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VERTICAL DYNAMICS ANALYSIS OF A UTV VEHICLE, FROM A MODEL WITH 7 DEGREES OF FREEDOM IN IDEALIZED OBSTACLES

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Abstract: The stochastic vibrations are considerable in the operation of any UTV vehicle (Utility Task Vehicle), being necessary to analyze the dynamic behavior. The present work is based on the elaboration of a model with 7 degrees of freedom, *full-car model*, with the purpose of delimiting the behavior of the mode of vibration, under the excitations of certain obstacles modeled analytically. These modes are represented by the movements occurring in the sprung mass: *bounce*, *roll* and *pitch*; and those associated with unsprung masses: *wheel hop* of each of the 4 independent suspensions. Due to the excitation forces, there must be a dynamic balance between the internal forces acting, since there is displacement of the mass elements and there are deflections in the elastic and damping elements. The present work also provides the obtainment of the natural frequencies and the mode shapes, through the development of an algorithm, such that it provides the understanding of the movement of the proposed model. As a result, natural frequencies and satisfactory transmissibilities are obtained.
Keywords: Stochastic vibrations, off-road vehicles, modes of vibrating, excitation forces, transmissibility.

INTRODUCTION

The operation of a vehicle on any track is a process characterized by the performance of vibrations of a stochastic nature, that is, induced forces arise based on random occurrences. It is intuitive to realize that no surface is perfectly smooth, especially when it comes to *off-road routes*, delimited by the presence of various undulations. Currently, there is a growing search for *motorsports practices*, being carried out as recreational activities, in regions with unconventional routes. According to a report by MORDOR INTELLIGENCE (2020), the *global off-road market*, in 2021 with an investment of 14 billion dollars, must reach 18 billion dollars by the year 2026.

This way, it is considerable that the vehicle can overcome different types of obstacles, without any failure of its components or any offer of danger to users. For this, it is worth that the performance of the suspension system provides the storage, through elastic and inertia elements, and the dissipation, through damping elements, of the energy from stochastic vibrations induced by the path. According to Rao (2008, p. 06): "The Theory of Vibration deals with the study of oscillatory movements of bodies and the forces associated with them", being a useful method for understanding the occurrence of damage and, therefore, the modes of damage failure. It must be noted that the transmissibility of forces can cause discomfort to passengers and cause loss of vehicle drivability.

In fact, the use of simulation models, around the beginning of the 1980s, provided great leaps in the conception of mechanical projects, that is, it is no longer necessary to manufacture vehicles by "trial and error", as was the case in the beginning of the automobile industry. Determining the maximum loadings and lifetime of the machine, inhibits the need to create numerous test prototypes and, consequently, significantly reduces costs and the period of time required for product development.

OBJECTIVES

- Suggest a methodology for dimensioning the vertical dynamics of an *off-road vehicle,* applicable to any vehicle, based on a logical sequence;
- To model, analytically, obstacles present in competition tracks, as a way to generate the excitations responsible for the induction of mechanical vibrations;
- To develop a numerical algorithm,

based on a *full-car model,* in order to provide the quick performance of the calculations that result in the acting internal forces;

- Analyze the data obtained and make a final opinion regarding responses to excitations.

THEORETICAL BACKGROUND

The operation of a vehicle is accompanied by the presence of stochastic loadings. The forces are characterized as static, in which the amplitude remains constant or varies infinitely in the domain of time; or dynamic, denoted by significant fluctuations in this domain. Basically, any structure that is in static or dynamic equilibrium presents a respective equilibrium of all its constituent infinitesimal elements. Each system has an intrinsic rigidity, depending on its geometry and material, which behaves linearly or non-linearly. In short, the linear regime is guided by negligible variations in the rigidity of the structure. Being the non-linear regime, totally the opposite.

Vibrating systems, in general, can be subject to two conditions: free, in which the application of force is given only at the initial instant and then the system vibrates freely; or forced, with the continuous action of force throughout the operation. The latter being the type of vibration prevailing in a vehicle, due to the fact that there is constant excitation of the elastic components, from random forces. It is important to highlight that vibrating systems can be provided with damping, that is, at each cycle of oscillation, part of the energy is dissipated through dissipative forces. Whereas if such dissipation is negligible, the vibration is configured as undamped, in which the system, theoretically, does not present energy dissipation.

DYNAMIC ANALYSIS OF MULTIPLE DEGREES OF FREEDOM

The analysis of systems with a single degree of freedom is fundamental for understanding the behavior of discretized structures in multiple degrees of freedom, especially because they can be treated based on simple mass-spring-damper systems. Therefore, this concept is important for any model, as stated by Wright *et al.* (2015, p. 27): "The term 'discrete parameter (or sometimes concentrated mass)' implies that the system in question is a combination of discrete rigid masses (or components), interconnected by elastic's and damping's elements".

With regard to the multiple degrees of freedom, the nodal displacements mutually interfere with each other, that is, if the force is applied to only one "node" of the system, all the others tend to respond. This is explained by the fact that the system is in some kind of balance, causing internal forces to arise within the elements, as a way of providing balance with external forces. This way, each degree of freedom has a given coordinate for the description of its movement. Figure 1 shows a system with three degrees of freedom, under the action of any force.

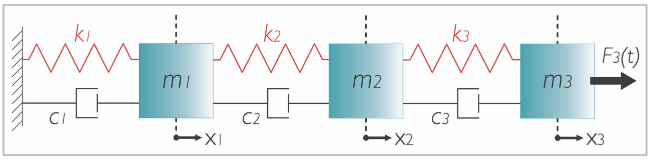

FIGURE 1. Forced system with three degrees of freedom. Source: Own authorship (2022).

