanding the relationship between knowing and knowing how to do, the school and the environment (MEN, 2006).

METHOD

Qualitative research under the case study approach, with a descriptive scope based on Brousseau's theory of didactic situations. The data collection instruments were observation through a grid with descriptive and interpretative annotations during the sequence (Table 1). Ten Activities were designed, an entrance and exit test, which involved questionnaires with the Google Forms tool. Each Activity -individual group-was mediated by WhatsApp and Geogebra) Table 2).

The data collected was analyzed under phase two and four of the Artigue Didactic Engineering experimental process. In the a priori analysis, the description that the student was expected to carry out in each Activity was made, taking into account previous knowledge and characteristics of the environment. The a posteriori analysis validated the hypotheses made in the a priori analysis. The data was compared, refuting the validity of the Activities in the design stage (Artigue, 1995).

The Activities were sent to seventeen students enrolled in tenth and eleventh grades, being an incidental sample. The Activities were sent home to the students, since at the time of the application of the sequence it was the methodology used by the educational institution at the time. Communication between teacher-students was through the WhatsApp application or phone calls when possible. The flipped classroom methodology was used at the time prior to the meetings, since it was a requirement for the students to study the material before. Geogebra was used to design some Activities as a technological tool.

RESULTS

GENERAL RESULTS

The design of the Activities brought together a set of tasks that formed the didactic sequence. The medium: WhatsApp, Google Forms, Geogebra (used not directly), informative documents: workshops became the instrument that allowed to identify the level of mathematical competence related to the modeling and representation of objects, which led to the design of tasks that improve these skills. The flipped classroom model became the ideal methodology to achieve the application and evaluation of this research.

ACTIVITY 1. CHECKING MY KNOWLEDGE!

The entrance test had as objective to identify the level of mathematical competences: modeling and representation of mathematical objects, in which the students of the institution are. Activity 1 consisted of seven Questions that required students to apply knowledge about function, identifying and associating dependent and independent variables within a real situation. Graphically represent a problem and characterize the quadratic function by

Observation sheet	
Date	
Degree	
Number of students who are present.	
Activity.	
Duration	
Media	
Student-teacher communication.	
Guidance request.	
Strengths in mathematical processes (variational thinking)	
Failures in mathematical processes (variational thinking).	
Evidence material review	
Response to Questions raised.	
Presentation of ideas in groups.	
Development of Activities in assigned times.	
Timely submission of progress for feedback	
Interpretive annotations	

Table 1. Observation grid

Didactic sequence	
Activity	Goal
Checking my knowledge!	Identify level of mathematical competence: modeling and representation of everyday situations using mathematics.
Features and more!	Review concepts related to real functions, elements, characteristics, emphasizing the quadratic function.
Read understanding	Evaluate basic concepts about functions: characteristics, properties, graphic representation and classification.
Identifying Changes: Quadratic, function-Geogebra!	Analyze the interpretative competence of the students when observing changes of the quadratic function in the Cartesian plane, observing the variations in initial conditions or given parameters, Geogebra.
Algebraic expression vs graphical representation!	Analyze the level of performance of the students of the when identifying representative values of a parabola from the meaning to determine the equation of the quadratic function.
Discovering quadratic functions!	To review the exercise process of the students when determining the quadratic equation and its elements from initial conditions, applying mathematical algorithms.
Relating graph-equation!	Analyze the level of interpretative and argumentative competence of the students when transforming information from the parabola, establishing a mathematical relationship that allows them to determine the equation of the function.
Reality vs Mathematics!	Analyze the level of competence of formulation and modeling of students when designing and executing solution plans that allows them to identify variables, relate them by establishing mathematical models.
Modeling everyday life!	Analyze the level of competence of formulation and modeling of the students when designing and executing solution plans, identifying variables and relationships, establishing mathematical models.
Finish the challenge...what I learned!	To know the knowledge of students around the concept of function as a relationship between two variables. Identify the level of mathematical competence - modeling and representation of everyday situations.

Table 2. Operationalization of Activities

recognizing elements and noticing changes, as well as modeling situations that required the concept of function in their solution.

Figure 1 shows the poor relationship between school knowledge and context, despite being asked to evaluate a task they perform daily based on the context in which they live, they were unable to use the mathematical object in their response.

