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DIOPHANTHO'S ARITHMETIC: A CONTRIBUTION OF GREEK MATHEMATICS AS A STRATEGY FOR TEACHING EQUATIONS IN BASIC EDUCATION

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Abstract: In this article, we investigated and discuss the didactic potential of historical problems in mathematics, aiming to locate classical problems and their possible formalizations, so that we could understand their elements and compare them. The investigation and study of equations from the work Arithmetic of Diophantus of Alexandria (3rd century), allowed us to select problems of a historical nature in an integration process, aiming to offer Basic Education teachers, notes and suggestions for the exploration of this type. of problems as a means of overcoming learning difficulties in the classroom. Since the use of the History of Mathematics promotes an integration of the mathematics of the past with the mathematics of the present day and provides a way of treating the contents and contextualized mathematical knowledge.

Keyword: Historical problems. Diophantus Arithmetic. Teaching equations.

INTRODUCTION

In the 8th century BC, the communities of Ionia, on the western coast of Asia Minor, stimulated by the geographical location that facilitated contact with other peoples, developed trade, crafts and navigation. There was also, between 800 and 750 BC, the reappearance of writing, derived from the Semitic alphabet used by the Phoenicians, probably because they used the sea for trade and had contacts with the Greeks.

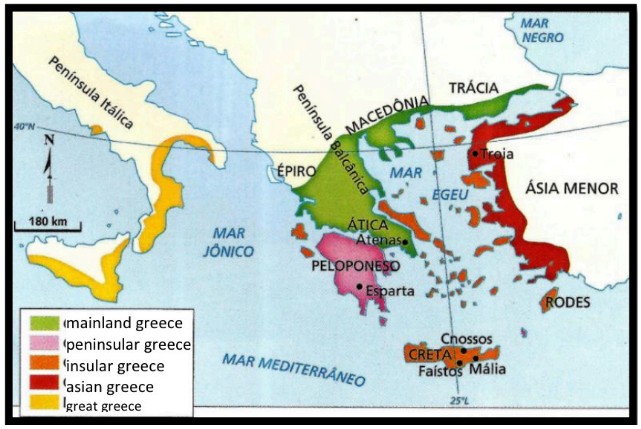


Figure 01: Map (1) of Ancient Greece. Historical Atlas.

Source: ENCYCLOPAEDIA BRITANNICA (São Paulo, 1977. p. 165).

It was in Ionia that the merger of several villages into one took place for the first time, giving rise to the polis (city-state), in a process called synecism, which later spread to other territories of Greece. The territory of the polis was reduced and the soil was not very fertile. On each was the Acropolis, a fortified hill and religious center; the central place where the public buildings, the market and the square were located, where citizens gathered to form the Ecclesia (political assembly); the port and the rural territory. The population clustered around the Acropolis or spread out in the countryside, constituting, however, countryside and city, a single unit.

Figure 2: Acropolis of Athens in Greece

Source:https://pt.wikipedia.org/wiki/Acr%C3%B3pole_de_Athens.

The Cities of Sparta and Athens represent the classic type of cities, respectively oligarchic and democratic. In Sparta, power always remained in the hands of the land-owning citizens, the sparcists. In Athens, political struggles led to the extension of citizenship to all free Athenians, thus making them democrats, despite the existence of a large number of foreign slaves. The poverty of the soil that did not produce enough food for the growing population, debt slavery and the increasing concentration of land in the hands of the aristocracy were factors that led to a wide migratory movement of the Greeks during the 8th to 6th centuries BC, in towards the Black and Mediterranean Seas.

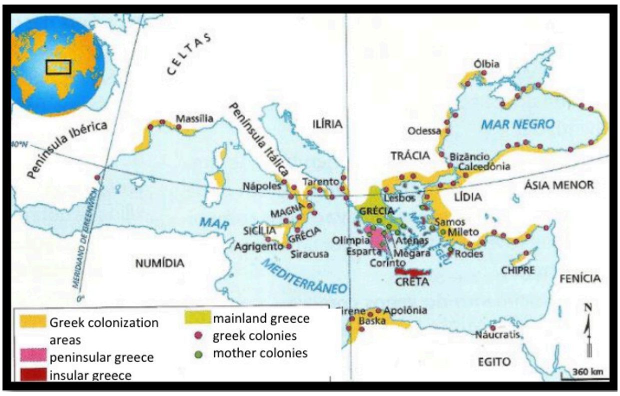


Figure 3: Map (2) The Greek Colonial Expansion.

Source: HILGERMANN and KINDER (1992, p. 46).

According to Boyer (1996), today we use the phrase "Greek mathematics" as if it indicated a homogeneous and well-defined body of doctrine. Such a view can be very misleading however, as it would mean that sophisticated Archimedes-Apollonius type geometry was the only species the Greeks knew about. We must remember that mathematics in the Greek world spanned a time span from at least 600 BC to AD 600 and that it traveled from Ionia to the tip of Italy and Athens, to Alexandria and to other parts of the civilized world.

