# International Journal of Human Sciences Research

# PISA MATHEMATICS 2012: AN ANALYSIS OF ITEMS IN THE SUBAREA CHANGES AND RELATIONSHIPS

#### Maria Isabel Ramalho Ortigão

Doctor in Education by the institution: Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio). Professor at the institution: Departamento de Estudos Aplicados ao Ensino, University of Education of the institution: Universidade do Estado do Rio de Janeiro (UERJ), Rio de Janeiro, RJ, Brazil http://orcid.org/0000-0001-7269-592X

#### Carlos Augusto Aguilar Júnior

Doctor in Education by the institution: Universidade do Estado do Rio de Janeiro (UERJ). Professor of the institution: Universitário Geraldo Achilles Reis (COLUNI UFF) of Universidade Federal Fluminense (UFF), Niterói, RJ, Brazil https://orcid.org/0000-0003-0199-0360

#### Geraldo Eustáquio Moreira

Doctor in Mathematics Education by the institution: Pontifícia Universidade Católica de São Paulo (PUC/SP). Professor of the institution: Faculdade de Educação da Universidade de Brasília – UnB, Brasília, Brasil https://orcid.org/0000-0002-1455-6646

All content in this magazine is licensed under a Creative Commons Attribution License. Attribution-Non-Commercial-Non-Derivatives 4.0 International (CC BY-NC-ND 4.0).

A first version of this text, more simplified and reduced, was presented at the IX SPEM – IX Research Seminar in Mathematics Education in Rio de Janeiro, which took place in 2020 in a virtual way.



Abstract: The text presents the results of an analysis of the public items of Mathematics of Pisa 2012 in relation to the required contents and skills. Specifically, through a documentary research, it analyzes three items in the Changes and Relationships subarea. The choice for this subarea is due to the fact that the proficiency subscale has a high concentration of students at the lowest levels. The on-screen discussion is part of a broader research that aims to understand the results of Pisa in Mathematics and is guided through different methodological approaches. The results obtained reveal a close harmony between the proposals of Pisa and BNCC for the area of Mathematics. It is noteworthy that half of the public items in the Changes and Relationships subarea are centered on proposals that involve the application of formulas and expressions. Such results can support other questions and investigations about the relationship between curriculum and assessment in Mathematics Education.

**Keywords:** Pisa; Math; Large-scale assessment; Changes and Relationships.

#### INTRODUCTION

Brazil has been participating in the International Student Assessment Program – Pisa – since the beginning of the year 2000<sup>1</sup>. Such participation stems from the Brazilian effort to build an educational evaluation policy related to the "perspective of evaluating knowledge and skills that allow continuous adaptation to a constantly changing world" (FRANCO; BONAMINO, 2001, p. 25). The adhesion of Brazil to Pisa is closely aligned with the managerial proposals for Brazilian education, especially from the 1990s onwards, with the constitution of the Basic Education Assessment System - SAEB, in which the idea of "measuring" the quality of teaching offered in education systems has gained relevance. Although Brazil has been participating in Pisa since its first edition, its results have only recently started to integrate, in a way never seen before, the political discourse and the political agenda of Brazilian education (MACEDO, 2014; ARRUDA et al., 2020).

Pisa is a large-scale assessment program applied to a sample of 15-year-old students, an age that, according to the organizers of the Pisa Consortium, corresponds to the end of compulsory basic education in most countries. The intention is to assess the extent to which these students have acquired essential knowledge and skills for full participation in modern societies.

The Program is part of the global agenda of the Organization for Economic Cooperation and Development – OECD – and seeks to produce educational indicators that allow the comparison of participating countries in terms of their national curricula and the preparation of students for the constantly changing world of work. It also lends itself to evaluating the functioning of educational management by bringing to the grammar of institutions and public education systems the meanings and "current discourses on a culture of evaluation, the calculation of competences and the measurement of performances" (MENDES; SEGABINAZZI, 2018, p. 851).

The Pisa assessment is triennial and organized in cycles; with each edition the focus is centered on one area of knowledge (Reading, Mathematics and Science), which means that more items from that area are included in the test (approximately 54%), and fewer items from the others (23% for each one). This greater number of items allows the specific area of knowledge to be examined

<sup>1</sup> PISA is led by the Organization for Economic Co-operation and Development – OECD, an entity that brings together 34 member countries, in addition to others that are invited to participate. In Brazil, the National Institute of Educational Studies and Research Anísio Teixeira (INEP) is the body responsible for planning and implementing evaluation in Brazil. Pisa, since 2000, has involved more than 90 countries and more than 3 million students worldwide.

in more detail and depth. The Mathematics area was the focus of the editions that took place in 2003 and 2012. A new edition of Pisa was scheduled to take place in 2021, with Mathematics as the focus of the evaluation. However, due to the difficulties faced by the Covid-19 pandemic, the member and associated countries of the OECD decided to postpone the application of Pisa-2021 to 2022 and of Pisa-2024 to 2025.

At the base of the creation of Pisa is the desire that its results be used by governments to define or redirect educational policies, aiming at improvements in basic education and a more effective training for young people. Barriga (2018) argues that Pisa is an international strategy for modeling student learning that initially denies indispensable cultural differences, in addition to real social and economic differences. Despite his criticisms of the Program, the author (p. 34) defends taking Pisa as a pretext for improving students' learning conditions in the classroom. To this end, Barriga (2018) recommends the convenience of developing a didacticpedagogical analysis of Pisa, its instruments and results, in addition to the development of political and technical analyses, in order to understand the potential and limitations of the Program.