The results of the Activity showed the ignorance of the mathematical object under study, function. The students failed to identify variables in a situation and determine the relationship between them (dependent and independent variable). There was evidence of a lack of meaning in school knowledge with the environment, they were unable to analyze them around mathematics to predict or explain the phenomenon studied (Henríquez, Pinto and Solar, 2020). Understanding a verbal statement by recoding it using mathematical language is a pending task for schoolchildren (Figure 2), they were unable to model situations using equations that would show the relationship between the variables (Huapaya, 2012).

Activity 1 validated that the students are at a low performance level in competencies: modeling and representation of mathematical objects, despite being students close to finishing their school stage. These results made it possible to design and implement eight Activities with tasks related to these skills in order to note the progress of the students as the sequence was completed.

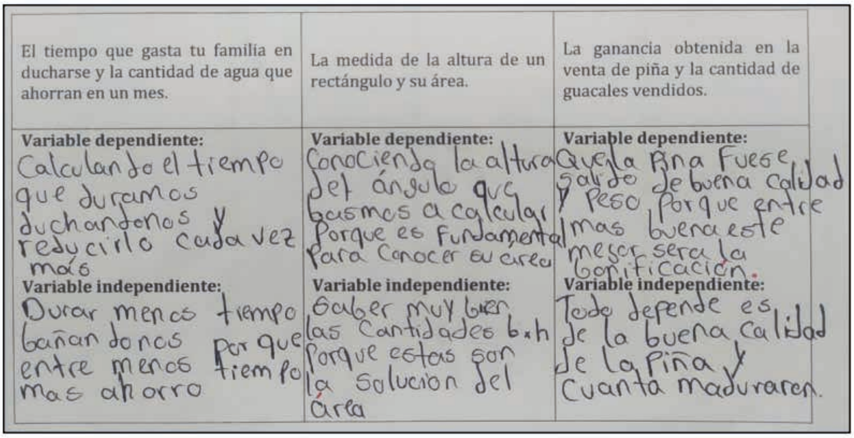
INTERMEDIATE ACTIVITIES

The flipped classroom model became the methodology to apply when we found ourselves in mandatory confinement. This required the student to prepare for the meeting and development of the Activities. Validating the student's interest in reading the information delivered in advance, allowed the students to be categorized according to their way of reading documents: Those who underlined lines, words or phrases; who left their document blank. In Figure 3, some of the techniques used by students when doing a reading.

This reflective and comprehensive reading was evaluated through a Google Forms form, since the fact of underlining ideas, words or Figures did not guarantee reaching the objective of the Activity. Although the test was only applied to thirteen of the students due to connectivity problems and lack of knowledge of the use of the tool, it was possible to show that the flipped classroom model requires self-learning, and this was not yet strengthened in schoolchildren. Their answers were not very accurate, leaving as evidence a not very reflective and comprehensive reading of the document delivered as a requirement to be able to carry out the following tasks.

The use of technological tools becomes a means to strengthen interpretative competence every time it allows observing, validating and communicating changes generated in a mathematical object. In this case, the shared videos in Geogebra allowed the student to observe changes that were generated in the parabola by modifying the initial conditions of the parameters. The results showed that although the students were able to account for changes and describe what was happening in the parabola using simple expressions "to the right", "upwards", "small", "it gets larger", they did not manage to establish mathematical relationships at the same time. Observe patterns in the parameters.

Figure 4 shows that the students describe very particular movements, in some cases limiting the observations to some not very global visual characteristics of the parabola by turning them numerical and local (Duval, 2006). The students showed a low level in interpretive competence.



El tiempo que gasta tu familia en ducharse y la cantidad de agua que ahorran en un mes.	La medida de la altura de un rectángulo y su área.	La ganancia obtenida en la venta de piña y la cantidad de guacales vendidos.
Variable dependiente: Calculando el tiempo que duramos duchandonos y reducirlo cada vez más **Variable independiente:** Durar menos tiempo bañandonos porque entre menos tiempo mas ahorro	**Variable dependiente:** Conociendo la altura del ángulo que vamos a calcular porque es fundamental para conocer su area **Variable independiente:** Saber muy bien las cantidades b×h porque estas son la solucion del área	**Variable dependiente:** Que la piña fuese salido de buena calidad y peso porque entre mas buena este mejor sera la bonificación. **Variable independiente:** Todo depende es de la buena calidad de la piña y cuanta maduraren.

Figure 1

Activity 1. Question 2.

