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THE EMPLOYMENT OF DIGITAL TECHNOLOGIES DURING THE RETROSPECTIVE PHASE IN THE SOLUTION OF GEOMETRIC PROBLEMS

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Abstract: In this work, we analyze the solution of two problems related to spherical geometry, present in official tests such as the National High School Examination, according to the stages of solving a mathematical problem proposed by George Pólya. In the retrospective phase, we used *Google Earth* to reconsider and reassess the results obtained and we presented the mechanism of operation of this software. We conclude that the use of technological resources is essential in the retrospective phase of the solution of the selected problems, being in line with what the National Common Curricular Base establishes on the use of digital technologies in the teaching of mathematics.
Keywords: BNCC. Spherical Geometry. Google Earth. Mathematics Teaching.

INTRODUCTION

The art of problem solving is as old as the art of counting. According to Rooney (2012, p. 10): "The first records of mathematical activity – in addition to the art of counting – date back to 4,000 years ago. They came from the fertile deltas of the Nile (Egypt) and from the plains between the two rivers, the Tigris and Euphrates (Mesopotamia, now Iraq)". Among the mathematical activities present in these first records are the calculation of lengths and areas.

In elementary, high school and higher education mathematics classes, the teacher uses problems to motivate the study of a given topic and also to apply knowledge related to it. This way, problem solving as a methodology (POZO, 1998) introduces, develops and finalizes the interventions of the mathematics teacher in the classroom. For Lopes et al. (1994, p. 35), "The problems are proposed with the purpose of verifying the learning and application of concepts, algorithms, properties and other facts of Mathematics". As for Echeverría and Pozo (1988, p. 9):

> Problem solving is based on the presentation of open and suggestive situations that demand an active attitude from students or an effort to seek their own answers, their own knowledge. Teaching based on problem solving presupposes promoting in students the mastery of procedures, as well as the use of available knowledge, to respond to variable and different situations.

When using problem solving as a methodological resource, the mathematics teacher must be clear about the objectives. Schroeder and Lester (1989, p. 32-33) established three approaches to teaching problem solving: teaching about problem solving; teach for problem solving; teach through problem solving. These three approaches overlap and depend on the context in which mathematics is learned and applied.

There are many works in the literature in Portuguese addressing problem solving. Justulin (2016) mapped research on problem solving from five journals in Mathematics Education. In this mapping, the author highlights seven thematic foci, or themes of interest, in problem solving, one of the themes being the use of technology or electronic equipment. In turn, Proença et al. (2022) selected six master's dissertations to investigate teaching proposals based on problem solving and categorize students' difficulties in solving problems. In the "Monitoring" category, the subcategories "Evaluate the response" and "Review the solution" stand out.

The National Common Curricular Base (BRAZIL, 2018) emphasizes, in its fifth general competence, the use of digital technologies in problem solving.

> Understand, use and create digital information and communication technologies in a critical, meaningful, reflective and ethical way in the various social practices (including school ones) to communicate, access and disseminate information, produce knowledge, solve problems and exercise protagonism and authorship in life personal and collective

(BRAZIL, 2018, p. 9).

For Elementary Education, the BNCC designates in its fifth specific competence in mathematics: "Using mathematical processes and tools, including available digital technologies, to model and solve everyday, social and other problems in other areas of knowledge, validating strategies and results" (BRAZIL, 2018, p. 267). This same competence appears in the Curriculum of the Parana State Network - CREP (PARANÁ, 2021, p. 5) as the eighth specific competence of mathematics for Elementary School - Final Years.

Still, when we think about problem solving it is impossible not to mention the work of Pólya (1995). George Pólya (1887-1985), Hungarian mathematician, was professor of mathematics at ETH Zurich, where Albert Einstein graduated in Mathematics and Natural Sciences, from 1914 to 1940 and at Stanford University from 1940 to 1953, having made important contributions in combinatorics, probability, number theory, mathematical and heuristic analysis[1] in Mathematics Education. In 1945, he published the book *How to solve it: a new aspect of mathematical method* (PÓLYA, 1945), in which he addresses the general idea of heuristics for mathematical and non-mathematical problems. The book has sold over a million copies and has been translated into over seventeen languages.

In *The art of problem solving*, Pólya establishes four phases in the solution of a problem:

1. understanding the problem;
2. establishment of a plan;
3. execution of the plan;
4. retrospect.