As a result of economic transformations and the expansion of wealth, the Greeks abandoned the gentile traditions and myths and developed an individualistic, rational and creative mentality, which is already clearly evident in the works of the Ionian scientists and philosophers of the 6th century BC, such as Thales (640- 562 BC), Anaximander (610-547 BC), Anaximenes of the Miletus school. They created logic and mathematics, claiming that the senses and reason are the true criteria for understanding the laws that govern the universe.

Socrates (470 BC) the greatest philosopher, born in Athens, was a teacher of Plato (428 BC), responsible for the organization and systematization of the study of Philosophy. Plato left 28 Dialogues, of which we will find excerpts related to Mathematics. In The Republic, one of his famous dialogues, there are several passages in which mathematics is mentioned, as, for example, in Book VII:

> It's easy to agree with that,' he observed; - geometry is, in effect, the knowledge of what always exists. Consequently, my noble friend, it draws the soul towards the truth and develops in it that philosophical spirit which raises to things above the eyes that we wrongly inclined towards things below. (PLATÃO, 2005. p. 336)

Education in Greece has undergone many changes over time. Until the 8th century BC, approximately, a teaching aimed at forming noble warriors prevailed. The boys of the aristocracy were sent to the palaces, where they were trained for war and learned values such as loyalty, honor and courage.

Over time, education began to prioritize sports training and the teaching of letters and calculations began. In the 5th century BC, there were two very different models of education: that of Athens, centered on integral formation, that is, on the development of body and spirit; and that of Sparta, centered on warrior formation.

In Athens, education was neither free nor compulsory. Families decided how to educate their children. Around the age of seven, boys from the wealthiest families took grammar lessons to learn to read and write; of music, when they learned to play instruments like the libra and flute; and they also learned to recite poems. Classes were taught by a master, usually a slave.

At fifteen, the boys went to gyms, where they practiced physical activities and had reading, writing, calculus, poetry and music classes. They also studied politics and philosophy, to argue with perfection and prepare to act in public life. The objective was to form the integral citizen.

Girls generally do not learn to read or write. They stayed at home, and their mothers taught them domestic gifts. Parents married their daughters at a very young age. The main purpose of marriage was to produce a child, preferably a male.

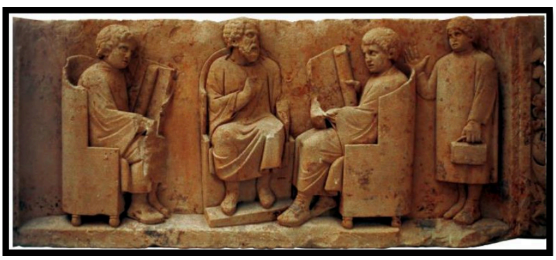


Figure 4: Roman relief from the 2nd century AD depicting a Greek teacher and his students. The work was found in Neumangen-Dhron, Germany.

Source: Rhineland State Museum, Trier, Germany.

For Plato, mathematical beings are real, objective entities, totally independent of our knowledge, they have well-determined properties, some known and many unknown. These beings are not, of course, physical or material objects. They exist outside of space and time. They are immutable and eternal - they were not created, they will not change, nor will they disappear.

Aristotle (384-322 BC) was a disciple of Plato and also a teacher of Alexander the Great. He was a philosopher and a biologist, but he was always aware of mathematical matters. He was assigned a treatise on Indivisible Lines, which consisted of straight line segments, for which there is no common unit of measurement. He was the founder of Logic and it can be said that by his allusions to mathematical concepts and theorems, Aristotle is also considered a contributor to the development of mathematics in his day.

With Alexander the Great (356-323 BC), son of Philip of Macedonia, Greece endeavored to spread the culture and humanist mentality of the Greeks far and wide. Alexander's conquests implanted, among the conquered peoples, a kind of cultural enlightenment, where the language, customs and art of the Greeks gained civilizing power.

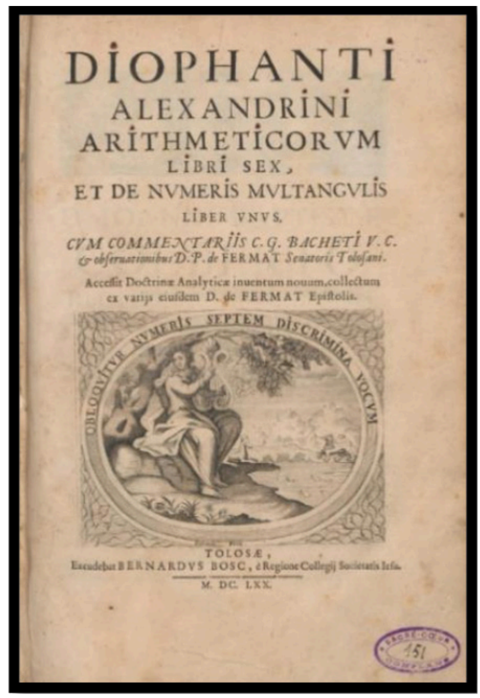
After the conquest of Egypt, in 332 BC, the foundations were laid for a new city, open to the new winds of Greek culture and art, free from the shackles of Egyptian pagan theosophy and independent of the cult of the dead, which so subjugated the life of the egyptian people.