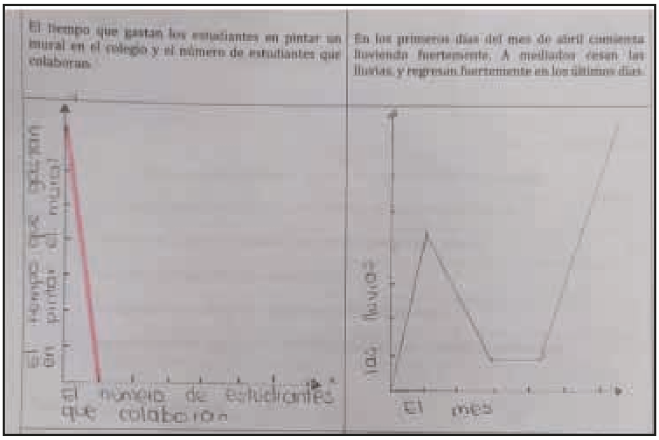


Figure 2

Activity 1. Question 3.

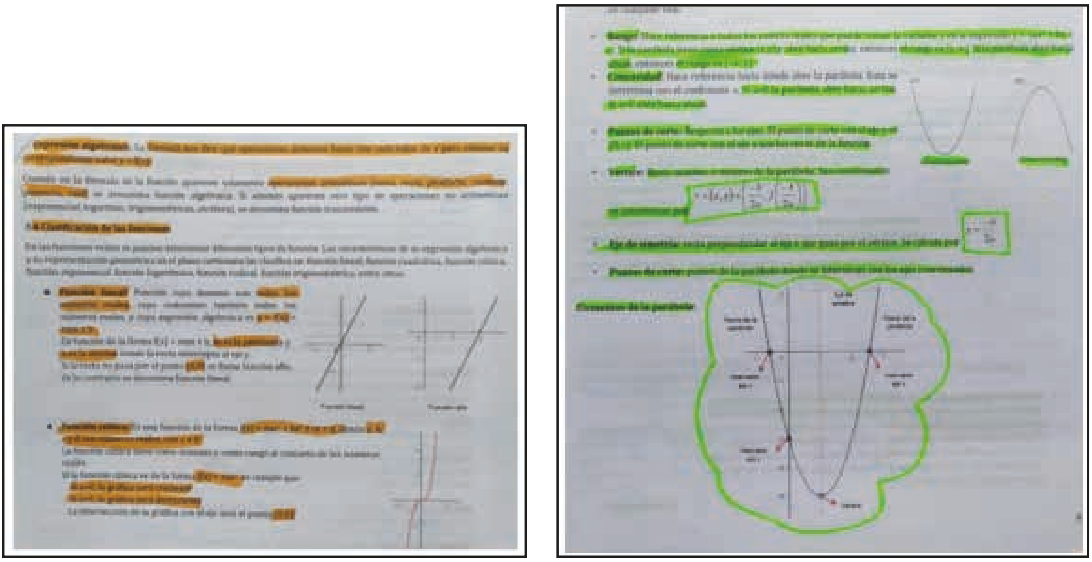


Figure 3

Activity 2.

Activities five, six and seven aimed to articulate the use of algebraic registers and graphs of the quadratic function, with the use of semiotic representation systems being a fundamental part of the mathematical thought process. Observing the graphical representation of the function and extracting elements such as the y-intercept, the value of the quadratic parameter, and the vertex, would lead students to write the requested algebraic expression. Figure 5 shows how the students failed to articulate the two representation systems, omitting the value of the quadratic parameter for the existence of the parabola, and ignoring the meaning of the intercept of the parabola with the axis and forgetting the ordered pair in the same equation.

In Activity 7, with the purpose of reviewing the interpretative and argumentative competence of the students when transforming the information delivered into a graph, the students had to select the most accurate equation of the quadratic function according to the given geometric representation. For this, the students must have read the strategies delivered to identify elements and relate them in the equation, without ruling out the possibility of finding new solution plans to the problem.

In Figure 6 it can be seen how some students managed to establish relationships between the concavity of the parabola with the sign that the quadratic parameter identified in the equation must have as the value that accompanies the variable with the largest exponent. They related the y-intercept of the parabola to the parameter c in the equation.

Other students preferred a procedural task using the canonical equation of the quadratic function y= a(x-h)2 +k to find the correct expression from the given initial conditions; conceptual, algebraic and procedural errors were present in their solution plan, revealing the little understanding in the generalization of the letter as an object (Flores and Auzmendi, 2016).

Activities 8 and 9 required collaborative work. Groups were formed through WhatsApp as the only means of communication at the time of confinement. The objective of these two Activities was to evaluate the competence in modeling problem situations through the quadratic model. The first Activity showed the movement of a car from one point to another. The student had to analyze the movement, finding relationships between what he sees and thinks, in order to later be able to record it according to the Questions posed (Briceño and Buendía, 2016).