The first phase comprises two stages: familiarization and improvement of understanding; the second phase encompasses the prerequisites needed to solve the problem; the third phase establishes an execution script, while the fourth phase is the verification and reassessment phase. Pólya emphasizes the importance of the last phase, which occurs after the problem is solved.

> Even reasonably good students, once they get the solution to the problem and the proof written, close the books and move on to another subject. In so doing, they miss an important and instructive phase of the work of resolution. If they look back on the complete resolution, reconsidering and re-examining the end result and the path that led to it, they can consolidate their knowledge and improve their problem solving skills. A good teacher needs to understand and convey to his students the concept that no problem is completely exhausted. There is always something left to do. With study and deepening, we can improve any resolution and, in any case, it is always possible to improve our understanding of the resolution (PÓLYA, 1995, p. 10).

This way, we selected two problems present in official tests to re-evaluate the solution in the retrospective phase using digital technologies. The first problem is question 55 of the National High School Examination (ENEM), of the 2002 math test, present in the yellow notebook on page 21 (BRAZIL, 2021). The second problem is question 13 of the mathematics test of the Federal Institute of Education, Science and Technology of Pernambuco - IFPE, present in the selection exam of the first semester of 2020 for the PROEJA integrated technical courses[2] (IFPE, 2020).

The two selected questions address knowledge of plane Euclidean geometry. However, the reassessment process, or the retrospective phase according to Pólya, allows us to compare the results based on concepts

1 Science whose object is the discovery of facts.

2 National Program for the Integration of Professional Education with Basic Education in the Modality of Youth and Adult Education.This program is aimed at young people/adults over 18 who have not completed high school or have not finished high school.

of spherical geometry. In this process, we used Google Earth (GOOGLE, 2022), a computational application that renders a 3D representation of planet Earth based on satellite images. Before analyzing the two problems, we present the working mechanism of this digital technology.

GOOGLE EARTH

Google Earth is a software that connects to GPS location satellites (*Global Positioning System*), allowing geolocation in real time from any point on the Earth's surface. This application uses the geographic coordinates of points on the Earth's surface as a reference, which makes it possible to measure the distance between any two points and trace routes for travel, walks and physical activities, such as walking, running and cycling. This technology is present in devices smartphones and notebooks, being widely used in vehicle tracking in delivery systems and urban transport applications.

The GPS is probably the most common example of the application of the concepts of Einstein's theory of relativity (ALMEIDA, 2022). The GPS constellation around the Earth is composed of 32 satellites, 24 of which are functional satellites – Figure 1(a) – with atomic clocks in your interior. Each of these satellites weighs almost a ton, measures about 5 meters in diameter and has an average lifetime of 10 years. Spare satellites are activated in case of problems or technical failures. Any point on the Earth's surface can be located by at least four satellites (UNICAMP, [S.d.]).

The connection between the receiving device and the satellites occurs through electromagnetic radio waves that propagate in the air with a speed close to the speed of light, which is equal to c = 299,792,458 m/s. Localization occurs in a process called trilateration. In this process, three satellites define the position of the receiver on the Earth's surface, while a fourth satellite indicates the altitude at which the receiver is in relation to sea level. The intersection of the catchment areas of the three satellites is the point at which the receiver is located – Figure 1(b). To determine the distances between the receiver and each satellite, the GPS calculate in the equation $\Delta s = v_m \Delta t$ – where Δs, v_m and Δt symbolize, respectively, the variation in distance, the average speed and the variation in time - the time difference between the emission and reception of the signal sent by the satellite and multiply this difference by the speed of light.

Temporal relativity (WOLFSON, 2005) in GPS is subtle, as the satellites that scan the Earth's surface are at a height of 20 km and travel in the atmosphere at an average speed of 14,000 km/h. The motion of these satellites with respect to time has a practical time difference on Earth of approximately 38 μs (microseconds, 1 μs = 10^{-6} s) per day. This way, the satellites' atomic clocks are programmed to delay 38 μs per day, matching their times to the times on Earth. In time, this difference is imperceptible. However, in the GPS system and in the calculation of distances, the difference between the points where the receiver really is and where it appears in the GPS system would be 11 km.

Modern smartphone devices, which send and receive signals to GPS satellites, display information in the form of geographic coordinates on a map. This way, this information is available in location apps (GoogleMaps, Waze) and sports apps for walking or cycling (RunKeeper, Strava), as well as apps for augmented reality games (PokémonGo).