In fact, when Alexander died, his power was shared by his two greatest generals: Seleucus, from whom the Seleucid dynasty derives, will retain the northern part of the empire, with headquarters in Antioch; the south, with predominance of Egypt, will be for Ptolemy I (367-282 BC), and will give place to the dynasty of the Lagids (303-30 BC). All of them did their best to spread and impose Hellenism, but it was the Ptolemies who, along the Mediterranean, in the western part of the Nile Delta and opposite the island of Pharos, would build the new city of Alexandria; it would be like the radiating seat of the force of Hellenism and human rationality, which it imposed. Man with his intelligence would be the propeller and measure of progress, culture, religion and art. This way and in this line of ideas.

With the aim of promoting Hellenism and all its culture, the famous Library of Alexandria was built. It would have been in the middle of the 3rd century BC (about 252 BC), when Ptolemy II (308-246 BC), Philadelphus, ruled Egypt. There the entire emporium of knowledge would gather: literature, history, philosophy, religion, art, mathematics, astrology, medicine. Callimachus (305-240 BC) was the librarian who produced the first catalogue, which covered 120 papyrus rolls. It is estimated that it had between 400,000 and 1 million papyri.

The city of Alexandria became a great center of investigation of knowledge, the first institute that recorded the knowledge of civilizations, the greatest city that the Western world had known, undoubtedly an intellectual economic center of the Hellenistic world. People from all countries flocked to Alexandria to live, trade and learn. It was a city where the Greeks, Egyptians, Syrians, Hebrews, Nubians, Phoenicians, Romans, Gallic and Iberians traded goods and ideas. Writers, poets, artists and scientists from all over were regimented there to enrich its Museum and Library. Names of important scholars have given their contributions: Euclid (323-283 BC), Aristarchus (310-230 BC), Archimedes (287-212 BC), Nicomedes (280-210 BC), Eratosthenes (276-194 BC), Apollonius (262-194 BC) in 604 AD the library of Alexandria was destroyed in a fire.

In 1621, Bachet de Méziriac (1581–1638) publishes the Greek text of Arithmetica together with a Latin translation and some of his notes on Diophantus' problems and solutions. A copy of this edition is acquired by Pierre de Fermat (1601–1665), a man of law by profession (he was an adviser to the superior court of Toulouse), a mathematician by passion. Fermat will note in the margins of his copy of Arithmetica results on natural numbers, inspired no doubt by his study and reading of this work, but completely new and of an astonishing beauty and depth, and unparalleled hitherto. Fermat confines himself to stating, in these margins and in letters to other mathematicians, these results, and only a sketch of his proof in Number Theory is known. The best mathematicians of the 17th century, especially L. Euler (1707–1783),


DIOPHANTI
ALEXANDRINI
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LIBRI SEX,
ET DE NVMERIS MVLTANGVLIS
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Figure 5: The Arithmetic of Diophantus by Bachet de Méziriac with commentaries by Pierre de Fermat published in 1670.

Source: Gallica Bibliotèque Numérique. Available at http://www.e-rara.ch/zut/content/pageview/2790613 - Bibliothèque Nationale de France. Accessed on December 03, 2013.

Some published versions of Arithmetic: in 1621, Claude Gaspard Bachet de Méziriac publishes a bilingual Greek-Latin version; in 1893 a critical edition by Paul Tannery Diophanti Alexandrini Opera omnia cum graecis commentariis; In 1910, Heath published Diophantus of Alexandria: A Study in the History of Greek Algebra.

According to Roque (2012) Diophanto's best-known contribution is to have introduced a way of representing the unknown value in a problem, designating it as arithmos, from which the name "arithmetic" comes. Arithmetic contains a collection of problems that integrated the mathematical tradition of the time, in book I, he introduces symbols, which the author calls "abbreviated designations", to represent the different types of quantities that appear in problems.[1]

Diophantus used the symbol analogous to the Greek letter ζ to represent the unknown; for the square of the unknown he used Y, which he called dynamis (square); for the unknown cube he used KY and called it Kybos; for the power of exponent four he used Y and called it dynamis-dynamis; for powers of exponent five and six he used, respectively, KY (dynamis-kybos) and KY K (kybos-kybos). (HEATH, 2012, p. 129).

symbols Diophantines	Description	Notation Modern	Description
ζ	arithmos	x	unknown
Δ^{Y}	dynamis	x^2	Square
KY	Kybos	x^3	Cube
$\Delta^{Y}\Delta$	Dynamis-Dynamis	x^4	4th Power
ΔKY	Dynamis-Kybos	x^5	5th Power
KY K	Kybos-Kybos	x^6	6th Power

Table 1: Diophantine Symbols.

Source: Made from Roque (2012).