In question, the number of degrees of freedom predicts the amount of equations to be applied. Thus, discrete systems are represented in matrix form, as a way of facilitating the performance of mathematical operations, in which the constants are presented as matrices and the variables as vectors. The Equation of Motion, Equation 1, can be changed to any type of discrete system, where [M], [K] and

[C] are the matrices of mass, stiffness and damping, respectively. Furthermore, $\{\ddot{x}\}$, $\{\dot{x}\}$ and {x} are the acceleration, velocity and nodal displacement vectors, in that order, with the loading also in this notation and represented by the vector denoting the excitation force $\{f(t)\}$.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\} \quad (1)$$

UNDAMPED FREE VIBRATIONS

A corresponding number of modes of vibration is admitted, as far as the degrees of freedom are concerned. As in systems with one degree of freedom, the response to free vibration can be obtained by attributing the solution to the Equation of Motion itself, resulting in the equation of equilibrium. Considering the hypothesis that the system is not being excited and the damping is negligible, one can arrive at the expression that relates the natural frequencies, to the respective nodal displacement profiles, as seen in Equation 2. This denotes a problem of eigenvalues and eigenvectors.

$$([K] - \lambda_i\,[M])\,\{X_i\} = 0 \quad (2)$$

The problem admits that there are two types of solution: a non-trivial one, such that the calculation of the determinant satisfies Equation 3, which is called the system's frequency equation; and a trivial one, where $\{X_i\}$ equals zero. For each non-trivial solution, there is an eigenvalue (λ_i) that meets the equation, which is prescribed by the square of the natural frequency. Each eigenvalue allows obtaining a X_i corresponding eigenvector, Equation 4, responsible for predicting how the nodal displacements occur, that is, it denotes the denotes the mode shape. For each eigenvalue, there is a compatible eigenvector, with both configuring an "auto pair" belonging to the mode of vibration and which is obtained from the so-called Modal Analysis.

$$det\,([K] - \lambda_i[M]) = 0 \quad (3)$$

$$([K] - \omega_i{}^2[M])\{X_i\} = 0 \quad (4)$$

ANALYSIS OF FORCED VIBRATING SYSTEMS

For the execution of the analysis of the dynamic response to the excitation, there are two methods that guarantee the resolution of the problem: The Hypothesis of Modal Superposition (or Hypothesis of Ritz), restricted to the linear regime, in which each mode of vibration presents a certain "factor of participation" in the general response; and the Direct Integration Method (*Full Method*), in which the integration process is carried out by means of an appropriate numerical algorithm.

From the direct integration method, two classes emerge: the implicit methods, in which the response at a given instant only depends on *input*s of that same instant; and explicit methods, whose response at a given future time is based on *inputs* from previous times. In this context, implicit methods are commonly used in dynamic analyses, as they are characterized as "unconditionally stable", that is, their use is always endowed with the convergence of results. However, they present higher computational time.

On the other hand, the explicit methods are ideal for such cases in which the propagation of vibrations takes place in sufficiently small-time intervals, in order to visualize how the system behaves with greater precision, in addition to having a lower computational cost. However, this method has the inconvenience of being "conditionally stable", that is, it depends on the appropriate *timestep* for the results to converge. The Central Difference Method, for multiple degrees of freedom, is a commonly used explicit method, in which the nodal (or mass) displacements are given by Equation 5 and the nodal velocities and accelerations by means of Equations 6 and 7, respectively, according to RAO (2008, p. 380).

$$\{x_{i+1}\} = \left[\frac{1}{(\Delta t)^2}[M] + \frac{1}{2\Delta t}[C] \right]^{-1} \cdot \left[\{f_i\} - \left([K] - \frac{2}{(\Delta t)^2}[M]\right)\{x_i\} - \left(\frac{1}{(\Delta t)^2}[M] - \frac{1}{2\Delta t}[C]\right)\{x_{i-1}\} \right] \quad (5)$$

$$\{\dot{x}_i\} = \frac{1}{2\Delta t}\left[\{x_{i+1}\} - \{x_{i-1}\} \right] \quad (6)$$

$$\{\ddot{x}_i\} = \frac{1}{(\Delta t)^2}\left[\{x_{i+1}\} - 2\{x_i\} + \{x_{i-1}\} \right] \quad (7)$$

Transient analyzes are applied to vibrating systems that respond to an excitation that is imposed "step-by-step". The choice of *timestep* is of paramount importance, in which BATHE (1996, p. 814) recommends adopting it according to Equation 8, where ω_{max} is the highest natural frequency of interest, in radians per second, and that comes from the smallest natural period (T_{min}).

$$\Delta t \leq \frac{1}{20} T_{min} = \frac{1}{20} \cdot \frac{2\pi}{\omega_{max}} \quad (8)$$

For a transient analysis, knowing the natural frequencies of the system is fundamental to be able to define the correct *timestep*. Also, it is important to understand the excitation frequencies of the system, in order to verify if such value is small enough for the adequate treatment of the *loadsteps* (ΔF), that is, discrete increments of force, seen in Figure 2.

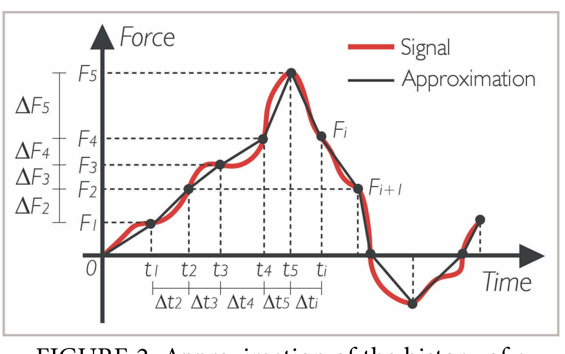

FIGURE 2. Approximation of the history of a loading source. Adapted from Rao (2008).

DYNAMIC RESPONSE TO ANY LOADING

Any periodic behavior can be treated by the Fourier Series, which is based on the sum of infinite terms, containing sine and cosine functions. The negotiation is carried out by approximating the loading, using the superposition of duly known sinusoidal functions, that is, a "sum of harmonic functions", according to Equation 9.

$$F(t) = \frac{a_o}{2} + \sum_{i=1}^{\infty}\left(a_i \cdot \cos(i \cdot \omega \cdot t) + b_i \cdot sen(i \cdot \omega \cdot t)\right) \quad (9)$$

In cases where the loading is not represented by a periodic function, that is, it does not present the repetition of its configuration in a given period, the treatment must be carried out through the Fourier Transform. Thus, it is considered that non-periodic loadings present the oscillation period that tends to infinity. This means that the Fourier Series becomes an integral that is manipulated based on limit operations. According to Alves Filho (2008, p. 232): "We can observe that if the period grows indefinitely, the Fourier Series coefficients (a_o, a_i and b_i) will decrease indefinitely, as long as the integrals exist for $T \rightarrow \infty$".

Therefore, the spectra of non-periodic functions are of continuous nature and the spectra of periodic functions are of discrete nature. In discrete time signals, the loading can be treated through the algorithm of the FFT (Fast Fourier Transform). Figure 3 (*a*) shows the frequency spectrum for periodic loading; in Figure 3 (*b*) for non-periodic loading.