MOTIVATING PROBLEMS

Solving mathematical problems requires specific skills, including mastery of concepts

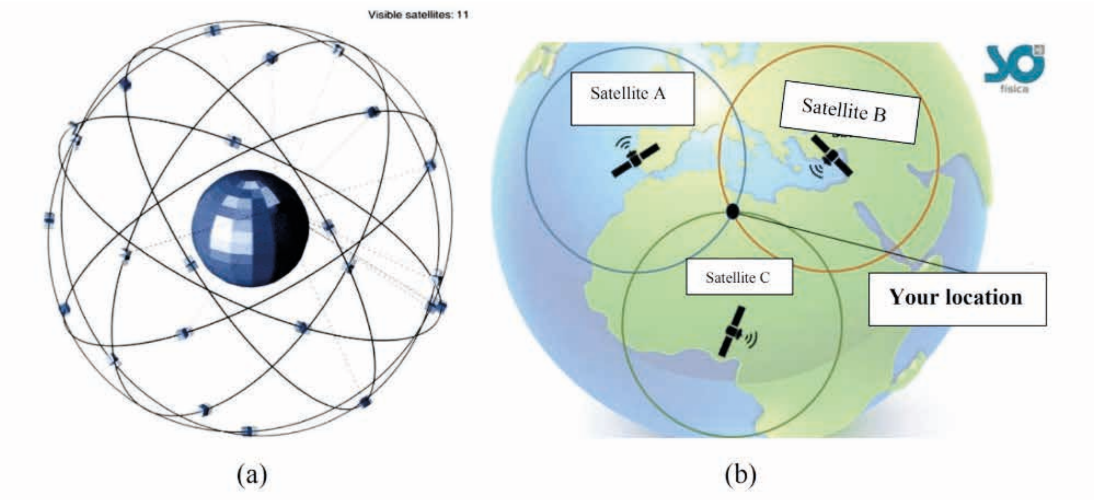


Figure 1: Trilateration in the GPS system: (a) satellite system; (b) geolocation.
Source: (a) Unicamp ([S.d.]); (b) Virtuous Information Technology ([S.d.]).

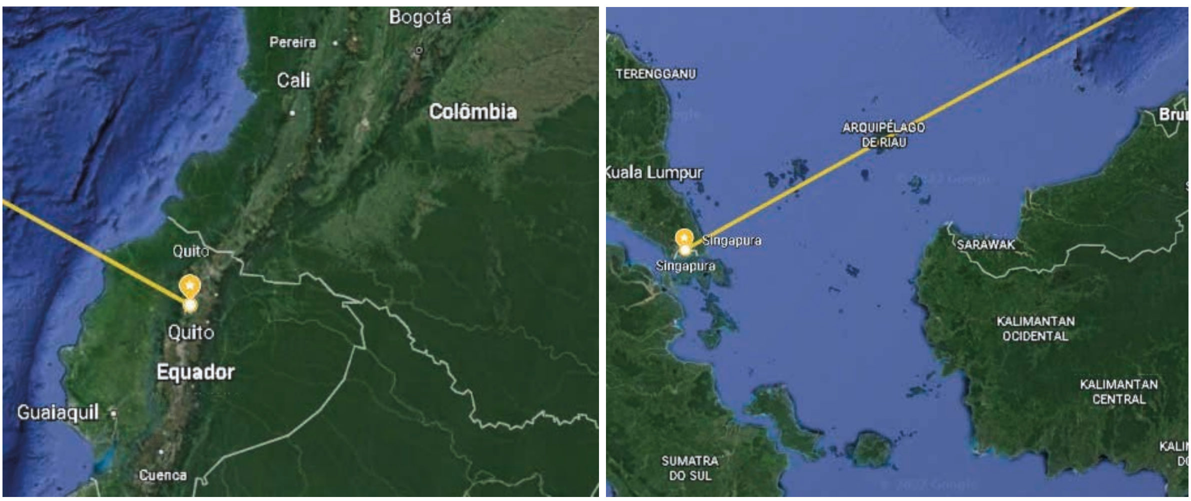


Figure 2: Location on Google *Earth*: (a) Quito; (b) Singapura.
Source: The authors using *Google Earth* (2022).