In the same direction, Fernandes and Gonçalves (2018) developed a longitudinal study in order to understand the results of Portuguese students in Pisa from 2000 to 2015, taking into account the diversity of public education policies and programs that have materialized them. The authors observed improvements in results without aggravating differences between students, as "the number of students with low performance, according to the proficiency scales of the program, has been decreasing significantly over the years" (p. 56). For the authors, the improvements observed are more associated with public policies initiated in the 1980s and the actions inherent to their implementation than with participation in Pisa. In their conclusions, they state that the definition of an agenda for education and the development of programs to combat inequalities and for equity need to be oriented towards a diversity of specific curricular areas associated with the continuous training of teachers. For them, "there is more life beyond Pisa" (p. 58).

In Spain, a wide-ranging teacher training project from the perspective of Pisa, the GAPPisa Project, was carried out in public schools in the city of Barcelona with the aim of offering a theoretical and practical basis on the assessment of learning and enabling teachers to modify their assessment instruments of the traditional test type (MACIAS; MONEREO, 2018; MONEREO, 2009; MONEREO et al., 2009). For the GAPPisa coordinators, the final objective of the training proposal was not to analyze the results of Pisa, but to conduct a process of transformation, involving the entire school team, so that the evaluation was no longer punctual and was integrated into practices. teachers' pedagogies. According to Macias and Monereo (2018, p. 88), "changing assessment instruments is not enough; to effect a more substantial change, we must analyze what affects the teaching identity itself". For them, in future studies, it is necessary to analyze how the participation of teachers in projects such as GAPPisa is affected by the practices they experience and the discourses that are constructed and how they modify their conceptual, strategic and attitudinal positions when evaluating student learning.

This manuscript is part of the perspective of contributing to a pedagogical understanding of the results of Pisa. To this end, it discusses the content of the public items of the assessment carried out in 2012, the most recent in which Mathematics was the prioritized area of knowledge. Specifically, the results of an analysis of public items in the Mathematics subarea of Changes and Relationships (BRASIL, 2013) are presented here. Our objective was to know which topics/themes/skills are covered in the items of the Pisa Mathematics test. We believe that studies of this type provide knowledge of the operationalization of the evaluation and the verification of the alignment between the items and what the documents that guide the evaluation.

The text is part of a broader research whose objective is to analyze the assessment in Mathematics within the scope of Pisa 2012 and how the results of Brazilian students are distributed across geographic regions, genders and socioeconomic levels. The research as a whole is developed through different approaches, methodological involving document analysis, statistical analysis and analysis of the perception of high school students about the test items. In this article, we restrict the scope to the analysis of Pisa 2012 items, which deal with the theme Changes and Relationships. From a methodological perspective of document analysis, we sought, when describing some of the items, to establish articulations and approximations between the pedagogical processes of the Mathematics classroom and external assessments. Next, we discuss the assessment in Mathematics in the Program and then we present the results of the analysis of the items. We close this text with some considerations.

## ASSESSMENT IN MATHEMATICS IN PISA

The assessment in Mathematics in Pisa is carried out through open-response items (sometimes referenced by constructedresponse items) and multiple choice, considering three dimensions: content, process and context. Content is defined primarily in terms of broader mathematical concepts and

secondarily in relation to subject-specific curriculum content categories. These contents are organized into structuring areas, namely: Quantity; Space and Form; Changes and Relationships; Uncertainty. For the organizers of the assessment, the first three constitute the essence of any Mathematics curriculum in Basic Education. The fourth (Uncertainty) meets the broader character of Mathematics competence and connects with the needs of the citizen's daily life. There is a close proximity between the organization of contents of Pisa and those proposed in the document Base Nacional Comum Curricular - BNCC (BRASIL, 2017), although the nomenclatures used are slightly different.

The second dimension is defined by general mathematical skills, which include the use of mathematical language, choice of models and procedures, and problem-solving skills. These competencies are organized into three classes: performing simple operations, establishing connections to solve problems and mathematical reasoning, generalization and discovery. The third dimension comprises the situations in which Mathematics is used, ranging from particular contexts to those related to broader scientific and public issues (BRASIL, 2013).

According to Ortigão and Aguiar (2012, p. 8),

To reflect the breadth of knowledge, skills and competences being evaluated, the concept of literacy is used, which refers to the student's ability to apply their knowledge, analyze, reason and communicate efficiently, as they expose, solve and interprets problems in different situations. Literacy development is a lifelong learning process and therefore broader than the historical notion of the ability to read and write.

For each knowledge area assessed in Pisa, there is a continuous scale in which students' performance levels and their positioning on the scale and subscales are represented by the number of points achieved (proficiency). In the Mathematics assessment, the scale and subscales comprise six levels, and students who do not reach the minimum score required to place themselves in Level 1 are classified as 'below Level 1'. The following table presents the mean and percentage distribution of students on the Pisa 2012 Mathematics scale and subscales.

It is observed that, in two subscales (Space and Form and Changes and Relationships), more than 40% of the students who participated in Pisa 2012 are below the minimum level considered in the scale. And, in all subscales, more than 70% of students did not reach level 3. According to the Pisa 2012 report (BRASIL, 2013, p. 19), at this level students are able to perform clearly described procedures, select and apply simple problem-solving strategies, interpreting and using simple representations based on different sources of information and reasoning directly from them. They are also capable of making short communications that report interpretations, results and reasoning.