Figure 7 shows responses given by students during group work. The graphical representation of the situation led the students to use straight lines for horizontal displacement and a curved line to represent the trajectory of the car through the mountain. They identified significant points of the situation, and some showed the possible inflection of the parable.

The relationship between the variables of the proposed phenomenon led the students to think only of the distance traveled as the spaces that were given from one point to another. They left out the movement of the car in each time interval without relating speed as a variable present in the situation. In Figure 8, it is evident how some students managed to recognize variables such as time, speed distance, allowing themselves to travel significantly in the distance-speed relationship. Graphically, the movement of the car for the students continued to be a totally linear model, being the only way of considering the behavior of variables.

In activity 9, the problem situation was close to the reality of the students, it required designating a space for planting, tasks carried out by the families in their economic activities. The objective of the activity was similar to the previous task, to model the phenomenon

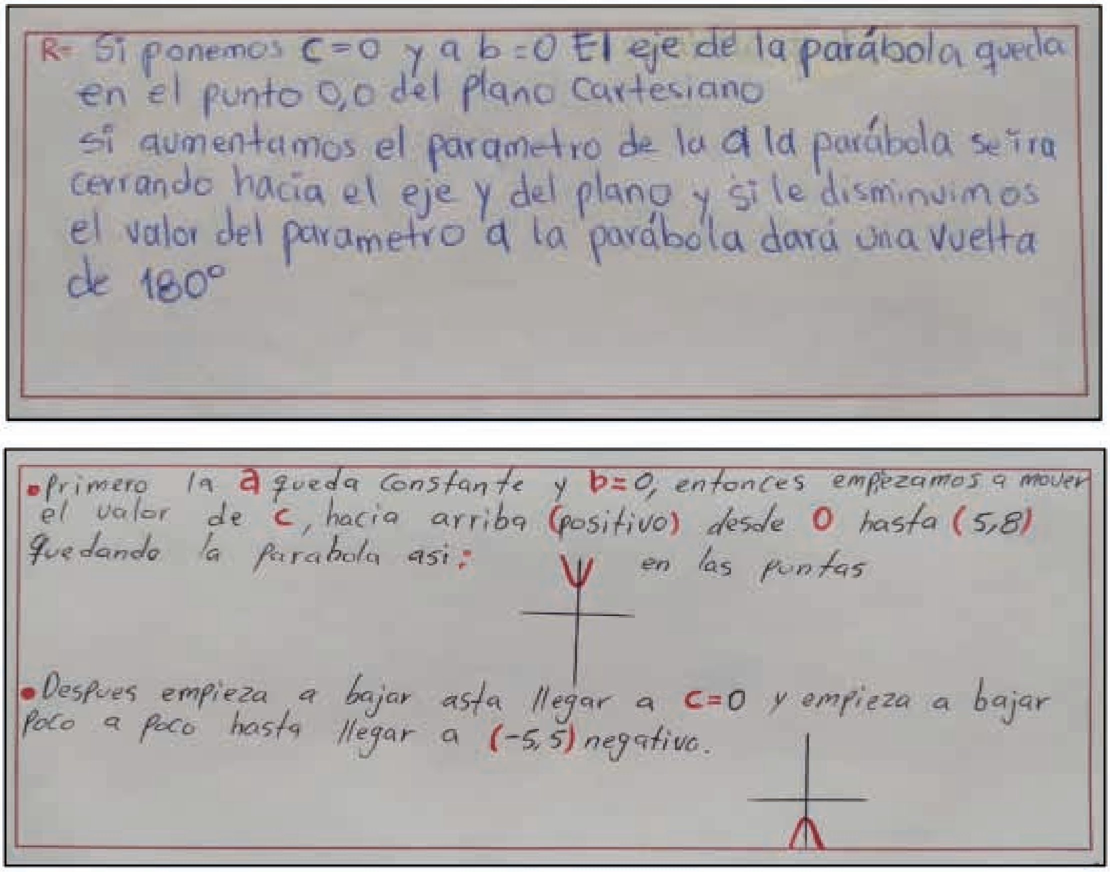


Figure 4

Activity 4. Question 1 y 2.