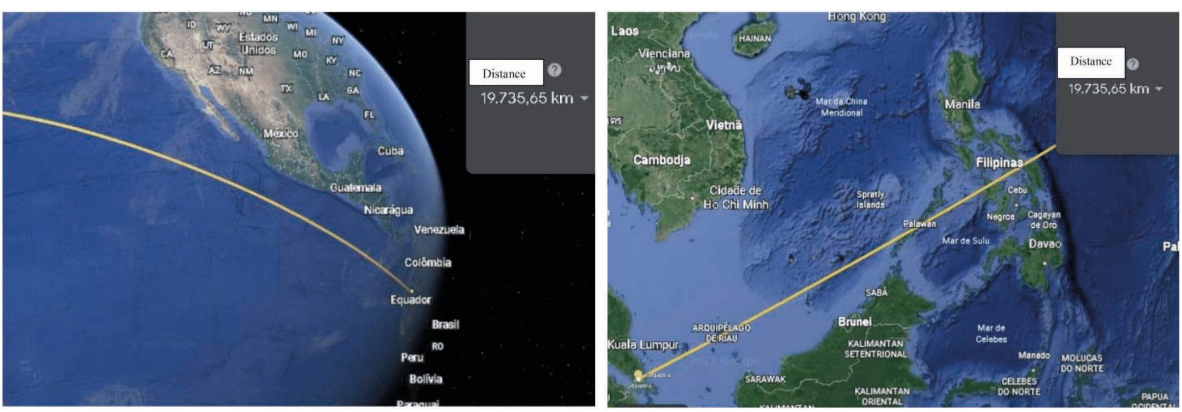


Figure 3: Distance between the cities of Quito and Singapore according to the *Google Earth*.
Source: The authors using *Google Earth* (2022).

and relationships. These skills are essential for students to obtain satisfactory results in official exams, such as the ENEM (National High School Examination), the National Student Performance Examination (ENADE), particularly the exam applied to students of Mathematics-Teaching Degree Course, and exams for entry into technical and higher-level courses.

Researching some of these exams, we selected two geometric problems that allow us to extrapolate the spectrum of mathematical knowledge required in the solution. By reassessing it, we can confront geometric concepts using digital technologies, such as Google Earth. An introduction to the use of this application, mainly regarding the structuring of a project, the location of points on the Earth's surface and the use of the tool "Measure distance and area", can be found in Motta e Nós (2022) and Nós e Motta (2021).

PROBLEM 1: QUESTION OF ENEM

> The cities of Quito and Singapore are close to the equator and at diametrically opposite points on the globe. Considering the radius of the Earth equal to 6370 km, it can be stated that a plane leaving Quito, flying an average of 800 km/h, excluding stopovers, arrives in Singapore in approximately:
>
> (A) 16 hours.
>
> (B) 20 hours.
>
> (C) 25 hours.
>
> (D) 32 hours.
>
> (E) 36 hours.
>
> Question 55 of Test 1 - Yellow of ENEM 2002 (BRAZIL, 2002, p. 21).

In Problem 1, the words *radius* and *diametrically opposite* they are the "clues" for us to use geometric concepts related to the circumference to establish an initial plan for the solution.

This way, using $\pi \approx 3.14$ and r = 6370 km, where r is the measure of the Earth's radius,

we determine the length C of the circumference:

$C = 2\pi r = 2(3.14)(6370) = 40{,}003.6\ km.$

As the cities of Quito and Singapore are diametrically opposed, the distance d between them is given by half the length C, that is:

$$d=\frac{C}{2} = 20{,}001.8\ km.$$

To complete the solution, we need the physical relationship between the time variation quantities Δt, distance d and average speed v_m. So, using d=20,001.8km and v_m=800 km/h, we conclude that:

$$\Delta t = \frac{d}{v_m} = \frac{20{,}001.8\ km}{800\ km/h} \approx 25\ h.$$

Therefore, the solution to the problem is given by alternative C.

As we reassess the solution, some questions naturally arise:

1. Why is the distance between the two cities not linear? Why do we calculate the circumference of a circle?

2. Is the distance between Quito and Singapore really around 20,000 km? How can we validate this result?

These questions allow us to confront concepts of Euclidean geometry and present concepts of spherical geometry, such as the notion of distance between two points on a spherical surface and, consequently, the concept of a spherical line (MOTTA, 2018; DORIA, 2019; BRANNAN; ESPLEN; GRAY, 2012). In this task, Google Earth can be used to reassess the solution. The initial step is to locate the two cities in the application, as shown in Figure 2.