The Space and Shape subarea comprises a set of phenomena present in the physical and visual world: patterns; property of objects; position and orientation; representation of objects; encoding and decoding of visual information; dynamic interaction with real forms and their representations. In this subscale, 40.3% of Brazilian students were not able to use the mathematical skills required by the simplest assessment tasks, which placed them below level 1 of the scale. A more indepth discussion of the Pisa results in this subarea can be found in Pereira and Moreira (2020), Ortigão et al. (2018) and Lima (2016).

The Quantity subarea, according to the Pisa documents, involves understanding numbers and their operations, which are proposed in different situations and contexts. Quantitative reasoning is the essence of the area of quantity, and it is necessary to understand, for example, the multiple representation of numbers, mental and computational calculation, estimation and evaluation of the reasonableness of results. That is, this subarea encompasses numerical phenomena, relationships and quantitative patterns.

Indeterminacy and data encompass the recognition of the place of variation in processes, taking into account the quantification of that variation; recognition of indeterminacy and error in measurement; and knowledge of probabilities. This subarea also includes forming, interpreting and evaluating conclusions drawn in situations in which indeterminacy is a central aspect. It is observed that in 2012 Brazilian students performed the best on this scale, with 26.5% below level 1, when compared to the other subareas (PEREIRA; MOREIRA, 2020).

provides information Table 1 that could help schools and teachers to rethink pedagogical plans with a view to learning, as well as teaching systems in the (re)formulation and monitoring of public policies aimed at education, as pointed out by Vaz, Aguilar-Júnior and Nasser (2021). These authors seek to articulate discussions regarding the role of school assessments (both those carried out by/in schools, internal, and external, far reaching), emphasizing that both can present a formative role for learning. Specifically in relation to external evaluations, the authors state that the results obtained therein allow

> (...) on the one hand, that public managers can formulate and monitor the functioning of public policies aimed at education, in a more macro/global perspective of the processes. On the other hand, the pedagogical and teaching bodies, in the microcontingencies and in the context of the practice developed in the school routine, can realign their pedagogical proposals and their didactic and even curricular plans. (VAZ; AGUILAR-JÚNIOR; NASSER, 2021, p. 150)

Scale and Subscales of Math	Average (Dp)	Below Level 1	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Global scale	391,5 (2,1)	35,5	21,7	8,3	3,1	0,8	0,0	0,0
Space and shape	380,0 (2,0)	40,3	30,6	18,8	7,3	2,4	0,6	0,1
Quantity	392,9 (2,5)	36,5	27,0	20,2	10,5	4,3	1,3	0,2
Indeterminacy and Data	402,1 (2,0)	26,5	35,1	25,5	10,0	2,5	0,3	0,0
Changes and Relationships	371,5 (2,7)	46,3	24,0	16,5	8,4	3,3	1,1	0,3

(Dp) = Standard deviation of the mean

Table 1: Global average and in the content categories and percentage distribution of students at the scalelevels and subscales in Mathematics - Pisa 2012

Source: Pisa 2012 Microdata – Brazil. Self elaboration.

Subarea of Mathematics	Quantity of items
Space and shape	14
Quantity	18
Indeterminacy and Data	13
Changes and Relationships	11
Total	56

Table 2: Distribution of Pisa 2012 Mathematics (public) items according to subareaSource: BRAZIL (2013). Self elaboration

Theme/Content	Number of public items
Equation/Functions/Application of formulas	5
Velocity/Kinetics	5
Identification of mathematical models	1
Total	11

Table 3: Number of items according to content/skills - Pisa 2012 - Subarea Changes and Relationships.Source: OECD/INEP (2013). Self elaboration

As part of the broader research, we focused on the analysis of the public items of Mathematics of Pisa 2012. The items of Pisa are constructed and analyzed based on the Item Response Theory - TRI, a theoretical-methodological approach that takes into account the answers given for each item. According to Vianna (1987), the IRT allows comparisons both between different populations, as long as they are submitted to tests that have items in common, and between individuals who are part of the same population submitted to totally different tests. Common items make up the Item Bank and are not disclosed by the consortium that manages Pisa. However, with each edition, a set of items is released which, according to the Program's proposal, are examples that illustrate the operationalization of the evaluation in the specific area of knowledge. In the 2012 edition, 23 items on Mathematics were released to the general public, which can be accessed on the INEP or OECD website. It must be noted that many of these items have a common heading followed by two or three questions, which are considered different items. As a result, there are a total of 56 public items for Pisa 2012 Mathematics.

The following table presents the distribution of these 56 items in relation to the subareas of Mathematics. The classification was based on the analysis of the contents/skills required in each of them.

The distribution of items in subareas is relatively equitable, although there is a slightly higher quantity in the Quantity subarea. This distribution generally follows what traditionally occurs in Mathematics curricular organization, as observed by Silva and Mandarino (2010) and Fonseca and Mandarino (2010). The authors analyzed curricular programs of mathematics teaching networks and textbooks and found a predominance of topics in Numbers and Operation (sub-area named as Quantity within the scope of Pisa) in relation to the other sub-areas of Mathematics.