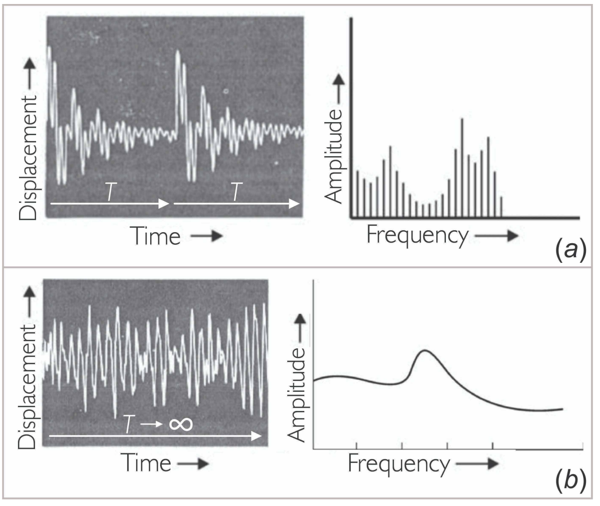

FIGURE 3. Frequency spectra processed using Fourier analysis. (*a*) Periodic loading. (*b*) Non-periodic loading. Source: Adapted from Alves Filho (2008).

VERTICAL DYNAMICS

In all cases of requests, it is essential to use elastic elements, such as springs (k), to store mechanical energy and consequently control the relative displacement between the wheels and the chassis. The tire is also considered as a type of elastic element and is endowed with an equivalent elastic constant (kt). In addition to these, the damping elements, denoted essentially by dampers (c), are components responsible for the partial dissipation of energy, at each executed cycle.

The Vertical Response Analysis is based on dynamic models composed of mass-spring-shock absorber systems, allowing to understand how the dynamic response of the vehicle occurs, through the different excitation conditions. This is a dynamic balance, which can be modeled through the Equation of Motion and which allows considering the inertial force arising from the vertical movement of the vehicle's masses, in addition to the elastic and damping forces arising from the suspension components.

The dynamic model endowed with 7 degrees of freedom, is denoted by considering the vehicle "completely", being called *full-car model*. This model presents good accuracy in the dynamic responses obtained and, consequently, greater complexity in its formulation. Furthermore, it allows *pitch* and *roll* movements to be analyzed simultaneously with vertical *bounce* and *wheel hop* movements. As seen in Figure 4, applying excitation to just one tire causes the entire system to vibrate simultaneously.

> A vehicle's fundamental *bounce frequency* is in the range of up to *1.0 Hz.* [...] High-performance cars, which sacrifice ride comfort in exchange for better handling characteristics, have their suspension spring stiffness selected for the natural frequency range of 2 to 2.5 Hz. (NICOLLAZI *et al.* , 2012, p. 282).

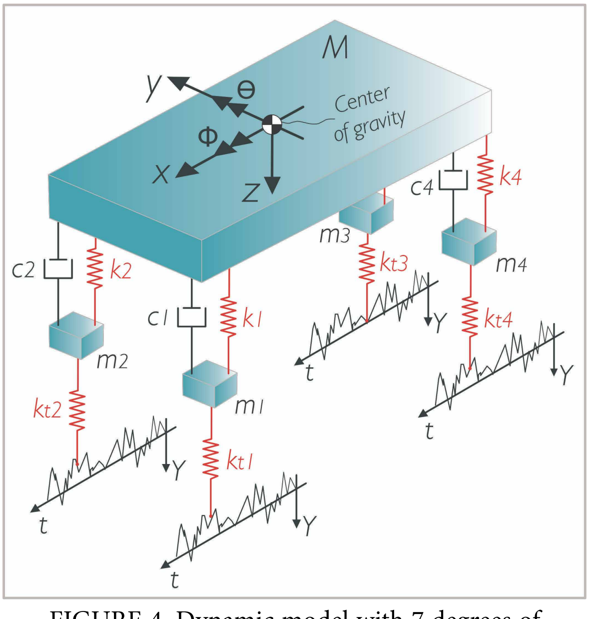

FIGURE 4. Dynamic model with 7 degrees of freedom. Source: Adapted from Nicolazzi *et al.* (2012).

In order for the spring-shock absorber system to act collinear to the vertical axis, the elastic constant is geometrically corrected by the installation ratio (IR), implying an equivalent stiffness, called *wheel center rate* (k_w) and which is calculated by Equation 10, as prescribed by MILLIKEN *et al.* (1995, p. 114). The same applies to damping (c_w), in Equation 11.

$$k_w = k.(IR)^2 \quad (10)$$
$$c_w = c.(IR)^2 \quad (11)$$

METHODOLOGY

The present work consists in the elaboration of a methodology for the analysis of the vertical dynamics of *off- road vehicles*. Therefore, the loading history is obtained through mathematical functions, delimiting displacement profiles similar to reality. The execution of the steps, Figure 5, starts from the elaboration of an algorithm in the MATLAB® *software*, according to NASCIMENTO (2022, p. 102), starting from the parameters of the BJT06 vehicle, of the Baajatinga Baja SAE Team.

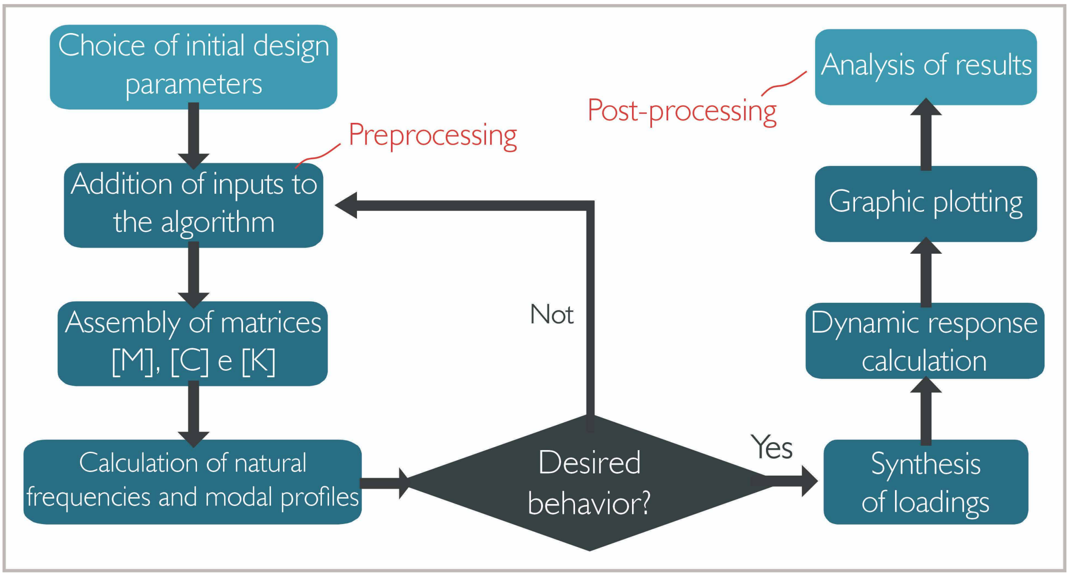

FIGURE 5. Algorithm execution flowchart. Source: Own authorship (2022).

