v. 2, n. 22, 2022

International Journal of Human Sciences Research

HAND IN HAND WITH UNDERSTANDING: CONSTRUCTION OF REGULAR POLYHEDRONS WITH TRIANGULAR FACES

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Abstract: The main objective of this work is to present a teaching proposal to approach Plato's polyhedra through structured didactic sequences, based on exploratory situations as a teaching methodology and a reflection on the procedures used during the manufacture of solids involving different techniques as a tool. education. The tasks were designed using different materials present in everyday life (paper, straws, toothpicks and string) to assemble and explore Plato's polyhedra that have triangular faces - tetrahedron, octahedron and icosahedron from the plans, from the reading of schemes to the assembly of the skeletons of the respective solids, and the folding in paper (origami). They were presented, in classrooms, to students of different levels of education, from Basic (6th and 9th grades, 2nd grade of High School) to Higher (mathematics and pedagogy students). The work developed with high school students aimed to differentiate between faces and sides, edges and sides, angles, polygons and the identification of Euler's formula, and with the undergraduates, to study and reflect on the different ways of approaching the solid in order to realize that different resources promote different approaches and mathematical meanings. The work was organized aiming to provide experiences and prioritizing reflections on the construction of spatial geometry concepts, approaching both the construction of solids and the analysis of their properties evidenced according to the material used. Although the presentation of different alternatives for the classroom may generate some concern with the objectives of mathematics, its appropriation, and with didactic-pedagogical aspects that cover the teacher's work, the experience of tasks of this type offers an opportunity for students to learn and question specific vocabulary in addition to developing spatial perception, visualization and observation and representation of mathematical relationships.

Keywords : Initial and continuing teacher education; Physical manipulative materials; Origami; schemes.

INTRODUCTION

The main objective of this work is to present a teaching proposal to approach Plato's polyhedra through structured didactic sequences, based on exploratory situations as a teaching methodology and a reflection on the procedures used during the manufacture of solids involving different techniques as a tool . of teaching. In general, schools do not have a laboratory, and the work of teachers is essentially based on textbooks, activities were developed using different materials present in everyday life (paper, straws, sticks and strings) to assemble and explore the three Plato's polyhedra that have triangular faces – the tetrahedron, the octahedron and the icosahedron from the plans, from the reading of schemes to assemble the skeletons of the respective solids, and from the paper folding (origami).

Another objective of this work is to propose support material to teachers through a script of tasks that can help them in classes on the subject. The work was organized aiming to provide experiences and prioritizing reflections on the construction of spatial geometry concepts, approaching both the construction of solids and the analysis of their properties evidenced according to the material used. Although the presentation of different alternatives for the classroom may generate some concern with the objectives of mathematics, its appropriation, and with didactic-pedagogical aspects that cover the teacher's work, the experience of activities of this type offers the opportunity for students to learn and question specific vocabulary, in addition to developing spatial perception, visualization and observation and representation of mathematical relationships.

It is clear that the teaching of mathematics has undergone some transformations in recent decades. New curricular proposals seek to insert and integrate different modalities of methodologies and approaches. However, traditional living teaching (VTE), in which students are usually organized in individual lines watching the teaching of the teacher, is still a predominant presence.

With the advent of Information and Communication Technologies (ICTs), people's lives have changed, being an unprecedented event in history. The way we deal with information and the way we acquire knowledge is changing radically, impacting the teacher-student relationship, the way students relate to school subjects, the way the family has followed the growth of their children and in the way of doing research. Students incorporated the use of these technologies both to search for information and to record classes. The photo of class notes on the board has been frequent.

Thus, more than ever, it becomes urgent that the situations presented to students encourage doing and discussing, that allow discovery and analysis, systematization and generalization instead of just being exercise classes. However, in order to place students as protagonists of their learning, it is necessary to review not only the content, but the chain of the same and think of different ways to present it.

According to Fainguelernt (1995),

> [...] it is essential that the mathematical content presented to students puts them ahead of the widest possible variety of situations that arouse their interest and contribute to their intellectual development (FAINGUELERNT, 1995, p.45).