Next, we present the results of the analysis of the items in the Changes and Relationships subscale. We adopted a documentary research, of an exploratory nature, to conduct the choice and analysis of the items, since the main references used were primary sources characterized "by the direct relationship with the facts to be analyzed" (GONSALVES, 2001, p. 32). This decision was motivated by the fact that it was the subarea in which students had the greatest difficulty, that is, lower averages and greater concentration of students in the lower levels of the subscale, as can be seen in Table 1.

#### CHANGES AND RELATIONSHIPS IN THE MATHEMATICAL ASSESSMENT OF THE PISA TEST

According to the National Pisa Report 2012 (BRASIL, 2013), the assessment of mathematical literacy in the Changes and Relationships subarea requires that the individual understand the fundamental types of change and recognize when such changes occur, in order to use mathematical models that can describe them. and predict them (p. 32).

The Changes and Relationships subarea, called Algebra in the National Common Curricular Base - BNCC (BRASIL, 2017), encompasses a set of skills, such as the identification of regularities in sequences, whether numerical, figurative or otherwise. Curriculum documents, such as the BNCC, indicate that activities involving regularities favor the development of a special type of thinking, as they allow the student to perceive rules for the formation of sequences and the elements that constitute them. This subarea also includes the study of mathematical equality/inequality, generalization, variable, the study of equations and inequalities, proportionality, functions and aspects related to growth/degrowth, among other topics. According to the BNCC document, the Algebra thematic unit aims to

> the development of a special type of thinking - algebraic thinking - which is essential for using mathematical models in the understanding, representation and analysis of quantitative relationships of quantities and also of mathematical situations and structures, making use of letters and other symbols. For this development, it is necessary that students identify regularities and patterns of numerical and nonnumerical sequences, establish mathematical laws that express the interdependence relationship between quantities in different contexts, as well as create, interpret and transit between the various graphic and symbolic representations, to solve problems through equations and inequalities, with understanding of the procedures used. The fundamental mathematical ideas linked to this unit are: equivalence, variation, interdependence proportionality. and In summary, this thematic unit must emphasize the development of a language, the establishment of generalizations, the analysis of the interdependence of quantities and the resolution of problems through equations or inequalities (BRASIL, 2017, p. 270).

The excerpt above is in harmony and dialogues with what was already foreseen in the Pisa 2012 reference matrix for the Changes and Relations subarea. In fact, Pisa is considered a program that induces curricular reforms that have taken place in several countries (FERNANDES, 2019; BARRIGA, 2018; MACIAS; MONEREO, 2018; BAUER et al., 2015; BAUER et al., 2013). In the Brazilian case, it could not be different. There is evidence of a close approximation between what Pisa evaluates and what the BNCC proposes for the field of Mathematics Education (ANDRADE, 2021). In a recent work on the development of

skills with argumentation and demonstration in Mathematics, Caldato and Aguilar-Júnior (2020) discuss the approximations between the definition of mathematical literacy provided for in the PISA 2012 reference matrix and the BNCC. According to the authors, the BNCC text, which prioritizes pedagogical work based on learning rights through the construction of certain skills and competences, is closely related to the definition of mathematical literacy contained in the Pisa 2012 matrix, that is, the individual student's ability to formulate, employ and interpret mathematics in multiple contexts.

It is important to highlight that the commitment to the development of important areas addressed in Pisa has mathematical literacy as a considerable axis for individual development, guiding the student in the formulation and interpretation of mathematical ideas in a variety of contexts of everyday life (LIMA et al., 2020).

In mathematical terms, this means modeling these changes and relationships with appropriate functions and equations, as well as creating, interpreting, and transitioning between the various graphical and symbolic representations. The subarea is characterized by mathematical manifestations of change, functional relationships and dependence between variables (BRASIL, 2013; 2017).

Regarding the themes that make up the Changes and Relations subarea, when we analyze the 11 public items, we find three themes. The following table presents the classification of items according to the themes and contents proposed in each one.

The themes proposed in the items are presented in a balanced way between Equations/Functions/Application of formulas and Velocity/Kinetics. However, the identification of mathematical models is significantly less predominant. Public items on equations and functions are based on problem situations that involve the application of formulas and assess cognitive processes required for the student to demonstrate skills related to know-how, even if in an incipient way. Let's see the example below.

Figure 1 deals with an item that explores the idea of average speed. The student must be aware that the velocity of a body is given by the relationship between its displacement in a given time. In summary, the average velocity of this body is the variation of its position in relation to a reference during a certain time interval and the unit of measurement, in meters per second (m/s), according to the International System of Measurements (SI).

To correctly answer the question, the student must relate the quantities, dividing the distance by the duration of the route in each reported route and this way, he will determine Helena's average speed.

On the first route, 4km/10min is equivalent to 0.4km/min. On the second route, 2km/5min is also equivalent to 0.4km/min. In both courses, although it was not necessary, making the necessary conversions in accordance with the SI, this is equivalent to 6.67 m/s, using the rounding feature. Therefore, the respondent must come to the conclusion that Helena's average speed during the first 10 minutes was equal to the average speed during the next five minutes, therefore marking option (B) of the item. Based on the available statistical data, 42,020 students answered this question, among which 21,523 (51.2%) got the item right, which is considered insufficient, given that the question is at an elementary level.

It is worth noting that the question explores, on the other hand, the proportional reasoning: walking 4 km in 10 minutes is equivalent to doing 2 km in 5 minutes, since the relationship between the distance covered and the time spent to cover it remains the same. constant. It is also noticed, when analyzing the question, that the alternative (A) was the answer most chosen by the respondents, 25.6% (10,753) of the valid answers. This result indicates that, perhaps, the student does not perceive the proportionality relationship between the measures presented (there is a halving of the time spent and the distance covered).