According to Eves (2008, p. 209), Diophantus had abbreviations for the unknown, power up to exponent six, subtraction, equality and inverses. Our word "arithmetic" comes from the Greek word aritnmetike which is composed of arithmos ("number") and techne ("science"). Heath pointed out quite convincingly that the symbol used by Diophantus for the unknown was probably derived by fusion of the first two Greek letters of the word arithmos, namely e. Over time this symbol came to resemble the Greek final sigmaz *apζ*. Although there are doubts about this, the meaning of the notations for the powers of the "cube" unknown. The symbols of the following powers are easily explained: thus, dunamis () of the unknown, (square-square), (square-cube) and (cube-cube). ΔYNAMIΣ$\Delta^{Y}\Delta K^{Y}K^{Y}K$.

Diophantus' symbol for "minus" resembles an invert with the bisector drawn on it. The explanation that has been given is that this symbol would be composed of, letters of the Greek word leipis which means "less". All the negative terms of an expression were gathered and the minus sign was written before them. Addition by juxtaposition was indicated; and the coefficient of the unknown or of any power of the unknown was represented by an alphabetic Greek numeral, immediately following the symbol to which it was supposed to be linked. And when there was a constant term, then you used, an abbreviation of the Greek word monades, which means units, followed by the appropriate numerical coefficient. Thus, we would write and, expressions that, literally, can be read like this: unknown cubed 1, unknown squared 13, unknown 5 e (incognito cubed 1, unknown 8) minus (unknown squared 5, units 1). This is how rhetorical algebra became syncopated algebra. *V Λ and I(ΛΕΙΨΙΣ)*$^{O}_{M}$ *(ΜΟΝΑΔΕΣ)* $x^3 + 13x^2 + 5x$ *and* $x^3 - 5x^2 + 8x - 1$$K^{Y}a\Delta^{Y}ai\gamma\zeta\varepsilon$ $K^{Y}a\zeta\eta$ ⋔ $\Delta^{Y}\overset{O}{\underset{M}{}}a$

Diophantus' symbols mark the passage from rhetorical algebra, in which expressions are written entirely in words, to syncopated algebra, in which some expressions are written in words and others are abbreviated (STRUIK, 1989).

1. The abbreviation method represented the word used to designate these quantities by their first or last letter according to the Greek alphabet.

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According to Klein (1968, p. 146), the signs used by Diophantus were mere abbreviations. For this reason, the procedure practiced by Diophantus was called syncopated algebra, which is a transition from rhetorical algebra to modern symbolic algebra.

Many of the problems dealt with in Arithmetic lead to equations of the 1st and 2nd degrees, to one or more unknowns, determined or not; others refer to cubic equations, but for these Diophanto chooses the data properly so that the solution is easy to obtain. There are also algebraic problems that Diophantus solves using geometry and problems about right triangles with rational sides. For the proposed problems, only positive rational solutions are accepted. It is the problems "about" solving equations that most interest us in our approach. Problems such as "dividing a given number into two others, knowing their difference" and their solving strategies. That allow us, from their comparisons, a more effective teaching in solving elementary and high school equations in basic education.

METHODOLOGICAL THEORETICAL ASSUMPTIONS

Problem solving is an important didactic/methodological strategy for teaching mathematics. However, in the classroom, it appears that an exaggerated use of rules, and resolutions through standardized procedures, demotivate both students and teachers. The use of routine problems does not develop creativity and autonomy in mathematics.

> Using historical aspects related to the mathematical content to be taught is important to know the development of mathematical concepts; an importance that is accentuated when we think of a Mathematics Teaching that aims at the recognition and, if possible, the contextualization of the contents (BRANDEMBERG, 2018, p. 1).

Today, in order to learn how to solve "mathematical problems", in general, repetitive exercises are used in the classroom to fix the contents that have just been studied, in an abuse of standardized procedures in solving similar problems. This activity does not develop in the student the ability to transport himself from the reasoning used to the study of other subjects or even related problems.

The search for new alternatives of didactic transposition for the teaching of Mathematics suggests that we took the history of Mathematics as an ally. The alliance consists of working on the historical development of certain contents in order to locate pedagogical possibilities that overcome the difficulties encountered by mathematics teachers and students (MENDES, 2001) (BRANDEMBERG, 2010).

Problem solving, in a work organized from the elaboration of activities of a historical nature, we believed, to be an important contribution to the teaching and learning process of Mathematics, by developing in the student the abilities of an "advanced mathematical thinking" or, at least, more elaborate, which is not restricted to the application and resolution of routine exercises that simply value learning through reproduction or imitation.

> By presenting the resolution processes from particular examples, collected from original sources, our "historical texts" can provide from each era in the resolution of these problems; guaranteeing us an opportunity to share a mathematics that has historically been consolidated as a human sociocultural production, where aspects of everyday life, school and academia, when merging, create the possibilities of comparing the resolution strategies or the other (BRANDEMBERG, 2018, p. 3).