Among so many possible variations, we have problem solving, conventional exercises, resorting to history, modeling a problem situation, games, videos or movies, among others. Free explorations and guided investigations allow us to think of them as possibilities to be developed in Geometry classes, both with physical and virtual manipulative materials, such as Geogebra.

According to Kindel (2021),

> Situations that allow exploring and discovering Mathematics are important tools to introduce and deepen mathematical concepts and are essential that they are proposed to students by their teachers. One of the ways that this exploration and discovery is possible is the proposition, in the classroom, *of* investigative activities for the study of certain contents (KINDEL, 2021, 525).

The activities require, however, that the teacher prepares himself to think and modify the whole dynamics of the classroom, from the spatial organization of the chairs, to the moment in which there will be an exchange of discoveries for the whole class, made by the students. To carry out these activities, students need to be organized in pairs or groups so that together they can interact in the explorations and investigate the proposed situations, discussing and reflecting on their findings, clarifying doubts, observing and describing the objects analyzed while developing creativity. and skills that favor the construction of concepts.

As a teacher in Basic Education, and currently in Higher Education, working in the initial training of teachers in pedagogy and degree courses in mathematics, it is possible to identify that there are gaps in the training of university students and that contribute to their having difficulties in understanding some concepts. and to teach their students.

Scheffer (2006) defends an approach that discusses "the use of media, such as folding and dynamic *software* , in approaching aspects of plane geometry" (p. 93). The author also highlights,

> [...] the importance of studying geometric concepts and objects starting from the experimental and inductive, to reach the exploration of activities that cover the study and construction of equilateral triangles, [...] and other polygons on the computer screen (SCHEFFER , 2006, p.96).

As polygons are constitutive elements of spatial geometric objects, an approach to them brings oral and written conjectures to the center of discussions in the classroom, as it offers students the opportunity to question and question themselves about the discoveries and visualizations made. In this way, students reinforce the spatial perception of the objects analyzed through the observations, descriptions and representations presented in the different work proposals.

Thus, according to Scheffer (2006),

> [...] the teaching of Geometry cannot be reduced to the mere application of formulas and results established by some theorems, without the concern of discovering ways for their demonstration, as well as for the deduction of their formulas (SCHEFFER, 2006). , p.96).

It is still worth remembering that the written record of explorations and investigations influences mathematical learning because during the writing process the student can explain his discoveries and difficulties, his conjectures and conclusions as well as his difficulties, his feelings related to the experiments or the classes of mathematics, raise questions and descriptions about the path taken in the analysis of the material.

In order to experiment investigative situations in the classroom, and verify the results obtained, this work, at first, sought to survey materials that could be applied in a didactic sequence using different approaches to the construction of polyhedra.

In agreement with Scheffer (2006), the proposal presented here,

> [...] turns to deepen the understanding and understanding of how mediation by the media highlighted here, through various interactions, promotes the attribution of mathematical meanings of geometry, mainly looking at the pedagogical practice in the classroom (SCHEFFER , 2006, p. 94).

In addition, it seeks, through the use of materials already available in school environments and the teacher's knowledge, to work on aspects of geometry. It is worth remembering that the tasks presented here were performed both by high school students and by mathematics undergraduates, evidently with different objectives. For the first audience, the objective was to build concepts and for the second, to propose a reflection on the importance of a more reflective, more dynamic and more diversified approach.

GEOMETRY AND PLATONIC POLYHEDRONS

It is necessary, initially, to observe that Geometry, from the Greek *"geo"* , which means earth, and *"metria"* , which means measure, was born as an empirical or experimental science in Ancient Egypt. Para cto build houses and pyramids it was necessary to observe and predict the movement of the stars, share fertile lands were some, among many activities that depended on geometric calculations carried out by the Egyptians.

Other peoples, like the Chinese, also developed knowledge in the area, but it was the Greeks, about five centuries before the Christian Era, between 600 and 300 BC, who systematized all the knowledge they had. In this period, geometry established itself as an organized system . Much of this is due to *Euclid* who published, around 325 BC, *The Elements* , a thirteen-volume work in which an unprecedented system in the study of Geometry is described, and the Platonic polyhedra are presented in some of these chapters.