According to BNCC (BRASIL, 2017), the study of Algebra topics, as well as Numbers, Geometry and Probability and Statistics, can contribute to the development of students' computational thinking, since they need to be able to translate a given situation. in other languages – how to transform problem situations presented in the mother tongue into formulas, tables and graphs and vice versa. These abilities are present in the context of Pisa, as can be seen in the example item presented below.

The second example is presented in Figure 2, which shows a situation related to Mount Fuji. The objective is to calculate the start of time taken for a walk, given two different speeds, a single total distance to travel, and an end time.

When starting to solve the above question, it is imagined that the student perceives that the statement contains information about the approxim

ate measure of the length of the Gotemba trail (9km). Then, you must realize that, when carrying out the walk, the hikers need to return by 8 pm (or 8 pm) and that the trail is 18 kilometers (round trip). It is also observed that the speeds for going up and down the trail are different (1.5km/h for going up and 3km/h for going down). At the end, there is an indication of what is unknown in the activity, that is, what, through the information presented in the previous sentences, must appear as an answer to the question: the last time when Toshi can start the walk so that he returns to the 8 pm (or 8 pm).

A simple scheme can be used to solve the question, which, apparently, is of a high



Figure 1: Pisa 2012 public item exploring the idea of average speed

Source: OCDE/INEP (2013).



Figure 2: Climbing Mount Fuji Source: OCDE/INEP (2013). level, as it requires reasoning and technique. Let's see: the Gotemba trail is 9km long, or 9,000m. Toshi ascends at a speed of 1.5km/h and descends at 3km/h. Speed is related to the hour it will take Toshi to go up the mountain, as well as the speed to the hour it will take him to go down the mountain. Every 1 hour it increases by 1.5km, obtaining a time of 6 hours to climb; to go down, every 1 hour they increase 3km, taking 3 hours to go down. Thus, the ascent time (6h) added to the descent time (3h), totals 9 hours. Subtracting 20:00 (8:00 pm) from 9:00 am, the answer is found, which is 11:00 am; the issue is resolved.

The different algorithms used to solve the question may indicate that the mathematical domain Change and Relations, which aims to calculate the beginning of the time spent given two different speeds, a total distance to travel and a final time, involve more elaborate social contexts and the processes of formulate are not always achieved, given that, according to the report (BRASIL, 2013), this item had an accuracy rate of less than 10%, which is worrying from the point of view of the subarea studied.

The third and last example chosen to compose this text refers to sailing ships and the question discussed is question 4. The objective of this question is to solve a complex real situation involving cost savings and fuel consumption. It must be calculated after how many years the savings with the cost of diesel oil could cover the cost of the kite sail, and the student must present the calculations to support his answer.

It must be noted that the question presents a motivating text, seen as a thematic trigger, "Sailing ships", followed by four questions (4 items). We present here only item 4.

The question above reflects on the high cost of diesel oil, at 0.42 zeds per liter; informs that the owners of the Nova Onda vessel are considering equipping the vessel with a

kite sail with the potential to reduce diesel consumption by around 20%. Next, the ship's data are presented, such as: type; length; wingspan; maximum capacity and speed. It also informs that the consumption of diesel per year, without a kite sail, is approximately 3,500,000 liters and that the cost to equip the Nova Onda ship with this kite sail is 2,500,000 zeds. Question 4 is then presented: "After how many years could the savings with the cost of diesel oil cover the cost of the kite sail? Present the calculations to support your answer.

According to the Pisa Report 2012 (BRASIL, 2013), the situation presented is real and complex, whose mathematical domain involves understanding changes and relationships, in a scientific context with a process of formulating. We note that the question involves "unknown currency" (zeds), English terms (kite sail), percentage and simple algorithm for the solution.

The solution to the issue can be simplified as follows: considering that the annual consumption of diesel without a kite sail is 3.5 million liters and the price is 0.42 zed/liter, the costs for diesel without a kite sail sail would be 1,470,000 zeds. Now, if 20% of this value is saved with the kite sail, we have that: 20/100 =0.2, which must be multiplied by the costs for diesel without a kite sail, that is, by 1,470,000 zeds. So 1,470,000 x 0.2 = 294,000 zeds per year. Finally, we have that 2,500,000/294,000 » 8.5. This time, after 8.5 years the kite sail becomes (financially) profitable.

Other public items of the Pisa test can be analyzed in a similar way and serve as a tool for teacher training, expanding not only what and how the Program evaluates, but leading teachers to reflect on the pedagogical aspects of educational assessment, among other issues.

As mentioned before, among the four subareas of Mathematics in Pisa 2012 (Space and Shape; Quantity; Indeterminacy and Data;

#### **Question 4: SAILING SHIPS**

Due to the high cost of diesel fuel, at 0.42 zeds per filter, ship owners

Nova Onda are thinking of equipping their ship with a kite sail. A kite sail like this is estimated to have the potential to reduce diesel consumption by around 20%.



The cost to equip Nova Onda with a kite sail is 2500 000 zeds. After how many years could the savings on the cost of diesel fuel cover the cost ofkite sail? Provide calculations to support your answer.

Figure 3: Sailing ship Source: OCDE/INEP (2013).