Basically, the natural frequencies and the respective mode shapes are obtained and then the dynamic response is calculated using the Central Difference Method. Finally, there is the graphic plot of the magnitudes: response to the displacement of the masses and response to the force in the elastic elements. The design of the model is based on certain considerations that simplify its application: the vehicle only performs uniform rectilinear movements, with inertial references; lateral and longitudinal forces are neglected; the CG (Center of Gravity) is symmetrical in terms of the transverse axis; the dynamic model is based on linear vibrations; the installation ratio is considered; the chassis is rigid and the tire is in punctual contact with the patch.

MODEL DEVELOPMENT WITH 7 DEGREES OF FREEDOM

The first stage consists of agreeing on the directions for each displacement, since the masses present relative movements to each other. In the degrees of freedom of rotation, there is a dynamic balance between the inertial moments and the moments arising from the elastic and damping forces. Similarly, the translational degrees are denoted by the balance between the vertical forces. Figure 6 shows the FBD (Free Body Diagram) for each degree of freedom.

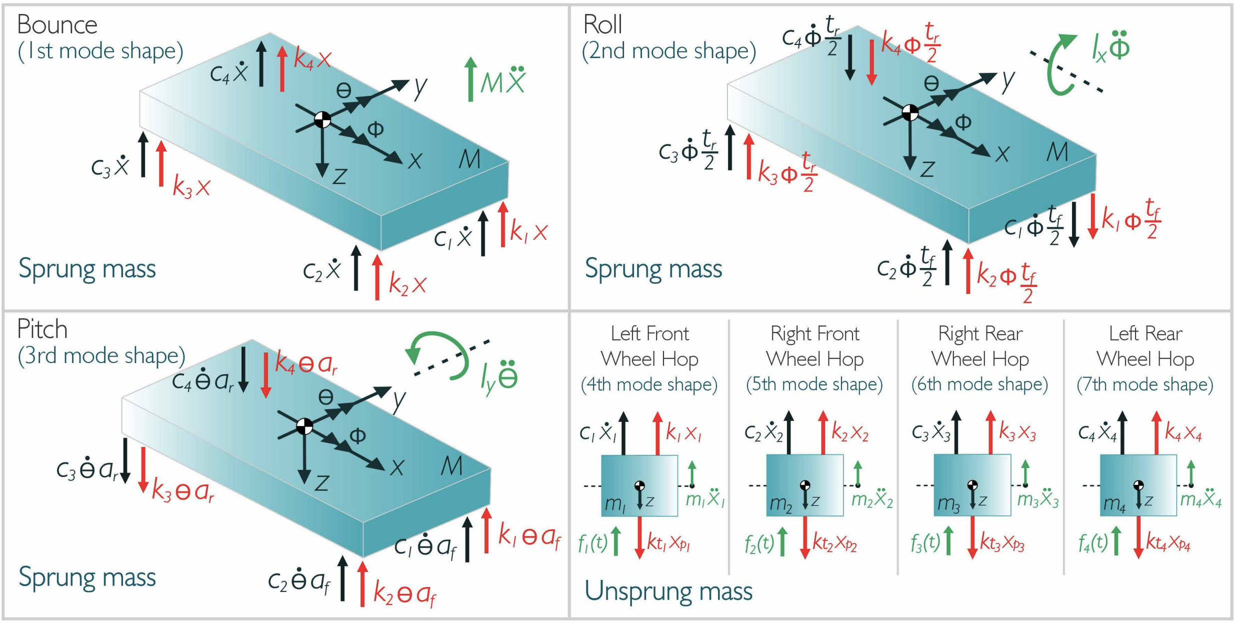

FIGURE 6. Free-body diagram, for the modes of vibration. Source: Own authorship (2022).

The excitation forces (*f(t)*), promote the displacement of the masses and the deflection of the elastic elements. Energy is introduced and distributed among these 7 coupled modes, with a dynamic balance between internal and external forces, as evidenced in Equations 12 to 18.

$$M\ddot{x} + c_1\dot{\delta}_1 + c_2\dot{\delta}_2 + c_3\dot{\delta}_3 + c_4\dot{\delta}_4 + k_1\delta_1 + k_2\delta_2 + k_3\delta_3 + k_4\delta_4 = 0 \quad (12)$$

$$I_x\ddot{\phi} - c_1\frac{t_f}{2}\dot{\delta}_1 + c_2\frac{t_f}{2}\dot{\delta}_2 + c_3\frac{t_r}{2}\dot{\delta}_3 - c_4\frac{t_r}{2}\dot{\delta}_4 - k_1\frac{t_f}{2}\delta_1 + k_2\frac{t_f}{2}\delta_2 + k_3\frac{t_r}{2}\delta_3 - k_4\frac{t_r}{2}\delta_4 = 0 \quad (13)$$

$$I_y\ddot{\theta} - c_1a_f\dot{\delta}_1 - c_2a_f\dot{\delta}_2 + c_3a_r\dot{\delta}_3 + c_4a_r\dot{\delta}_4 - k_1a_f\delta_1 - k_2a_f\delta_2 + k_3a_r\delta_3 + k_4a_r\delta_4 = 0 \quad (14)$$

$$m_1\ddot{x}_1 - c_1\dot{\delta}_1 - k_1\delta_1 + kt_1\delta p_1 = f_1(t) \quad (15)$$

$$m_2\ddot{x}_2 - c_2\dot{\delta}_2 - k_2\delta_2 + kt_2\delta p_2 = f_2(t) \quad (16)$$

$$m_3\ddot{x}_3 - c_3\dot{\delta}_3 - k_3\delta_3 + kt_3\delta p_3 = f_3(t) \quad (17)$$

$$m_4\ddot{x}_4 - c_4\dot{\delta}_4 - k_4\delta_4 + kt_4\delta p_4 = f_4(t) \quad (18)$$