Euclid's studies focused on plane and

International Journal of Human Sciences Research ISSN 2764-0558 DOI 10.22533/at.ed.5582222219074

spatial figures , and among them we can highlight polyhedra. He gives a complete mathematical description of the Platonic polyhedra in the last book (Book XIII) of *The Elements* and presents them inscribed on a spherical surface. Centuries later, regular polyhedra inspired the German astronomer Johannes Kepler (1571 – 1630), who tried to find a relationship between the five solids and the seven planets known at the time: Mercury, Venus, Earth, Mars, Jupiter and Saturn.

Kepler proposed a cosmological model represented by polyhedra where you can see, from the inside out, an octahedron followed by the icosahedron, the dodecahedron, the tetrahedron and finally the cube. But, his model was totally disproved by later discoveries of the planets Uranus, Neptune and Pluto and with that the model was abandoned.

The word "polyhedron" is formed from two Greek words: " *polys* " meaning "various" (giving rise to the prefix poly) and " *hédrai* " meaning "faces" (giving rise to the suffix "hedron"). Among its infinite polyhedral forms there are some that, due to their symmetry, have long fascinated men, being used in religious and mystical rites, and as burial chambers (in the case of the Egyptian pyramids), where in some of them the remains of kings.

Polyhedra are classified according to the shapes and numbers of their faces, which may or may not be regular. Regular polyhedra are examples of "aesthetically harmonic" shapes.

Due to the way in which Plato (427 BC-34 BC), in a dialogue entitled *Timaeus* , used them to explain nature, these solids were called "Plato's polyhedrons". It is unknown whether *Timaeus* actually existed or whether Plato invented him as a character to develop his ideas. In these dialogues, in *Timaeus* , Plato associates each of the classical elements (earth, air, water and fire) with a regular polyhedron, namely:

> Fire – Tetrahedron (four triangular faces): the most mobile; the smallest body; the most acute angle.
>
> Earth – hexahedron (six square faces), the cubic form: it is the most stable element of bodies, the one with the widest base.
>
> Air – Octahedron (eight triangular faces): an intermediate figure, the intermediate body.
>
> Water – Icosahedron (twenty triangular faces): the least mobile; the biggest body. These four elements form the Universe, represented by the dodecahedron (twelve pentagonal faces), the closest figure to the sphere (DANTE, 2013, p. 204).

When we examine the formation and origin of the word geometry and polyhedron, two concepts to be addressed, we evidence the contribution of different peoples to its construction. Currently, a polyhedron is an object of Mathematics that can be defined with different levels of generality, such as the fact that a polyhedron is the union of a finite set of flat polygons with the following properties:

> It is always possible to go from the interior of a polygon to the interior of any other polygon by a path entirely contained in the polyhedron.
>
> Let V be any vertex of the polyhedron and F_1, F_2, ..., F in the $_n$ polygons that meet in V. We can go from any polygon F_i to any polygon F_j, with i, j = 1 , 2,...,n, without going through the vertex V .
>
> If two polygons are adjacent, then they are not coplanar (MAR, 2013, p. 6)

Note that in the definition of polyhedron presented earlier, some of its elements were also defined, namely: vertex, face, edge.

Polyhedra can be classified into: regular (concave and stellate or convex), semi-regular and irregular. Concave regular polyhedra also known as Plato's polyhedra have all faces formed by equal regular polygons. There are only five of them, the tetrahedron,

International Journal of Human Sciences Research ISSN 2764-0558 DOI 10.22533/at.ed.5582222219074

octahedron and icosahedron which have 4, 8 and 20 triangular faces, respectively; the hexahedron which has 6 square faces and the dodecahedron which has 12 pentagonal faces. There are 13 semiregular polyhedra, and all of them can be obtained directly or indirectly by operations performed on regular polyhedra. Irregular polyhedra are prisms and pyramids, commonly studied in high school.