#### SAILING SHIPS

Ninety-five percent of world trade is carried out by sea, in approximately 50,000 tankers, large freighters and container ships. A good part of these ships is powered by diesel oil.

Engineers are designing the development of wind support for ships. The proposal is to attach skysails to ships and use the force of the wind to reduce diesel consumption as well as the fuel's impact on the environment.



and Changes and Relationships), Brazilian students performed weaker in the latter, namely Change and Relationships, with 46.3% below level 1. It is observed that just over half of the students evaluated obtained more satisfactory results or above the minimum level and it must be noted that, among the 79 countries participating in this edition of Pisa, Brazil was in 74th place in the ranking of mathematical literacy (PEREIRA; MOREIRA, 2020; AGUIAR; ORTIGÃO, 2012).

### FINAL CONSIDERATIONS

In this text we propose to bring a discussion about released items from the Pisa 2012 edition, focusing on the analysis of three public items from the Changes and Relationships subarea that are related to real-world activities, manifested, mathematically, by the changes, functional relationships and dependence between variables. That is, "the world, whether natural or man-made, involves a series of temporary or permanent relationships between objects and circumstances in which changes take place, which in many cases may involve discrete changes, and in others, continuous changes" (BRAZIL, 2013, p. 32).

According to the Pisa 2012 report on the national results obtained in this edition (BRASIL, 2013), it is highlighted that the Changes and Relationships category requires the student to understand the observation of patterns in processes that involve the change of verified quantities, in order to build and use mathematical models for their description and prediction. Thus, the student is required to understand the main types of change, as well as the causes and consequences of these changes, using mathematical modeling to interpret such variations (BACCHETTO; PINTO JUNIOR, 2017). The student must then be able to "create, interpret and move between the various graphic and symbolic representations" (BRASIL, 2013, p. 32). We highlight that the average performance presented by Brazilian students in Pisa 2012 – 371.5 in the Changes and Relationships subscale, as shown in Table 1, above –, considering the proficiency scale contained in the Pisa matrix (BRASIL, 2013, p. 19), locates participants at level 1 of the scale (of 6 levels) and characterizes these students, with an average age of 15 years, as

> able to answer clearly defined questions, involving known contexts, in which all relevant information is present (...) and to carry out routine procedures according to direct instructions in explicit situations. Are able to perform obvious actions and immediately follow up on given stimulus.

The first two questions evaluated (one with an open answer and the other of the multiple choice type) are related to average speed and aim to verify if, in addition to the domain, including the calculation of the average speed from the magnitudes distance and time, the The evaluated student had the ability to make inferences and estimates and look for patterns that would allow him to find the correct answer. The results found in the database reveal that only 41.9% of the respondents were able to find the answer for question 1 (A cyclist Helena), considered an item of elementary level in terms of difficulty, while in question 2 (Climbing Mount Fuji), that required mobilization of more reasoning and technique (calculation of the time spent, considering different speeds; the initial and final times and other data of the utterance), the rate of correct answers was around 10%. The third question analyzed, "Sailing ships", involves unknown terms and currencies and the simple percentage calculation. It aimed to solve a real situation, seen as complex, involving cost savings and fuel consumption. The rate of correct answers in the question was around 6%, that is, much lower than expected, although, by the solution presented here, the

question was not so difficult.

Even noting that the national results of Pisa 2012 showed that Brazilian students achieved performance below have the OECD average, we find that the average performance in Mathematics has shown a slight improvement (OECD, 2013; ORTIGÃO, 2018). Similarly, when obtaining results below the world average, with better performance in the Indeterminacy and Data subarea (26.5% below level 1), and with a scandalous difference of almost 20 points when compared to Changes and Relationships (46.3 % below level 1) (PEREIRA; MOREIRA, 2020), in which they achieved the worst performance, we reflect on the causes and consequences of this disastrous result, since the contents worked in the Change and Relations subarea appear more frequently in the seventh and eighth years of elementary school. In other words, students are close to completing the edition of Pisa, which is generally for students aged 15 years old, the end of compulsory basic education in most countries participating in the assessment.

In this sense, based on the assumption that "Mathematics must and needs to be critical" (MOREIRA, 2020, p. 15), given that, from the Pisa perspective, the inseparability of Mathematics from everyday life is the mainspring to understand and bring its application to the real world, the little progress made by students in each edition of the Program highlights the huge gap between what is desired and what has been achieved. With this, it ratifies the need to improve the mathematical literacy achieved by Brazilians, which denounces the need for investments in Basic Education in different sectors, since the cascade effect reflects in the next stages of the population's life, whether in access to University, whether in daily activities, such as interpreting the world around them, the job market, the relationship between supply and

demand, among other aspects.

we understand Thus, that external evaluations are not, per se, "pure" and unbiased measures to define the quality of educational policies. We do not take the side of those who use these results to make harsh and heavy criticisms of public schools, questioning public investments in education in the face of low academic performance verified in these external evaluations. However, despite the negative emphasis in terms of the "quality" of the education offered that the results of these evaluations can lead to, such results can and must be thought of from a more formative and diagnostic perspective.

The data analyzed here and their discussion processes need to be returned to schools with the intention of pointing out the learning that has not yet been implemented, leading to a movement of school self-assessment, aiming at the production of didactic-pedagogical proposals that meet the needs found by the school community, ensuring a "quality" education, however polysemic the word may be.

#### THANKS

The present work was carried out with the support of the National Council for Scientific and Technological Development – CNPq, the Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro – FAPERJ and the Prociência program at the University of the State of Rio de Janeiro.