After locating, the "Measure distance and area" tool allows us to determine the distance between the two cities. Google Earth indicates that the distance between Quito and Singapore is 19,735.65 km – Figure 3, a value approximately 1.3% smaller than that calculated to solve the ENEM issue. In the discussion about the reasons for this difference, the teacher can list some questions for the debate: the irregularities of the ground (one of the satellites in the trilateration system determines the height in relation to sea level); the approximation used for the constant π; Earth has an equatorial radius and a polar radius, that is, it is not perfectly spherical.

PROBLEM 2: IFPE QUESTION

> The Vatican is a country recognized by the United Nations (UN). Located in the northern part of the city of Rome, it is considered the smallest country in the world, measuring 0.45 in size. If the Vatican were in the shape of a circle, what would be the square of its radius?
>
> (Use the approximation π=3)
>
> a) 0.13 km^2
>
> b) 0.14 km^2
>
> c) 0.15 km^2
>
> d)0.16 km^2
>
> e) 0.17 km^2
>
> Question 13 of the 2020.1 Selection Exam for PROEJA Integrated Technical Courses of the IFPE (IFPE, 2019, p. 7).

In problem 2, the words *circle, radius* and *extent* are the "tips" to establish a plan for the solution. Associating the word extension with the geometric concept of area, symbolized by $\mathcal{A}$, and employing π=3, we have to:

$$\mathcal{A}(circle) = \pi r^2;$$

$$0.45\ km^2 = 3r^2;$$

$$r^2 = \frac{0.45\ km^2}{3};$$

$$r^2 = 0.15\ km^2.$$

Therefore, the solution to the problem is given by alternative C.

In the reassessment of the solution, some questions are inherent:

1. Is the shape of the Vatican circular?
2. What was the source used to state that the area of the Vatican is 0.45 km^2F? Can we compare this data?

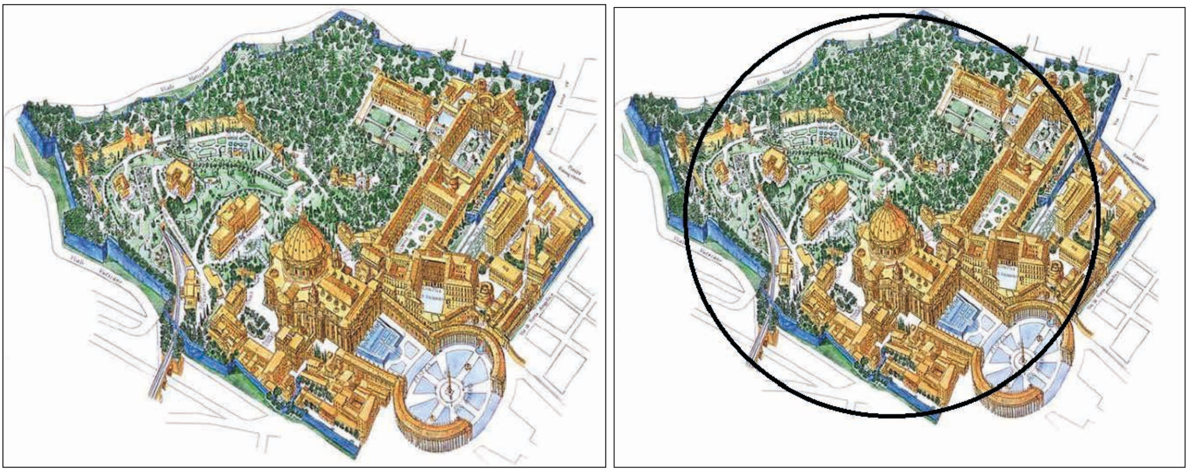
As for the first question, Figure 4(a) illustrates the format of the Vatican. Observing it, we find that the area of the Vatican can be approximated by the area of a circle – Figure 4(b). However, the most appropriate format is that of a polygon with many sides.

Regarding the second question, we were able to use Google Earth to approximate the Vatican area. The first step is to locate the country. Next, we need to mark the points to define the polygon whose area we want to measure. Figures 5 and 6 illustrate two approximations by polygons with few sides, almost rectangular. Figure 7, on the other hand, illustrates an approximation with many points, defining an outline closer to the actual shape of the Vatican.

In the multipoint approximation – Figure 7, Google Earth assigns to the Vatican an area of approximately , measurement about 15.6 % greater than that constant in Problem 2. How to justify the difference? The teacher can list some questions for discussion: the bibliographical reference, which cites the Vatican area, used in the elaboration of Problem 2; the number of points used to define the outline of the Vatican; ground irregularities (one of the satellites in the trilateration system determines the height in relation to sea level); Earth is not perfectly spherical.