Some teachers have already noticed that, when teaching spatial geometry classes, some conceptual confusions arise when naming the side and edge elements of polyhedra, because what is called the side of the polygon in plane geometry is now intersected with the side of another polygon. , becoming the edge, and while the polygon becomes the face of the polyhedron. The point is that from a mathematical point of view, when the intersection between two polygons has a common side, this is called an edge, but what remains "strong" for the student is that it is the side of the polygon. In this way, conflict is installed, after all, is it a side or an edge?

Kindel (2021), mentions that,

> In the case of spatial geometry we have an extra component because the figure has three dimensions (3D) but when you draw it on the board or see the drawing in the textbook it has only two dimensions with some rules to give a sense of 3D. For example, full and dotted lines. The fact is that what is apparently easy and obvious for the teacher, as he moves between objects_ plane figure and spatial figure, referring sometimes to the first and sometimes to the second indistinctly. That is, sometimes the teacher refers to the elements of the plane figure, sometimes he refers to them as elements of the spatial figure. For the student, this "passage" is not that simple (KINDEL, 2021, p. 521).

The difficulties present in three-dimensional objects involve the differentiation between side and edge, side and face and identification of angles in the plane and in space.

Considering the difficulties presented by the students, we sought to create a geometric context that would serve as a didactic instrument that could develop in the participants the development of geometric thinking , as well as serve as a base material so that they could work with their students in the future under a new perspective, that of active classes with learning based on investigative activities, using games, based on problem solving, among other approaches.

We cannot forget that many geometric concepts were born empirically. Given the above, the use of manipulable materials can be an interesting strategy to study geometry , as it helps to establish a connection between the physical world and thought.

GEOMETRY AND MANIPULABLE MATERIALS

The opinion has been generalized that the use of manipulative material can serve to articulate and contextualize the concepts with the students' cognitive reality, although this is still not the reality of many high schools. One of the main arguments used by teachers is that there is not enough time given the extension of the curriculum to be fulfilled. In other words, according to Scheffer (2006) " discussing different alternatives for the classroom generates a certain concern with the objectives of mathematics, its appropriation, and with didactic-pedagogical aspects that cover the teacher's work" (SCHEFFER, p. 93-94).).

But, increasingly, especially in geometry classes, there are already some initiatives involving different alternatives such as the use of structured and unstructured physical manipulative materials (scrap) and dynamic software such as Geogebra. At the moment, years 2020-22, with social distancing due to Covid 19 and its variants, the introduction of manipulative materials and other resources is gradually gaining ground.

Manipulating materials can be classified as structured, created for educational purposes, which can be found in wood or acrylic to represent geometric solids, and unstructured ones existing in everyday life (straws, string, barbecue sticks, styrofoam balls) and others (paper, glue, scissors, cardboard), also present in the school space. In our case, these are used to assemble the skeletons (structure of the edges) of the solids, make origami (of the faces) and the flattening of the solids.

In particular, we sought to present situations in which participants develop the ability to visualize, analyze and informally organize , and in which they could represent and interpret graphic situations. According to Van Hiele, these skills are important steps towards understanding and formalizing geometric concepts.

According to Kaleff (1998),

> Several researches in Mathematics Education point to the importance of encouraging, in educational environments, the development by the student of the ability to visualize both real-world objects and, at a more advanced level, mathematical concepts, processes and phenomena. For some researchers, this ability is as important or more important than the ability to calculate numerically and to symbolize algebraically (KALEFF, 1998, p.15).

Possessing this skill has been increasingly valued in recent years. Just look at the memes, the " *gifts* " and the collections of icons that are created daily to represent ideas and entire messages in the media and social networks. Never has the phrase "a picture is worth a thousand words" been so socially important.

The use of manipulative material serves as a conceptual metaphor on which both, teacher and students, lean to discuss what they observe and infer observations, facilitating communication between them. Discussing these different ways of seeing the same object enriches the discussion and contributes to the construction of mathematical knowledge, facilitating dialogue. It is different to see a tetrahedron in the material and a drawn representation of a tetrahedron. For many students the representation of a tetrahedron is a composition using several different triangles, hence the importance of varying the illustrated representation since both the technique and the illustrator's point of view change the way of representing the visualized object.