After correcting the installation ratio, the relative displacements (δ) between the sprung and unsprung masses, start to denote the deflections suffered by the springs. Analogously, there are relative velocities ($\dot{\delta}$) between such masses, which denote the damping velocities. The amplitudes of the obstacles (xp) are present in the relative displacements between these and the unsprung masses, that is, we have the elastic deflections of each tire (δp). Table 1 presents the expressions that allow obtaining such relative kinematic variables, where and are the distances between the front track width (t_f) and rear track width (t_r), respectively, to the CG.

relative movement	Acronym	Expression
Sprung mass and left front unsprung mass	δ_1	$x - \phi\frac{t_f}{2} - \theta a_f - x_4$
	$\dot{\delta}_1$	$\dot{x} - \dot{\phi}\frac{t_f}{2} - \dot{\theta} a_f - \dot{x}_4$
Sprung mass and right front unsprung mass	δ_2	$x + \phi\frac{t_f}{2} - \theta a_f - x_5$
	$\dot{\delta}_2$	$\dot{x} + \dot{\phi}\frac{t_f}{2} - \dot{\theta} a_f - \dot{x}_5$
Sprung mass and right rear unsprung mass	δ_3	$x + \phi\frac{t_r}{2} + \theta a_r - x_6$
	$\dot{\delta}_3$	$\dot{x} + \dot{\phi}\frac{t_r}{2} + \dot{\theta} a_r - \dot{x}_6$
Sprung mass and left rear unsprung mass	δ_4	$x - \phi\frac{t_r}{2} + \theta a_r - x_7$
	$\dot{\delta}_4$	$\dot{x} - \dot{\phi}\frac{t_r}{2} + \dot{\theta} a_r - \dot{x}_7$
Left front unsprung mass and obstacle amplitude	δp_1	x_4 - xp
Right front unsprung mass and obstacle amplitude	δp_2	x_5 - xp
Right rear unsprung mass and obstacle amplitude	δp_3	x_6 - xp
Left rear unsprung mass and obstacle amplitude	δp_4	x_7 - xp

TABLE 1. Expressions for calculating relative kinematic variables. Source: Own authorship (2022).

In order to obtain matrices expressions that facilitate the calculations, the variables related to Equations 12 to 18 can be substituted, so that all the constants can be put in evidence of the variables that denote the movement. The mass matrix, Frame 1, is formed by terms that come from the acceleration variables ($\ddot{X}$). In this case, only the main diagonal is non-zero, since these are inertial reference frames and there are no couplings. I_x and I_y are the moments of mass in *roll* and *pitch*, respectively, given by the product of the mass and the square of the radius of gyration.

M	0	0	0	0	0	0
0	I_x	0	0	0	0	0
0	0	I_y	0	0	0	0
0	0	0	m_1	0	0	0
0	0	0	0	m_2	0	0
0	0	0	0	0	m_3	0
0	0	0	0	0	0	m_4

Frame 1. Mass Matrix. Source: Own authorship (2022).

The stiffness matrix is formed by constants that accompany the relative displacement variables (δ e δp), Frame 2, in which the symmetry in relation to the main diagonal is noted, as it is a symmetrical vibratory system. Each cell denotes the existing interdependence between the pairs of degrees of freedom, constituting a force-displacement relationship between both, that is, the displacement imposed by one causes the force applied to the other and vice versa. Because the model is intended for vehicles with independent suspensions, the null cells are duly expected, since there is no coupling relationship between the unsprung masses themselves.

$k_1+k_2+k_3+k_4$	$\frac{t_f}{2}(k_2-k_1)+\frac{t_r}{2}(k_3-k_4)$	$a_f(-k_1-k_2)+a_r(k_3+k_4)$	$-k_1$	$-k_2$	$-k_3$	$-k_4$
$\frac{t_f}{2}(k_2-k_1)+\frac{t_r}{2}(k_3-k_4)$	$\left(\frac{t_f}{2}\right)^2(k_2+k_1)+\left(\frac{t_r}{2}\right)^2(k_3+k_4)$	$\frac{a_f t_f}{2}(k_1-k_2)+\frac{a_r t_r}{2}(k_3-k_4)$	$k_1\frac{t_f}{2}$	$-k_2\frac{t_f}{2}$	$-k_3\frac{t_r}{2}$	$k_4\frac{t_r}{2}$
$a_f(-k_1-k_2)+a_r(k_3+k_4)$	$\frac{a_f t_f}{2}(k_1-k_2)+\frac{a_r t_r}{2}(k_3-k_4)$	$a_f^2(k_1+k_2)+a_r^2(k_3+k_4)$	$k_1 a_f$	$k_2 a_f$	$-k_3 a_r$	$-k_4 a_r$
$-k_1$	$k_1\frac{t_f}{2}$	$k_1 a_f$	k_1+kt_1	0	0	0
$-k_2$	$-k_2\frac{t_f}{2}$	$k_2 a_f$	0	k_2+kt_2	0	0
$-k_3$	$-k_3\frac{t_r}{2}$	$-k_3 a_r$	0	0	k_3+kt_3	0
$-k_4$	$k_4\frac{t_r}{2}$	$-k_4 a_r$	0	0	0	k_4+kt_4

Frame 2. Stiffness Matrix. Source: Own authorship (2022).

The damping matrix is analogous to the elasticity matrix, but it is obtained through constants that follow the relative velocities ($\dot{\delta}$), as seen in Frame 3. It is important to highlight that there is also symmetry in relation to the main diagonal. Furthermore, because the damping of the tires is taken as negligible, only the damper constants are considered for the model formulation.