The importance of solving mathematical problems of a historical nature must enable students to mobilize knowledge and develop the ability to manage the information that is within their reach inside and outside the classroom. Thus, students will have opportunities to broaden their knowledge of mathematical concepts and procedures as well as the world in general and develop their self-confidence.

According to Dante (1991), it is possible through problem solving to develop in the student initiative, exploratory spirit, creativity, independence and the ability to elaborate logical reasoning and make intelligent and effective use of available resources, so that he can propose good solutions. to the issues that arise in their day-to-day, at school or outside.

> We must emphasize that the activities go beyond the simple step-by-step and mechanized routing. Rather, they must be connected to the everyday, school and academic aspects of mathematical culture. One of the implications of this process is the discussion based on the mistakes and successes produced in the search for answers and multiplying the paths or creative strategies of resolution that lead to new frontiers of mathematical knowledge (BRANDEMBERG, 2017, p. 28).

Students, when solving problems of a historical nature, can discover new facts and find several other ways to solve the same problem, arousing curiosity and interest in mathematical knowledge and thus developing the ability to solve situations that are proposed to them in addition to the possibility of knowing and compare the different resolution strategies and mathematical tools available in each era.

SOME SELECTED PROBLEMS FROM DIOPHANTUS' ARITHMETIC FOR CLASSROOM ACTIVITIES

We will present problems I-27 and I-28 of Book I of Diophantus' Arithmetic, they are the first to be reduced to complete 2nd degree equations and in the presentation of their resolution, Diophantus uses an artifice that allows them to be transformed into 2nd degree equations. incomplete degrees, whose resolution is immediate. The artifice consists in designating a certain unknown quantity by arithm. Then, the various unknowns of the problem are written as a function of this new unknown, and substitutions are made between the various equations, in order to reduce everything to a single equation, with a single unknown (the arthmo) never with a degree higher than the second. The problems considered are what Brandemberg and Mendes (2005) call problems of a historical nature.

> Thus, when considering an approach that uses historical problems and their solution methods in structured or at least semi-structured activities, we sought the relationships between the conceptual structures involved in the conception (formation and production) of these problems and the processes of solving them. aiming to illuminate connections between current and ancient knowledge, and that allow our students to have a greater understanding of the concepts involved, of a mathematics that is constituted in the most diverse sociocultural contexts of human activity (BRANDEMBERG, 2021, p. 25).

Note that the choice of arithm was not arbitrary. Instead, it was done in such a way that, in the end, an equation was obtained under the conditions mentioned above. After calculating the value of the arithm, it was easy to determine the various solutions to the problem.

> Diophantus' procedure is totally different, conceptually, from the procedures by the Egyptians, and from geometry. Indeed, here, an unknown (known as arithm, which means number) is highlighted in the calculations. This unknown is not, like in arithmetic processes, the point of arrival of calculations, it is no longer, as in the case

> of geometry, a static reference point in the development of the problem, but a quantity that is operated as if it were a number. known. (RADFORD, 1993, p. 37).

It seems to us that Diophantus suggests that one can follow the process of discovering the result. This is very visible in troubleshooting. Here, we initially selected only three problems and their respective activities, seeking to relate the history of Greek mathematics to the teaching of equations in basic education.

Problem I-1 - Divide a given number into two numbers of given difference.

a) **Resolution proposed by Diophanto (rhetorical)**:Let the number and the difference; find the numbers. Assuming the smaller number, the larger will be; therefore, the two added together give, which is worth. So is equal to. Next we'll subtract each of the members getting. Then the number will be. Then, 10040 *arithmo plus* 402 *arithmo plus* 401001002 *arithmo plus* 40402 *arithmo equals* 6030 *arithmo equals* 40 *arithmo equals*.

b) **A resolution using the abbreviations (more general)**: supposing ζthe smaller number, the larger will be; therefore, the two added together give, which is worth. So 100 is equal to. Then we will subtract each of the members getting to an equal number. Then the number will be. Then, ζ + 402 ζ + 401002 ζ + 40402ζ6030ζ *equals* 30 and ζ + 40 *equals* **70**.

c) **Resolution in modern notation:**Assuming the smaller number, the larger will be; therefore, the two added together give, which is worth. So is equal to. Then we will subtract each of the members getting to an equal number. Then the number will be: $xx + 402x + 401001002x + 40402x + 40 - 40 = 100 - 40\ 2x6030x =$ **30** *and* $x + 40 =$ **70**.

Proposed Activity: We will assume new values for the number and for the difference given in the problem, proposing to name the unknown, and thus, seek to identify generalizing expressions through related situations. This activity aims to develop investigative skills in the student, identifying the mathematical structures, the function of the unknown, and building an algebraic language to describe it symbolically, providing the student, based on Diophantus' ideas, with the creation of expressions that have regularities in the resolution. of problems.