CONCLUSION

In this work, we analyze the solution of two geometric problems proposed in official



(a) (b)

Figure 4: Vatican: (a) map; (b) area approximated by a circle.
Source: (a) Pinterest ([S.d.]); (b) the authors.

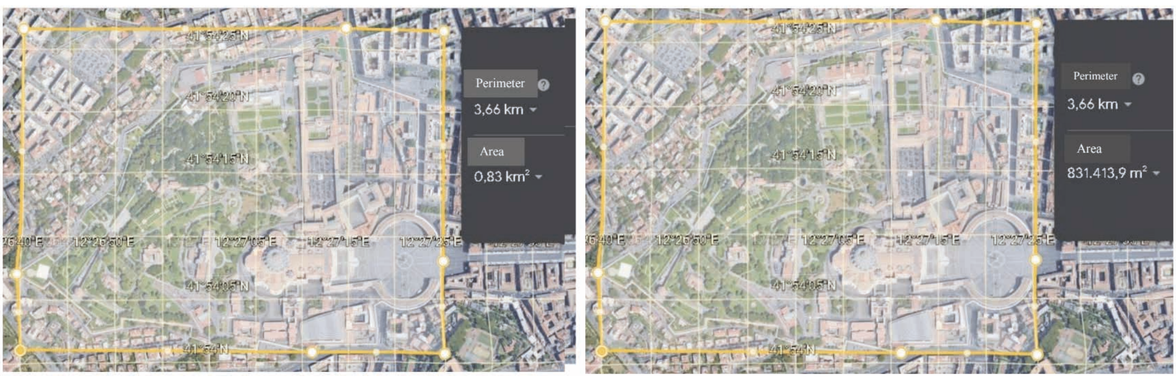


Figure 5: Perimeter and area of the Vatican: first approximation.
Source: The authors using *Google Earth* (2022).

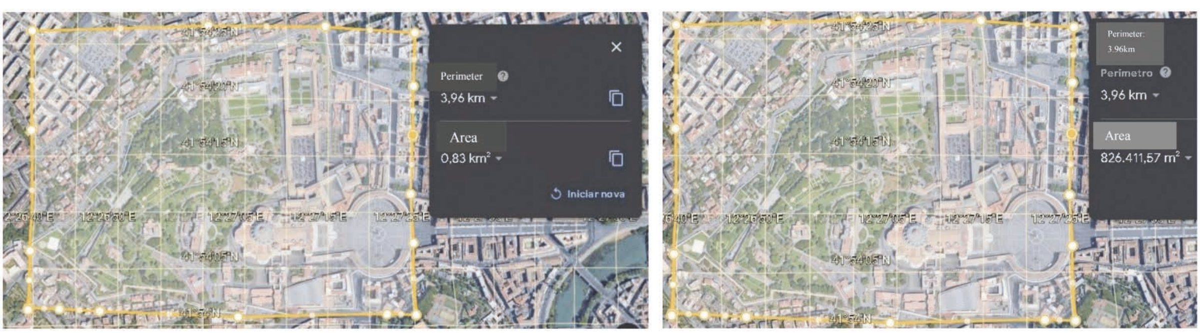


Figure 6: Perimeter and area of the Vatican: second approximation.
Source: The authors using *Google Earth* (2022).