$c_1+c_2+c_3+c_4$	$\frac{t_f}{2}(c_2-c_1)+\frac{t_r}{2}(c_3-c_4)$	$a_f(-c_1-c_2)+a_r(c_3+c_4)$	$-c_1$	$-c_2$	$-c_3$	$-c_4$
$\frac{t_f}{2}(c_2-c_1)+\frac{t_r}{2}(c_3-c_4)$	$\left(\frac{t_f}{2}\right)^2(c_2+c_1)+\left(\frac{t_r}{2}\right)^2(c_3+c_4)$	$\frac{a_f t_f}{2}(c_1-c_2)+\frac{a_r t_r}{2}(c_3-c_4)$	$c_1\frac{t_f}{2}$	$-c_2\frac{t_f}{2}$	$-c_3\frac{t_r}{2}$	$c_4\frac{t_r}{2}$
$a_f(-c_1-c_2)+a_r(c_3+c_4)$	$\frac{a_f t_f}{2}(c_1-c_2)+\frac{a_r t_r}{2}(c_3-c_4)$	$a_f^2(c_1+c_2)+a_r^2(c_3+c_4)$	$c_1 a_f$	$c_2 a_f$	$-c_3 a_r$	$-c_4 a_r$
$-c_1$	$c_1\frac{t_f}{2}$	$c_1 a_f$	c_1	0	0	0
$-c_2$	$-c_2\frac{t_f}{2}$	$c_2 a_f$	0	c_2	0	0
$-c_3$	$-c_3\frac{t_r}{2}$	$-c_3 a_r$	0	0	c_3	0
$-c_4$	$c_4\frac{t_r}{2}$	$-c_4 a_r$	0	0	0	c_4

Frame 3. Damping Matrix. Source: Own authorship (2022).

SYNTHESIS OF OBSTACLE PROFILES

The base excitations can be obtained by the product between the elastic constant of the tires (kt) and the amplitude of the displacement of the obstacle (xp), according to the vector-excitation of Equation 19. These are forces that act only on the referred contact patch of the tires with the ground and therefore there are no direct excitations in the modes associated with sprung mass. It is a somewhat idealized approach, but it aims to assimilate to the behavior of real excitations.

$$f(t)=\begin{Bmatrix} f_1(t)\\ f_2(t)\\ f_3(t)\\ f_4(t)\\ f_5(t)\\ f_6(t)\\ f_7(t)\end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ 0\\ kt_1\cdot xp_1(t)\\ kt_2\cdot xp_2(t)\\ kt_3\cdot xp_3(t)\\ kt_4\cdot xp_4(t)\end{Bmatrix} \quad (19)$$

The lags between the front and rear wheels are considered, since both pass through the obstacles at different instants of time. The phase angle can be calculated using Equation 20, which considers the "relative natural frequency" between the rear (ω_r) and front (ω_f) wheels and the time interval between obstacles (t_d), at a given speed.

$$\alpha=(\omega_r - \omega_f)\, t_d \tag{20}$$

Table 2 describes some of the obstacles conceived and the characteristics adopted. As for obstacles with rectangular profiles, "cow ribs", their synthesis can be simplified through the use of the "pulstran()" function, from MATLAB® itself. The addition of stochastic roughness profiles is done through the "unifrnd()" function, responsible for establishing a continuous distribution of random numbers, also being phase angles, contained in the domain from 0 to 2π.

Obstacles	Functions	main *inputs*	Values adopted
sinusoidal ($\Delta t = 5 \text{ X } 10^{-3}$s)	xp(t)=Y.sin(ωt + α)	unit wavelength	10 m
		Obstacle amplitude (Y)	0.1m
beef ribs ($\Delta t = 5 \text{ X } 10^{-3}$s)	Pulstran (rectpulses) xp(t)=Y. pulstran()	Obstacle total length	10 m
		Obstacle amplitude (Y)	0.2 m

TABLE 2. Modeled obstacles, using MATLAB®, and their respective properties. Source: Own authorship (2022).

SOLUTION OF THE 7 DEGREES OF FREEDOM MODEL

The Modal Analysis is performed using the "eig" function, available in MATLAB®. As for the forced response, the vehicle is considered with a speed of 10 m/s. The results are arranged in solution matrices, where each line corresponds to the time history of a mode of vibration. With the relative movements, it is possible to calculate the shock absorbers forces (F_{shock}) and the elastic forces in the springs (F_{spring}), Equations 21 and 22, in addition to the forces in the tires (F_{tire}), with Equation 23.

$$F_{shock}=c\,.\,\dot{\delta} \tag{21}$$
$$F_{spring}=k\,.\,\delta \tag{22}$$
$$F_{tire}=kt.\delta p \tag{23}$$

RESULTS AND DISCUSSIONS

ANALYSIS OF NATURAL FREQUENCIES

The values found for the natural frequencies (ω) are consistent with the recommendations by NICOLAZZI *et al.* (2012, p. 282), between 2.0 and 2.5 Hz. The frequencies were compared for the vehicle situation with a 70 kg driver and another situation without this presence, Table 3, with significantly higher values for the latter case. Note that the *bounce frequency* has the lowest value, indicating a lower tendency to rotation. As for the *wheel hop* frequencies, these have greater magnitudes, due to the lower inertia, especially in the front.

mode of vibration	ω(Hz) – With pilot (70 kg)	ω(Hz) – without the pilot
Bounce	1.93	2.47
roll	2.17	2.77
Pitch	2.41	3.06
Front *Wheel Hop (left and right)*	12.17	12.17
Rear *Wheel Hop (left and right)*	8.69	8.69

TABLE 3. Values of natural frequencies and damping factors. Source: Own authorship (2022).

The mode shapes, found in Table 4, are normalized by the highest value in each column, making it more intuitive to understand the proportion of each mass displacement. For the modes related to the sprung mass, there are small proportions of displacement by the unsprung masses, with the reciprocal when referring to the excitation of this.

	1st degree	2nd degree	3rd degree	4th grade	5th grade	6th grade	7th grade
Bounce	0.70	0	-0.74	0	-0.05	0	0
roll	0	-1	0	0.10	0	0	0
Pitch	-1	0	-1	0	-0.10	0	0
LFWH	0	-0.10	-0.26	-1	1	0	0
RFWH	0	0.10	-0.26	1	1	0	0
RRWH	0.17	0.10	0	0	0	1	1
LRWH	0.17	-0.10	0	0	0	-1	1

TABLE 4. Mode shapes normalized by the highest value in each column. Source: Own authorship (2022).

ANALYSIS OF RESPONSE TO DYNAMIC EXCITATION

Responses to obstacles are treated: sinusoidal and "cow ribs", because they provide specificities regarding behavior. The calculated forces total the sum of the elastic and damping forces, for a given suspension, where (ζ) is the damping factor. Inertial forces must be dealt with in mass elements. Note the reduction of sprung mass displacements, indicating Dynamic Amplification Factors smaller than 1. This does not occur with unsprung masses, since they have less inertia.

In sinusoidal excitations, it is noticed that the response is not endowed with a harmonic behavior, due to the existence of stochastic vibrations. Therefore, the response is also of such nature, Figure 7, with amplitudes that vary between 20 and 30 millimeters, 5 times smaller than those of excitation. This indicates that there is attenuation of the movement, as seen in the answers in green color.