Problem I-27 - Finding two numbers with sum and product given.

a) **Resolution proposed by Diophanto (rhetorical)**: Assume the sum is 20 and the product is 96. Assuming the difference between the two numbers is 2 arithmos, we started by dividing the sum of these numbers (which is 20) by two (getting 10). From this result, we considered an arithmos added to and subtracted from 10, respectively, each of the halves. As half of the sum is 10, taking the half subtracted 1 arithmos plus the added half of 1 arithmos obtaining 20, which is the desired sum. So that the product is 96, we multiply these same quantities, getting 100 subtracted from the square of the arithmos (a dynamis). We thus got at the conclusion that the dynamis must be 4, so the value of the arithmos is 2. The values sought will therefore be 10 plus 2 and 10 minus 2, that is, 8 and 12.

b) **A resolution using the abbreviations (more general)**: If these numbers were equal, each of them would be 10. We assume that the difference between them is 2ζ, that is, the two numbers sought are obtained by subtracting ζ from one of these 10 and adding ζ to the other. Since the sum does not change after these operations, we have 10 – ζ + 10 + ζ = 20. But we also know that the product of these numbers is 96, so we can write (10 – ζ) (10 + ζ) = 96. We observe, so what 10^2 - Δ^y = 96, and we concluded that the value of ζ must be 2. Therefore, the numbers sought 10 - ζ and 10 + ζ are, respectively, and the number: **812.**

c) **Resolution in modern notation:** Assuming, be the difference between them, then there exists, such that. Substituting into the equation we got. So z is equal to. *x and y* = 102 *xzx* = 10 – z and y = 10 + *zx.y* = 96(10–*z*).(10+*z*) = 96100 + z^2 = 962 Then, the numbers sought *x* = 10 - *z and y* = 10 + *z* are, respectively, and **812**.

Proposed Activity: As for this activity, we intended that students begin to interpret the different aspects that involve solving equations and that they can formulate from these ideas generalizations that allow solving the I-27 problem in different ways. So, for the proper understanding of this problem, as a suggestion we will go in the opposite direction of the resolution proposed by Diophanto. We start by proposing any two numbers to find the sum and product, and thus identify expressions in correlated situations.

Problem I-28 - Find two numbers whose sum is a number equal to 20 and the sum of the square is a number equal to 208.

a) Resolution proposed by Diophantus (rhetorical): Assume that the sum is 20 and the summed squares are 208. Assuming that the difference between the two numbers is 2 arithmos, we started by dividing the sum of these numbers (which is 20) in two (obtaining 10). From this result, we considered an arithmos added to 10 and subtracted from 10, respectively, each of the halves. As half of the sum is 10, taking the half subtracted 1 arithmos plus the added half of 1 arithmos obtaining 20, which is the desired sum.

b) So that the sum of squares is 208, we added the squares of these same quantities, we got (10 *minus arithmos*)^2 *plus* (10 *minus arithmos*)^2 *equals* 208 and 10^2 *minus* 2.10. *minus arithmos* (*minus* Dynamis) plus 10^2 *plus* 2.10. *arithmos plus Dynamis equals* 208, therefore 100 *minus* 20 *arithmos plus* Dynamis *plus* 100 *plus* 20 *arithmos plus* Dynamis *equals* 208 200 plus 2 Dynamis *equals* 208. We thus got at the conclusion that the value of the arithmos is. The values sought will therefore be 10 minus and 10 plus and the number: **222812.**

c) **A resolution using the abbreviations (more general)**: We wanted to find two numbers with a sum of 20 and the sum of the squares is equal to 208. If these numbers were equal, each of them would be 10. We assumedthat the difference between them is 2ζ, that is, the two numbers sought are obtained by removing ζ from one of these 10 and adding ζ to the other. Since the sum does not change after these operations, we have 10 –ζ + 10 + ζ = 20. But we also know that the sum of the squares of these numbers is 208.We observed, then, that (10–ζ)^2 + (10+ζ)^2 = 208100 – 20ζ + Δ^y + 100 + 20ζ + Δ^y = 208, therefore, and we concluded that the value of 200 +2Δ^y=208ζ must be. Then, the numbers sought 210 – ζ e 10 + ζ are, respectively, and**. 812**

d) **Resolution in modern notation:** Assuming,is the least of these numbers, then there exists,such that. Substituting into the equation; so the two add up to. So z is equal to. *x*+*y* =20 e x^2+y^2 = 208 *xzx* =10 – *z and y* =10 + *z*x^2 + y^2 = 208 (10–*z*)^2+(10+*z*)^2 = 208200 + 2z^2 = 2082. Then, the numbers sought *x* = 10 – *z and y* = 10 + *z* are, respectively, and **812**.

Proposed Activity: This activity aims to review mathematical aspects related to the square of the sum of two terms and the square of the difference, a subject worked in the 8th grade of elementary school. So, going in the opposite direction of the resolution proposed by Diophanto. We started by suggesting any two numbers to find the sum and summed squares, and from there apply the method to any pair of suggested numbers.

AN EXAMPLE OF PRESENTING HISTORICAL PROBLEMS IN TEXTBOOKS

Next, we presented the problem: Deciphering the enigma of Diophanto's age, selected in the textbook of the 7th year of elementary school, Sampaio (2010) when approaching the theme equations and inequalities of the 1st degree, describes the problem found in Diophanto's tombstone.