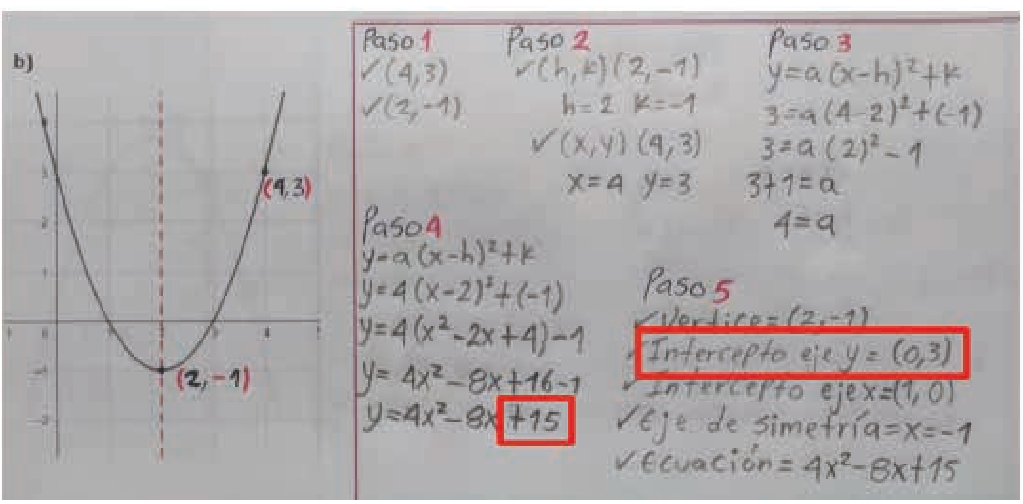


Figure 5

Activity 5. Question 1.