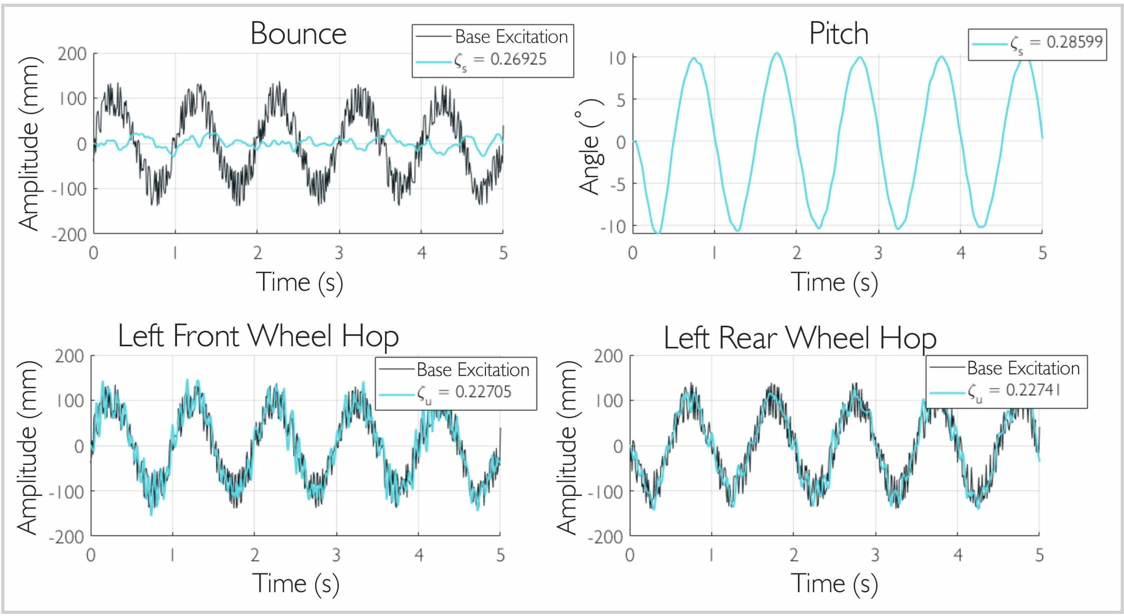

FIGURE 7. Displacements of masses for sinusoidal excitation. Source: Own authorship (2022).

"Cow ribs" cause more significant displacements to the sprung mass, Figure 8, due to the fact that there are discontinuities between the ground reference and the "undercut" regions. Such discontinuities cause abrupt displacements, in small time intervals, causing amplitudes close to those of excitation. It is noticed that in *bounce,* there is an equalization of these amplitudes, during the first oscillation (in 0.1 second), due to the fact that the damping has not yet attenuated the transient component of the movement, being even greater in *wheel hop.*

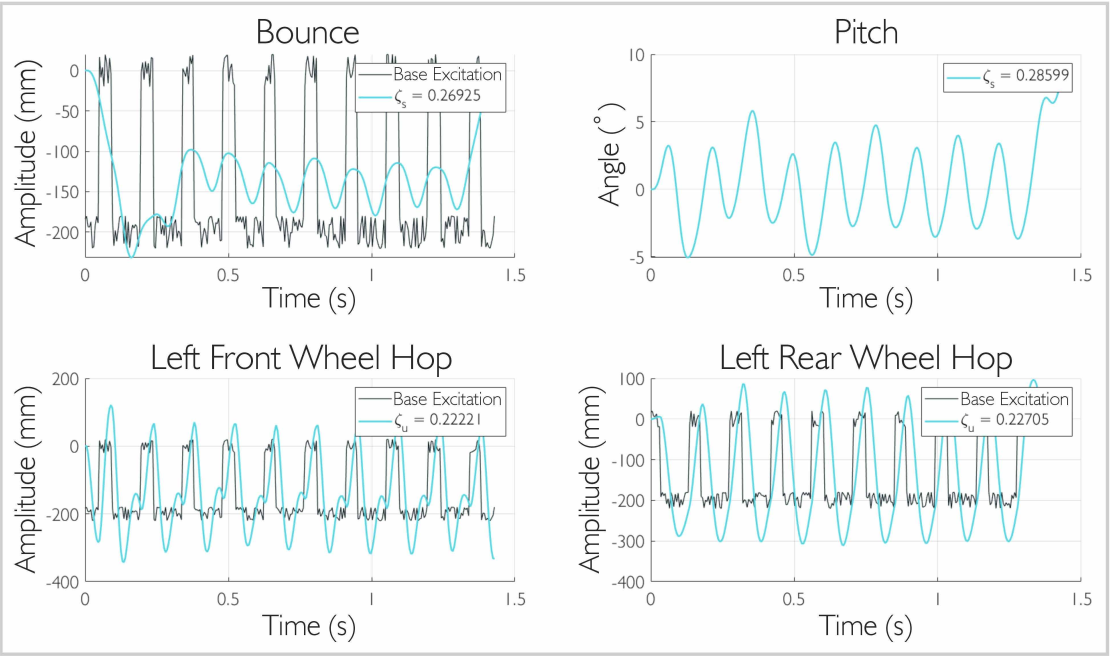

FIGURE 8. Displacement response for "cow ribs". Source: Own authorship (2022).

The magnitudes of the forces acting on the tires are greater than those imposed by the excitation, since the discontinuities cause amplitude peaks and the damping does not act efficiently to attenuate them. Unlike springs-shock absorbers, there is dynamic amplification in tires. In Figure 9, one can see the time lag between the referred force peaks, lagged by 180°.