Our intention when using historical data and characteristics, aims at greater interaction and a possible comparison of historical and current resolution strategies, always seeking to give more meaning to the contents (concepts) studied (involved) (BRANDEMBERG and MENDES, 2005).

The study allows us to analyze the strategies used in the problems, comparing them and noting some differences. The strategies used are diversified, falling into the categories of informal strategies. In addition, its analysis highlights the type of work that can be developed in the classroom under conditions that allow students to progress from the use of informal, poorly structured strategies to more structured and efficient strategies. On the contrary, the analysis of informal strategies shows a prevalence of the use of the traditional algorithm and shows a continuity in terms of the progression of strategies in the three problems. As we were able to evidence in the rewriting of Brandemberg (2020), based on Heath (1964), namely:

> We can, therefore, consider it as a historical activity, for example: a textual fragment, related to a Greek anthology, dating from the year 500, which initially is characterized as a riddle problem (playful) and which gives us an idea about the age and some characteristics of the life of Diophantus of Alexandria (200-284), which we reported in a translation obtained from another made by Moritz Cantor (1829-1920) and quoted by van der Waerden (1975, p. 278), as follows: "In this tomb lies Diophantus. It is wonderful! We can count the time of his life. God gifted him with the grace of being a boy for a sixth of his life, and adding a twelfth to that, he grew a beard. A seventh part later he was married, and five years later his son was born. Poor child; on reaching half his father's age he succumbed to fate. After facing his pain for four more years, he ended his mission on earth." Whence we concluded, after performing some algebraic calculation, that Diophantus lived for 84 years. (Heath, 1964). (BRANDEMBERG, 2020, p. 270).

The potential of working with the Resolution of Mathematical Problems of Antiquity as a strategy for teaching mathematics in basic education, as a means of being preponderant and exclusive tools to engage in problematization and logical reasoning. Up to this point, we can see that knowledge of the history of the development of mathematics allows us greater understanding when we realized that arithmetic leads us to an algebraic symbology as a language that facilitates the expression of mathematical thinking (BRANDEMBERG, 2010).

Thus, according to Brandemberg and Mendes (2005), starting from a historical approach, going through the possible stages of evolution of mathematics, we can contribute to the construction of algebraic thinking towards the formalization of symbolic language and, before that, alleviate difficulties related to the abstraction. As the algebraic language becomes familiar to the student, he can understand the function of

Deciphering the riddle of Diofanto age passed away

Let's consider with x the age at which Diofanto died and transcribe these expressions into mathematical language.

$$x = \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4$$

$$x = \frac{[14x + 7x + 12x + 420 + 42x + 336]}{84}$$

$$84x = 75x + 756$$

$$9x = 756$$

$$x = 84$$

Diofanto studies were based on the use of symbols to facilitate writing and mathematical calculations. The symbols created by him meant that expressions, hitherto written entirely in words, could be represented by abbreviations. Algebra begins to be understood as the study of solving equations.