Being able to pick up and place a solid supporting it in different positions provides a wealth of details that are not always noticeable in its representation on paper. We can cite the case of a straight prism with a pentagonal base, any face can be considered as a support base. The same goes for any other straight prism. However, for nomenclature purposes , the two congruent and similar faces, supported on parallel planes, are considered to be the base.

In the case of materials used for the study of spatial geometry, it is possible to identify aspects that are difficult to differentiate in the design. For example, when a tetrahedron is represented in the plane, two or more faces are drawn , and in perspective the others appear, but the drawing is a composition of different triangles.

Another aspect to be considered is the support that the material offers for students to take ownership of the contents and provides conditions for them to test their findings, elaborate conjectures and try to demonstrate their hypotheses.

In view of what was mentioned above, it is also necessary to point out that there are different materials, different possibilities for discoveries, visualizations and consequently conjectures. That is, when we assemble the skeletons, the edges are more easily visualized and the faces are hollowed out. In origami, the faces are highlighted and the edges become part of the context as a connecting link materialized by the connectors, while in the flat pattern, a drawing becomes a spatial object. The plane (2D) transmutes into something in space (3D).

According to Kindel (2021),

> Making origami is also a form of visual/sculptural representation being defined by the folding of a single sheet of paper, in addition to valuing the movement of the hands, stimulating the joints and the brain. Through the folds, students use their hands to follow a specific set of steps in sequence producing a visible result. Activities with Origami, in math classes, have a dynamic that values discovery, conceptualization, manipulative construction, visualization and geometric representation (KINDEL, 2021, p. 536).

If, on the one hand, origami is used to create shapes and objects, on the other hand, we can also unfold it and analyze the marks of the folds left printed on the paper and try to identify what is seen, which figures are formed by the intertwining of the different folds. In this sense, Kindel (2010) carried out a study with 8th grade students, at the time 7th grade, in which they identified different positions of lines and figures existing when the origami had been unfolded, and from there, the study of parallel lines cut by a transversal, content present in the curriculum of that series.

Furthermore, we cannot forget that to mathematize, one of the fundamental steps is the registration through writing.

According to Powell and Bairral (2006), "writing forces interlocutors to reflect, differently, on their mathematical experience. As we examine our productions, we develop our critical sense. Writing supports acts of cognition and metacognition (p.26)".

By recording the process experienced in assembling the solids, students can both describe their difficulty with writing, as well as describe the material, the making process and the mathematical discoveries. In this way, the teacher identifies what feelings students express in relation to the discipline, their difficulties with writing or their experience with the material. But if at first the writing is free and often describing their feelings, or the material little by little, at each new stage of the work, in a new register, their writing becomes more specific, it can then present mathematical *insights about the action.* experienced. Taking into account writing, it is also necessary to think about the way in which the task is proposed to students.

Given the above, we present what we think is a task and an activity. We understand by task, the proposal (exercise, problem, text, etc.) that is presented to the students and, as activity, the description of the students' engagement and solution for the accomplishment of the tasks. Thus, we understand that we propose tasks and that when they are understood, taken for themselves to be carried out, the students are in activity. The activity foresees several attitudes that come into action for the execution/preparation of the task: reading the material, analyzing the material received, talking with your groupmate, taking notes, searching for a solution strategy, searching by the consensus of the answers to be given, among others.

The tasks presented here can be classified into two types: exploration and investigation. Explorations, according to Ponte, are easy and open, while investigations are also open, but have a high degree of difficulty. However, it will not always be possible to distinguish this difference. So it will depend on who is active. According to Ponte (2010),

> This happens, most likely, because it is difficult to know at the outset how difficult an open task will be for a certain group of students. However, since we attach importance to the degree of difficulty of the tasks, it is preferable to have a designation for the easier open tasks and another

> designation for the most difficult ones (PONTE, 2010, p.22).