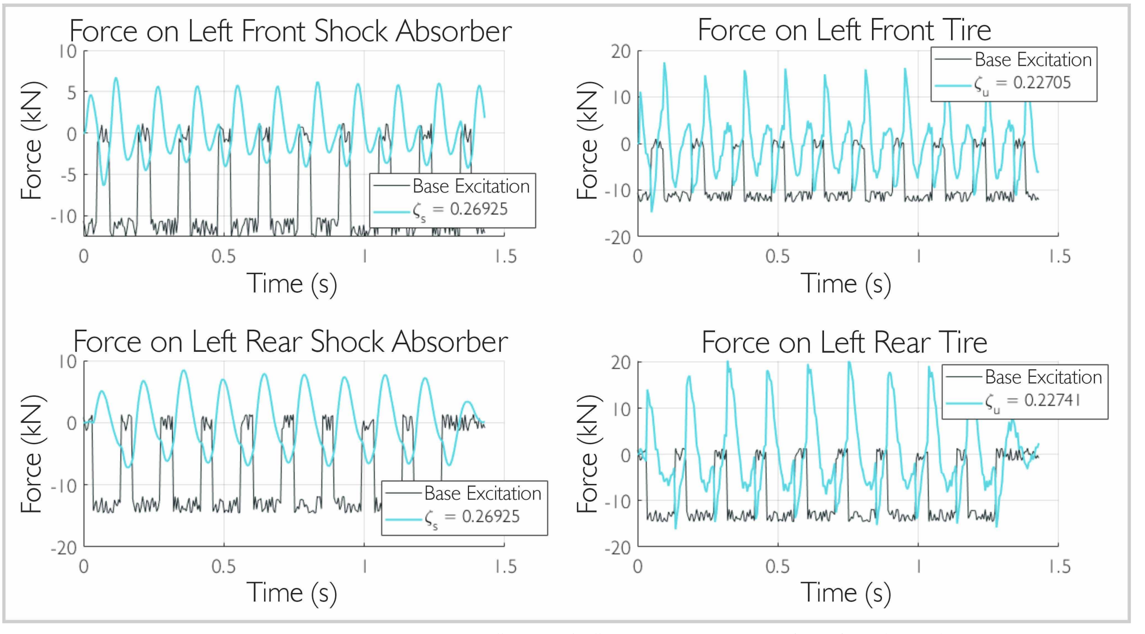

FIGURE 9. Force response to "cow ribs". Source: Own authorship (2022).

APPLICATION OF THE FOURIER TRANSFORM

The application of the Fourier Transform makes it possible to analyze the frequencies that most offer risks to the system. It was possible to synthesize a loading history from the obstacles mentioned here and from other obstacles also elaborated, as seen in Figure 10. The FFT (Fast Fourier Transform) solution algorithm, available in MATLAB®, allowed to obtain the normalized spectrum of frequencies, Figure 11, for the proposed loading history.

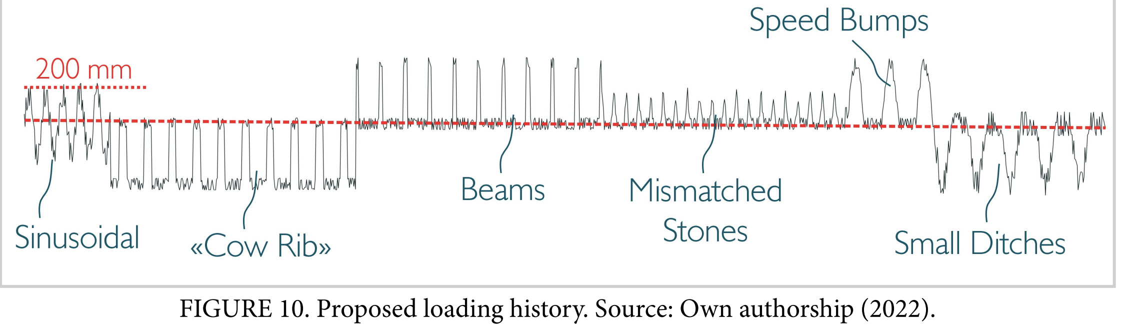

FIGURE 10. Proposed loading history. Source: Own authorship (2022).

It is possible to notice that all the natural frequencies are outside the adjacencies of the regions with higher amplitudes, that is, these correspond to frequencies significantly lower than the *bounce mode*. It is important to emphasize that the application of the FFT demands the use of quantified groups of points to be considered in the sampling. Therefore, 6144 points are adopted, resulting in a number of points close to the number of partitions in the time interval of the analyzed history. As expected, the graphic behavior is continuous.

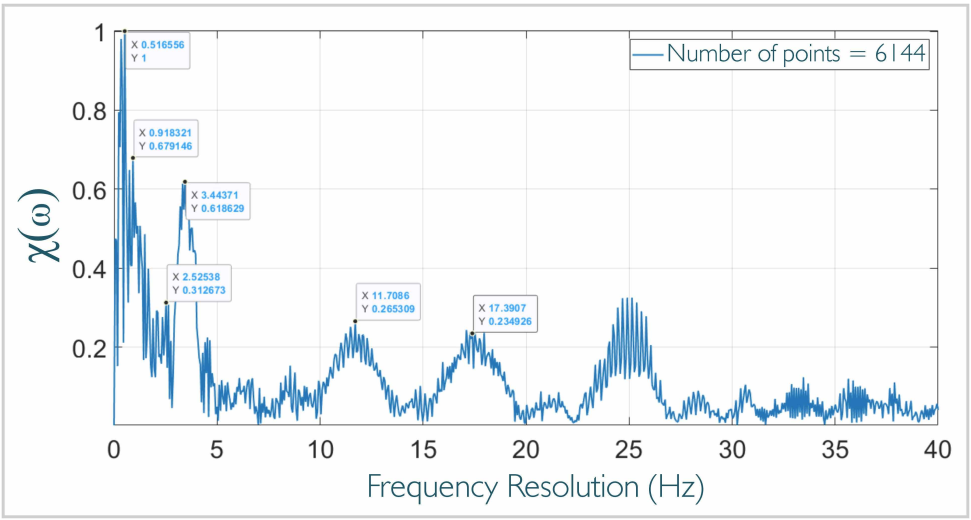

FIGURE 11. History FFT, with normalized amplitudes. Source: Own authorship (2022).

CONCLUSIONS

It is possible to conclude that the present work achieved the objective of developing a model with 7 degrees of freedom, *full-car model,* in order to delimit the dynamic behavior of a UTV vehicle. This is based on histories of stochastic excitations, elaborated through analytical functions. The application of the Equations of Motion allowed delimiting the matrices of mass, damping and stiffness, which are fundamental for carrying out the Modal Analysis and, consequently, obtaining the natural frequencies and mode shapes.

Only for "cow ribs" obstacles, the *bounce* movement was above half the amplitude imposed by the baseline excitation. About the *wheel hop modes*, the transmissibility was marked for both obstacles. This model is endowed with a good approximation of physical reality, as it considers *bounce, roll* and *pitch modes,* in addition to *wheel hop modes.* Therefore, it is concluded that the present elaborated model constitutes an efficient sequential tool for the *setup* of the dynamic-vertical parameters.

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