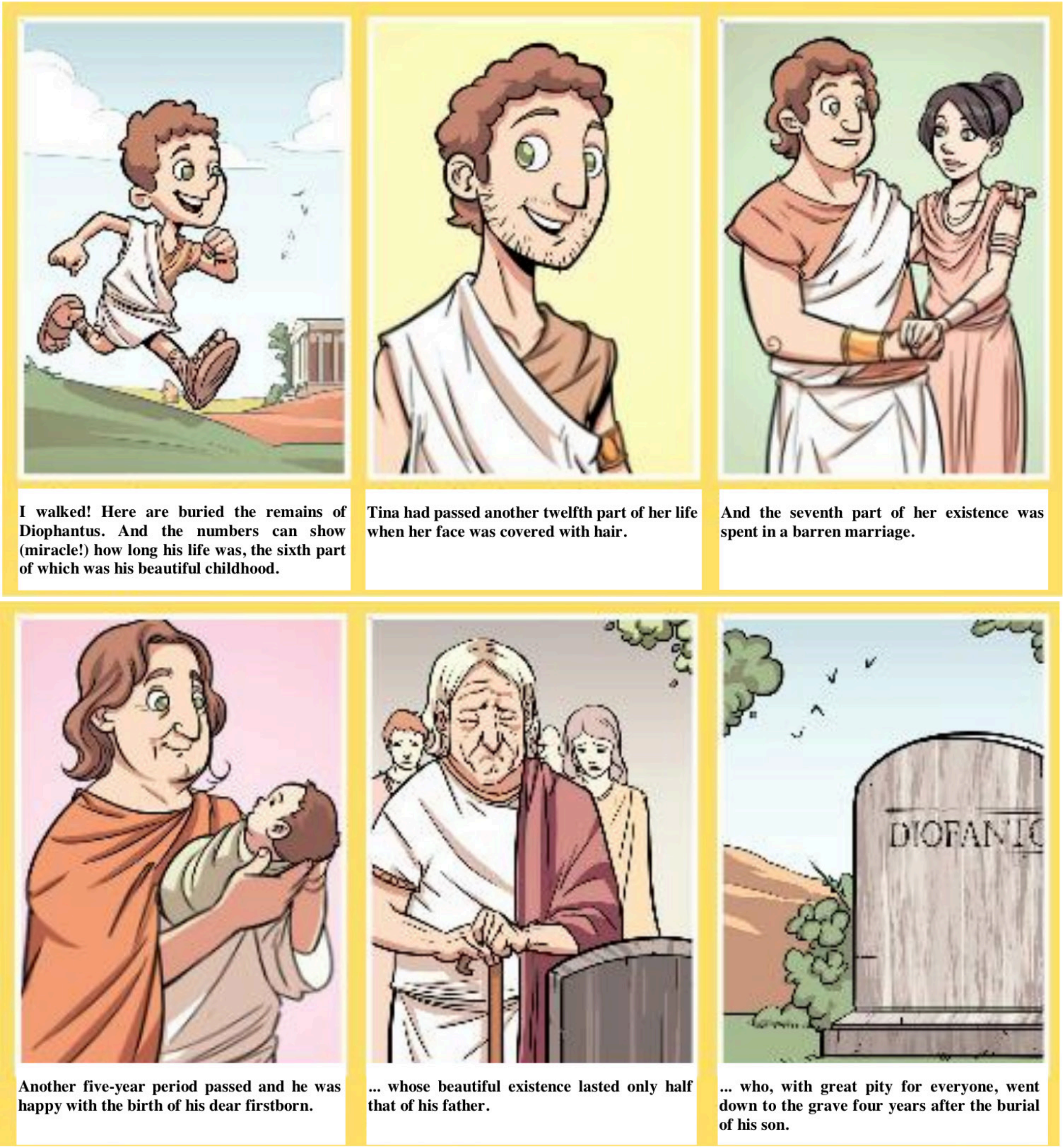

Figure 6: Deciphering the riddle of Diophantus' age in the textbook.

Source:Presented by Fausto Arnaud Sampaio, in the Jornadas – Mathematics Collection. Publisher Saraiva. 2010. p. 150 and p. 151.

generalization for the solution of problem situations. The information and historical problems allow reflections that help, both in the formation of the teacher and the students, and can also contribute to the re-elaboration of mathematical concepts (BRANDEMBERG, 2010).

FINAL CONSIDERATIONS

The History of Mathematics as a Methodological Component in Mathematics teaching processes brings to the classroom questions related to human needs that gave rise to mathematical concepts and theoretical productions resulting from the abstractions and generalizations obtained. The great challenge for mathematics teachers who seek to make use of the history of mathematics in the classroom is the transformation of historical information obtained through bibliographic research into teaching activities that provide students with a historical encounter with mathematical knowledge and in the elaboration of of pedagogical approaches that favor the reconstruction and assimilation of the concepts involved in these contents (MENDES, 2015) (BRANDEMBERG, 2018).

The knowledge of the history of mathematics, we claimed to be essential for every teacher in this area, because even if the historical information does not have direct application in the classroom, the understanding of the historical development of the concepts can positively influence the pedagogical practices.

An approach that involves historical investigations on social practices linked to the development of mathematics has to do with the teaching of mathematical content, as in addition to autonomy, it offers students mathematical concepts in critical and innovative thinking (MENDES, 2015) (BRANDEMBERG, 2018).

The history of mathematics in teacher education can contribute to the perception of the socio-cultural nature of mathematics, or even of its processes of abstraction, generalization and synthesis, in all its dimensions. However, a considerable portion of teachers who work in Brazilian Basic Education schools did not have subjects related to the history of mathematics in their training; it is up to them to search for this knowledge through continuing education courses and bibliographic research.

The use of the History of Mathematics alone does not solve all the problems of Mathematics Education, but it is observed that activities inspired by history motivate students to learn, humanize mathematics, lead to investigations and contribute to the understanding of mathematical content from recreation or rediscovery of concepts. (MENDES, 2015) (BRANDEMBERG, 2018).

Besides, according to Mendes (2015) and Brandemberg (2017), a historical approach to the construction of mathematical concepts can provide a view of mathematical production, and reveals that mathematics is a product of human culture, changeable over time. Knowledge of the history of algebraic development enables the perception of algebraic symbology as a language that facilitates the expression of mathematical thinking. Activities that contemplate the stages of evolution can contribute to the construction of algebraic thinking towards the formalization of symbolic language and, before that, alleviate difficulties related to abstraction (BRANDEMBERG, 2010).

As the algebraic language becomes familiar to the student, he can understand the function of generalization for the solution of problem situations. The investigations developed showed that the use of the history of mathematics in pedagogical practice goes beyond a motivating element, as the information and historical problems allow reflections that help both in the formation of the teacher and the students and can also contribute to the re-elaboration of mathematical concepts, in this specific case, algebraic concepts.

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