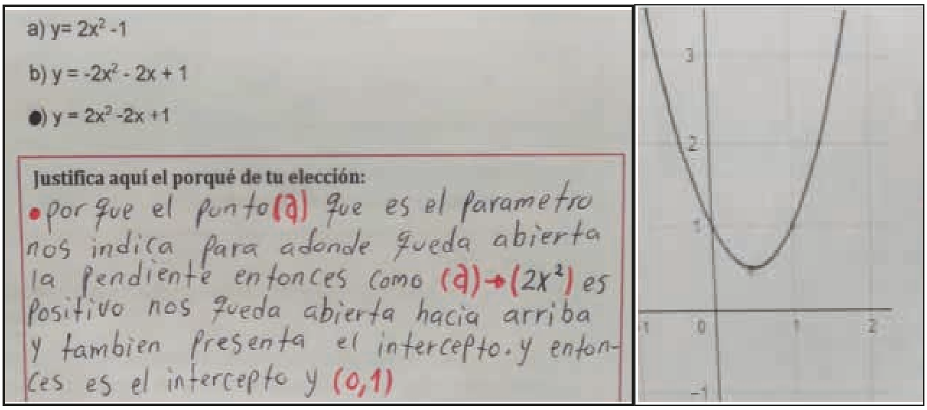


Figure 6

Activity 7. Question 1

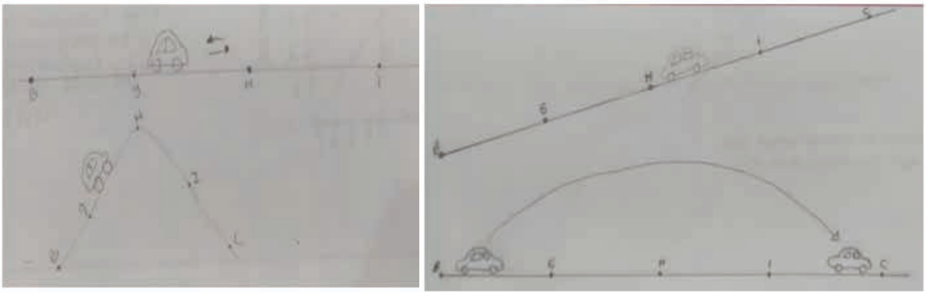


Figure 7

Activity 8. Question 1

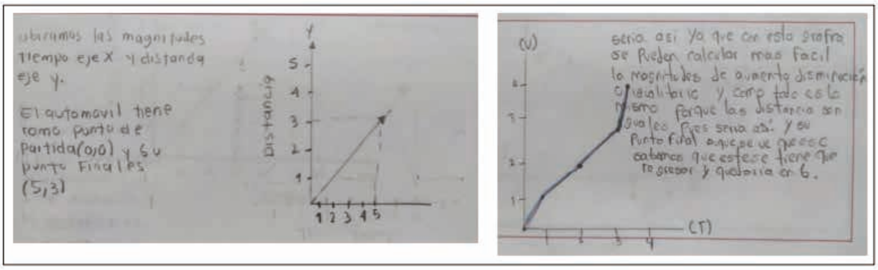


Figure 8

Activity 8. Question 4

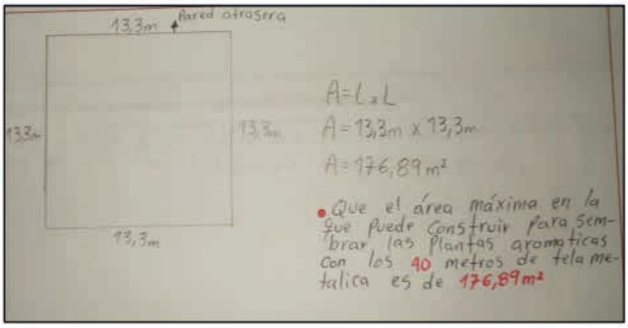


Figure 9

Activity 9. Question 1

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using basic concepts of area and perimeter. In Figure 9, the students preferred trial and error, a solution plan that led them to wrong answers by ignoring the initial conditions of the problem. The models delivered can be located at a very basic level for the grade of schooling.

Regarding teamwork, it was limited. WhatsApp, apparently, did not generate the necessary trust for them to discuss the most accurate mathematical model with their peers. This medium had not been used to carry out collaborative work in the work-at-home modality. Other factors, such as connectivity, generated a delay in the reception of messages by the members of the groups to validate the proposals made during the meeting time.

ACTIVITIES 10. FINISH THE CHALLENGE...WHAT I LEARNED

The last activity became the exit test, a task that would allow evaluating the effectiveness of the didactic sequence applied within the modeling and representation of mathematical objects competencies of basic education students. Some of the questions had already been raised in the entrance test, since it was necessary to review again the answers that the students gave after applying eight intermediate activities. In the development of the activity, more autonomous and conscious work on the part of the students was evidenced, designing more accurate solution plans than in the first activity, allowing to show progress towards the achievement of the general objective of the investigation.

Regarding the competence related to the representation of objects, the students managed to go more easily from one semiotic representation to another. They managed to determine the equation of the quadratic function from the initial conditions given in the geometric representation of it. However, the fact that there have been shortcomings in the application of algorithms and arithmetic and algebraic properties is not omitted. The difficulties and little significance of the notable products and empowerment did not allow them to correctly model the proposed situation.

The modeling competition continued to show a bias towards the linear model as the only possible model to represent problem situations. However, it was not possible to demonstrate that they understood the model as such by establishing the proportionality present in the variables. The quadratic model remains an absent mathematical object in the mathematical process.

DISCUSSION AND CONCLUSIONS

The task of designing new teaching strategies is a pending and urgent task in the classroom. This design requires the student to be able to reconstruct the new information and relate school knowledge to their own context throughout the process. The teacher's task is the planning and execution of didactic sequences that allow the classroom to be a tool that allows progress in a common thread towards a learning object. At the end of the application of the didactic sequence, it was possible to conclude that 28% of the students managed to advance to level 2 of performance, the remaining 72% continue in the first level, some trying to advance, others still with shortcomings that will not allow them to reach a better level in the missing school time. The quadratic model is a pattern unknown to the participants. The linear model is the graphical representation of phenomena they know, but this does not guarantee that they identify variables and the dependency between them that leads them to design a linear algebraic model.

The application of the flipped classroom model requires an induction process for those who are going to participate, since it is essential to know the roles that each one must play within the model. With the participation of students, it is necessary to generate awareness of their responsibility in the learning process, since self-learning is the basis of the effectiveness of the educational process.

The WhatsApp mobile application during the application of the didactic sequence functioned more as a tool for sending evidence of activities carried out. On the one hand, student-teacher communication is very limited, since schoolchildren do not feel the necessary confidence to write a design solution to a problem; The writing process in these types of applications leads them to omit words, change letters for images that limit their verbal or written expression. It is usually thought that adolescents have a great command over this type of applications and the tools they provide us, however in the case of the students who participated, some of them required the explanation of the teacher or another pair to create a group, assign the administrator role to a participant or simply leave a group.

The use of technological tools, such as the Geogebra dynamic geometry software used not directly with students, allows students to interpret the changes of a variable from initial conditions, establishing relationships between different forms of semiotic representation of a mathematical object. It is necessary to allow the handling of this type of applications in the classrooms, which today can be addressed without requiring being connected to the Internet, but which requires interest and willingness on the part of the student to be experimenting with new tools in the times allocated for it.

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