#### REFERENCES

AGUIAR, G.; ORTIGÃO, M. I. R. Letramento em Matemática: um estudo a partir dos dados do Pisa 2003. **Bolema**, Rio Claro, v. 26, n. 42A, p. 1-21, abr. 2012. (DOI https://doi.org/10.1590/S0103-636X2012000100002). DispoLevel em: https://www.scielo. br/pdf/bolema/v26n42a/02.pdf. Acesso em: 23 fev. 2021.

ANDRADE, W. M. **O Sistema Permanente de Avaliação da Educação Básica do Ceará (Spaece) e o currículo escolar**: implicações no 9º ano do Ensino Fundamental. 2021. 221f. Tese (Doutorado em Educação Brasileira) - Faculdade de Educação, Universidade Federal do Ceará, Fortaleza, 2021.

ARRUDA, F. S. *et al.* Letramento matemático: um olhar a partir das competências matemáticas propostas na Base Nacional Comum Curricular do Ensino Fundamental. **Ensino da Matemática em Debate**, São Paulo, v. 7, n. 2, p. 181-207, 2020. DispoLevel em: https://revistas.pucsp.br/index.php/emd/article/view/48745/pdf. Acesso em: 9 fev. 2021.

BACCHETTO, J. G.; PINTO JUNIOR, W. N. Oportunidade de aprendizagem de conteúdo em Matemática no Pisa 2012. **Estudos em Avaliação Educacional**, São Paulo, v. 28, n. 68, p. 418-442, maio/ago. 2017. DispoLevel em: http://publicacoes.fcc. org.br//index.php/eae/article/view/3905. Acesso em: 04 fev. 2021.

BARRIGA, A. D. A prova Pisa: idealização, cidadania global, imposição cultural e ausência de impacto pedagógico didático. In: ORTIGÃO, M. I. R. (Org.). **Políticas de avaliação, currículo e qualidade**: diálogos sobre o Pisa. Curitiba: CRV, 2018, p. 19-38. (Série: Temas em Currículo, Docência e Avaliação, v. 3).

BAUER, A. *et al.* (Orgs.). Vinte anos de avaliação de sistemas educacionais no Brasil: origens e pressupostos. Ciclo de Debates. v. 1. Florianópolis: Insular, 2013.

BAUER, A. *et al.* Avaliações em larga escala: uma sistematização do debate. **Revista Educação e Pesquisa**, São Paulo, v. 41, n. especial, p. 1.367-1.382, dez., 2015. DispoLevel em: https://doi.org/10.1590/S1517-9702201508144607. Acesso em: 7 set. 2020.

BAUER, A.; GATTI, B. A. (Orgs.). Vinte e cinco anos de avaliação de sistemas educacionais no Brasil: implicações nas redes de ensino, no currículo e na formação de professores. Ciclo de Debates. v. 2. Florianópolis: Insular, 2013.

BRASIL. Ministério da Educação. **Base Nacional Comum Curricular - BNCC**. Brasília: MEC, 2017. DispoLevel em: http://basenacionalcomum.mec.gov.br/images/BNCC\_EI\_EF\_110518\_versaofinal\_site.pdf. Acesso em: 2 set. 2020.

BRASIL. Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira. **Relatório Nacional Pisa 2012**. Brasília: Inep, 2013. DispoLevel em: http://download.inep.gov.br/acoes\_internacionais/pisa/resultados/2014/relatorio\_nacional\_pisa\_2012\_resultados\_brasileiros.pdf. Acesso em: 7 set. 2020.

CALDATO, J. C.; AGUILAR-JÚNIOR, C.A. Argumentação e Prova em Matemática: uma análise dos itens públicos do PISA 2012. *Revista Baiana de Educação Matemática*, v. 01, p. 01-24, e202021, jan./dez., 2020. DispoLevel em: https://www.revistas. uneb.br/index.php/baeducmatematica/article/view/10321. Acesso em 19 set. 2021.

FONSECA, V. G.; MANDARINO, M. C. F. O ensino aprendizagem de estatística no ensino médio por meio de projeto interdisciplinar. In: 5º ENCONTRO ESTADUAL DE EDUCAÇÃO MATEMÁTICA - V EEMAT. Out. 2010, Rio de Janeiro. **Anais...** Rio de Janeiro, SBEM-RJ, p. 56-66, 2010.

FRANCO, C.; BONAMINO, A. Iniciativas recentes da avaliação da qualidade da educação no Brasil. In: FRANCO, C. (Org.). **Avaliação, ciclos e promoção na educação.** Porto Alegre: Artmed, 2001. p. 15-28.

GONSALVES, E. P. Conversas sobre iniciação à pesquisa científica. 2. ed. Campinas: Alínea, 2001.

LIMA, P. V. P. *et al.* Brasil no Pisa (2003-2018): reflexões no campo da Matemática. **Tangram - Revista de Educação Matemática**, v. 3, p. 03-26, 2020.

LIMA, R. de L. **Avaliação em Geometria no Pisa 2012:** uma análise dos conteúdos e dos itens disponibilizados pelo INEP. 2016. 114f. Dissertação (Mestrado Acadêmico em Educação, Cultura e Comunicação) - Faculdade de Educação da Baixada Fluminense, Universidade do Estado do Rio de Janeiro, Duque de Caxias, 2016. DispoLevel em: http://www.bdtd.uerj.br/tde\_busca/arquivo.php?codArquivo=10804. Acesso em: 21 fev. 2020.