Tasks can also be classified in relation to the context used, which can be contextualized in a real situation or in terms of pure mathematics, in our case. In our case, it involves different representations of solids in which, with the proposal, we seek to instigate students to formulate questions and seek explanations. In this scenario, the students are responsible for the process, here placed in the different constructions and in the search for regularities from the comparison of the number of sides, edges and faces, in the observation and analysis of the folds that can be seen as relative positions of straight lines, one or more lines (segments) and the visualization of figures formed by them.

According to Skovsmose (2000),

> Scenario-based classroom practices for investigation differ markedly from exercise-based ones. The distinction between them can be combined with a different distinction, which has to do with "references" that aim to lead students to produce meanings for mathematical concepts and activities (SKOVSMOSE, 2000, p.7).

A proposal that considers moments of construction of solids made in different ways and moments for their analysis is an environment that offers resources to carry out investigations.

The presentation that follows is partially based on my work with mathematics education, as a teacher of the degree course in the discipline of Teaching Mathematics, in workshops offered in extension courses, in guidance of monographs of undergraduates working in high school and through the coordination of different projects, including the PIBID (2014-2017).

HANDS UNDER CONSTRUCTION: THE MODULES AND TASKS

Next, I bring several tasks that were developed with students from the 6th and 9th years of Elementary School II, from the 2nd grade of High School and with mathematics undergraduates from a public university in Baixada Fluminense/RJ in workshops, in study group discussions from Pibid in the period from 2014 to 2017 and in the Mathematics Teaching II subject, present in the curriculum.

The tasks were grouped four modules, namely: I- The names and music; II- Assembling the polyhedrons of triangular faces from the planning; III- Skeletons of Plato's polyhedra with triangular faces and; IV- Making folds to assemble the polyhedrons with triangular faces.

I THE NAMES AND THE MUSIC

All languages are alive and change over time, being influenced by other peoples or undergoing internal transformations depending on the uses. It is no different with the terms used in mathematics in which, for many of them, understanding their origin facilitates the understanding of the concept itself. Several activities were developed from a reflection involving the terms geometry, polygons, polyhedra and the names of the different polygons. In this module, two tasks are foreseen: (1) understanding the names; (2) making a connection with the music.

Task 1 Understanding the names

In general, with the licensee, I promote questions involving the meaning of the nomenclature used in geometry, specifically: geometry, polygons, polyhedra and the names of the different polygons. From there, I propose some developments such as: citing other words in your vocabulary that begin with the prefix geo having the same meaning used in the word geometry, looking up other words in the dictionary. Ditto, for poly, tri,

International Journal of Human Sciences Research ISSN 2764-0558 DOI 10.22533/at.ed.5582222219074

tetra, among others.

Task 2 Making a connection to the music.

The lyrics of the song Tribalistas present several terms in which the radical tri appears. Listening to it serves to arouse curiosity and establish a direct connection with the mother tongue.

Promote a conversation around the lyrics and suggest that they identify the words contained in the song that refer to the number three. Then, proposing a deeper study by researching other words in the dictionary and in the math book, can enrich the work even more.

Then the teacher can suggest a conversation circle about the letter and suggest that they identify all the words that are related to the number 3 (three); list, in your notebook, the ones that appear in the song and the ones you know; look up other words in the dictionary; establish connection with other words, whose meaning is also associated with quantities such as tetra, quadri, penta, hexa, etc.

II ASSEMBLING THE POLYHEDRONS WITH TRIANGULAR FACES FROM THE FLAT PATTERN.

This module provides five tasks: (1) Draw equilateral triangles using ruler and compass; (2) Reproduce on a larger scale the plans of the three solids to be studied; (3) Analyze which drawings represent flats of the octahedron and (4) Complete the table by counting the elements (vertices, edges and faces) of each solid.

Task 1 Drawing triangles with ruler and compass.

Follow the script to draw the equilateral triangle with sides measuring 5 cm:

1)Trace any segment in the middle of your sheet.

2)Mark a point anywhere on this segment and name it the letter A.

3)Measure 5 cm from point A and mark point B.

4)Take the compass and with the dry point at A, and opening equal to 5 cm, draw an arc through B.

5)Centering the compass on B with the same opening, 5 cm, draw another arc such that it passes through A.

6)The two arcs must intersect.