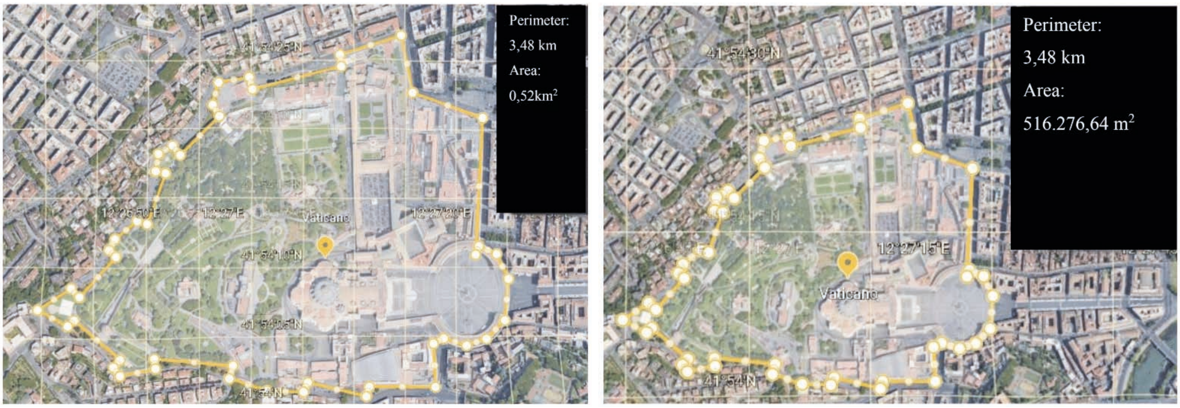


Figure 7: Perimeter and area of the Vatican: third approximation.
Source: The authors using *Google Earth* (2022).

Figure 8: Map of the US state of Colorado.
Source: Loeffler and Dietz (2022).

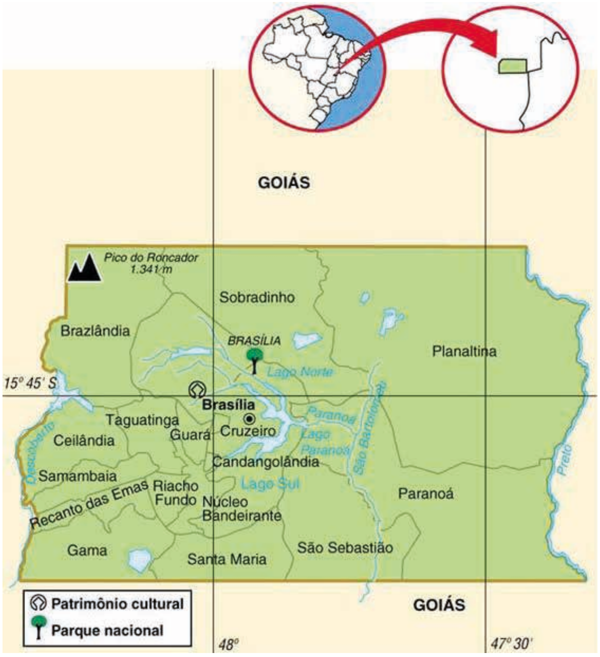


Figure 9: Map of the Federal District.
Source: Mendes (2011).

exams. In the retrospective stage, according to the stages of solving a problem proposed by George Pólya (PÓLYA, 1945, 1995), we reassess the results of plane Euclidean geometry according to conceptions of spherical geometry. In this process, we use Google Earth, a digital technology for geolocation.

The two selected problems are different in terms of difficulty: the ENEM question – Problem 1 – relates geometric concepts, such as the length of the circumference, with physical concepts, such as distance, time and speed, and geographic concepts, such as geolocation; the IFPE question – Problem 2 – addresses only the geometric concept of area of the circle. However, both allow exploring the calculation of distances and areas on the Earth's surface. The use of Google Earth enables/motivates this way to extrapolate valid "local" geometric concepts in Euclidean geometry (EUCLIDES, 2009) and to introduce concepts of non-Euclidean geometries (WOLFE, 2012; MOTTA, 2018; DORIA, 2019; BRANNAN; ESPLEN; GRAY, 2012; ALBON; NÓS, 2022).

This extrapolation of geometric concepts is important since the BNCC (BRAZIL, 2018) does not establish minimum parameters for teaching non-Euclidean geometries in Elementary and High School. On the other hand, the Curriculum Guidelines for Basic Education – Mathematics of the State Department of Education of Paraná (SEED) (PARANÁ, 2008) establish parameters for teaching non-Euclidean geometries in Basic Education.

Google Earth can be used in mathematics classes in Basic Education, including Elementary School - Early Years. At this level, students can "play" with the application marking points to best define the outline of a selected region and make associations with geometric figures. In the other levels, when using Google Earth to check large areas on the Earth's surface, we suggest that the teacher start by exploring regions with triangular shapes, such as the Bermuda Triangle or Devil's Triangle (MOTTA; NÓS, 2022; NÓS; MOTTA, 2021), or almost rectangular, such as the State of Colorado – Figure 8 – and the Federal District (Brasília) – Figure 9 (ALMEIDA, 2022).

We hope that this work will contribute to the discussion on problem solving, whether as a methodology or just as a motivating element in the teaching-learning process, and mainly regarding the use of digital technologies, thus meeting the general and specific competences of the BNCC for teaching mathematics.

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