MACEDO, E. Base Nacional Curricular Comum: novas formas de sociabilidade produzindo sentidos para Educação. **e-Curriculum**, São Paulo, v. 12, n. 3 p. 1.530-1.555, out./dez. 2014. DispoLevel em: https://revistas.pucsp.br/index.php/ curriculum/article/view/21666/15916. Acesso em: 9 fev. 2021.

MACIAS, E. M.; MONEREO C. M. Transformar a avaliação através do Pisa: o Projeto GAPPisa. In: ORTIGÃO, M. I. R. (Org.). **Políticas de avaliação, currículo e qualidade**: diálogos sobre o Pisa. Curitiba: CRV, 2018. p. 69-90. (Série: Temas em Currículo, Docência e Avaliação, v. 3).

MENDES, G. M. L.; SEGABINAZZI, M. Incluir, comparar e competir: serviços de avaliação externa em larga escala e inclusão escolar. **Revista Educação Especial**, Santa Maria, v. 31, n. 63, p. 849-862, out./dez. 2018. DispoLevel em: http://dx.doi. org/10.5902/1984686X33104. Acesso em: 15 fev. 2021.

MONEREO, C. (Org.). Pisa como escusa. Repensar la evaluación para cambiar la enseñanza. Barcelona: Graó, 2009.

MONEREO, C.; CASTELLÓ, M.; DURÁN, D.; GÓMEZ, I. Las bases psicoeducativas del proyecto Pisa como guía para el cambio en las concepciones y prácticas del profesorado de secundaria. **Infancia y Aprendizaje**, v. 32, n. 3, p. 421-447, 2009. DispoLevel em: https://www.researchgate.net/publication/233651476\_Las\_bases\_psicoeducativas\_del\_proyecto\_Pisa\_como\_guia\_para\_el\_cambio\_en\_las\_concepciones\_y\_practicas\_del\_profesorado\_de\_secundariaThe\_psychoeducational\_principles\_of\_the\_Pisa\_project\_as\_a\_guide\_for\_ch. Acesso em: 23 fev. 2021.

MOREIRA, G. E. (Org.). **Práticas de ensino de Matemática em cursos de licenciatura em Pedagogia**: oficinas como instrumentos de aprendizagem. São Paulo: Editora Livraria da Física, 2020.

ORGANIZAÇÃO PARA COOPERAÇÃO E DESENVOLVIMENTO ECONÔMICO (OCDE). **Country note – Results from Pisa 2012**. [s.l.]: OCDE, [s.d]. DispoLevel em: http://download.inep.gov.br/acoes\_internacionais/pisa/resultados/2013/ country\_note\_brazil\_pisa\_2012.pdf. Acesso em: 04 fev. 2021.

ORTIGÃO, M. I. R.; AGUIAR, G. S. Letramento em Matemática: um estudo a partir dos dados do Pisa 2003. **Bolema**, Rio Claro, v. 26, n. 42A, p. 1-21, abr. 2012. DispoLevel em: https://www.scielo.br/pdf/bolema/v26n42a/02.pdf. Acesso em: 9 fev. 2021.

ORTIGÃO, M. I. R. *et al.* Letramento em Matemática no Pisa: o que sabem e podem fazer os estudantes? **Zetetiké** (online), p. 375-389, 2018. DispoLevel em: http://https://periodicos.sbu.unicamp.br/ojs/index.php/zetetike/issue/view/1256/showToc. Acesso em: 2 set. 2020.

ORTIGÃO, M. I. R.; AGUIAR, G. S. Letramento em Matemática no Pisa. SEMINÁRIO INTERNACIONAL DE PESQUISA EM EDUCAÇÃO MATEMÁTICA, 5, out. 2012, Petrópolis, Rio de Janeiro. **Anais...** Brasília, Sociedade Brasileira de Educação Matemática, 2012. DispoLevel em: http://www.sbembrasil.org.br/files/v\_sipem/PDFs/GT08/CC66430259749\_B.pdf. Acesso em: 9 set. 2020.

PEREIRA, C. M. M. C.; MOREIRA, G. E. Brasil no Pisa 2003 e 2012: os estudantes e a Matemática. **Cadernos de Pesquisa**, São Paulo, v. 50, n. 176, p. 475-493, abr./jun. 2020. DispoLevel em: https://www.scielo.br/scielo.php?script=sci\_arttext&pid=S0100-15742020000200475&lng=pt&nrm=iso&tlng=pt. Acesso em: 2 set. 2020.

SILVA, L. A.; MANDARINO, M. C. F. Dificuldades no aprendizado de álgebra no oitavo ano do Ensino Fundamental. 5° ENCONTRO ESTADUAL DE EDUCAÇÃO MATEMÁTICA - V EEMAT. Out. 2010, Rio de Janeiro. Anais... Rio de Janeiro, SBEM-RJ, p. 84-96, 2010.

VAZ, R. F. N.; AGUILAR-JÚNIOR, C.A.; NASSER, L. Avaliações internas e externas para as aprendizagens: um debate necessário. In: SILVA, I. V.; KISTERMANN-JUNIOR, M. A. (Org.) **Avaliação da Aprendizagem**: possibilidades para a prática docente na educação básica. 2ª ed. ampliada e revisada. Taubaté: Editora Akademy, 2021. p. 137-156.