7)Note how many crossings there were. Choose one of them and call it C.

8)Connect the points A with the point C, thus determining the segment $\overline{AC}$ and then connect the point B with the same point C, determining the segment $\overline{BC}$.

9)Ready! You have an equilateral triangle whose sides measure 5 cm.

Following the steps above, you are now able to draw any equilateral triangle.

Task 2: Expanding the flats of triangular-faced polyhedra.

You are receiving the flattening molds of the three solids that we will study: octahedron, icosahedron and tetrahedron, respectively.

Figure 1: Plato's Polyhedral Planes.

Source: Author.

On a cardboard or cardboard, follow the script to enlarge the solids that will be cut and assembled afterwards.

Script to draw the base triangle of the flat patterns:

1.On card stock, with a ruler, trace a line segment $\overline{AB}$, in the desired size. If you want to make a tetrahedron whose sides measure 5 cm, then the segment must have this measurement.

2.After that, let's draw two circles centered at these points, A and B.

3.With center at A and radius AB construct the circle $C_1$and with center at B and radius AB construct the circle C_2. At the intersections of these two circles we mark the points C and D.

4.Then connect all the points, determining the segments AC, BC, AD and BD. Once that's done, we'll draw another triangle.

5.Now, with center at C and radius AC, draw a third circle C_3. At the intersection of $C_1$and C_3, we mark the point D_1 and at the intersection of C_2 e C_3, we mark the point D_2.

6.We trace the segments AD_1, CD_1, BD_2 and CD_2. We thus determine the flatness of a regular tetrahedron.

7.Observe each of the figures above and identify the need to draw an "ear" that will serve as a base for the collage when assembling the polyhedron. Draw on your figure too.

8.Cut out your template and assemble the tetrahedron.

9.Then, based on what you learned here, do Task 3.

Task 3: Design and assemble the octahedron and icosahedron

Use a cardboard to reproduce the flattening of the octahedron and the icosahedron, choose a measure value for the side of the triangle. Cut, fold, glue and you're done!

Task 4: Yes or No?

The following eleven figures are possible flattenings of the octahedron.

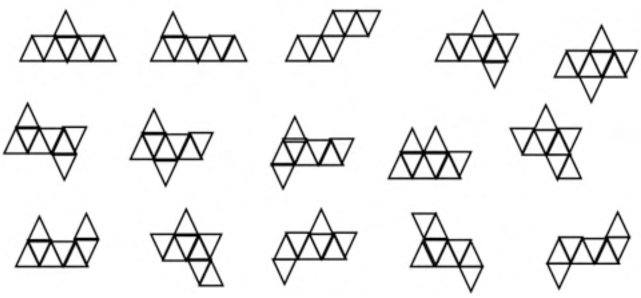

Figure 2: Possible octahedron flattening molds

1)Note that the flat patterns are missing the tabs to be able to paste and assemble the octahedron. Identify where they should be drawn so the solid can be closed.

2)Then, reproduce these figures, possible plans of an octahedron, on cardboard, in the size you want. But don't forget to add the tabs so you can glue and close the octahedrons later.

3)Cut them out, then fold them along the lines, apply glue to the flaps to close them. Your octahedron is ready!

Or was the figure you chose, possible planning, not enough to assemble the octahedron?

Reply:

a)Are all figures flats of the octahedron?

b)Did your mold form an octahedron?

c)Check if your colleagues' mold also formed an octahedron.

d)Is there any way to draw the eight triangles so that it's not the flattening of an octahedron? Check and explain your answer.

Task 5: Complete the table and compare the results.

1)Note the faces of the polyhedrons . What kind of faces do these solids have?

2)Generally speaking, the name of geometric solids depends on the number of faces. Knowing that tetra refers to the number four, octo to the number eight, and icosa to the number twenty, complete the following table noting the solids you have assembled. Describe these three Plato's polyhedra, try to identify the similarities and differences between them, comparing the shape and number of faces, the number of edges and vertices. To help with your analysis, complete the table below:

Number of faces (F) triangular	solid name	Number of edges A)	Number of vertices (V)
4			
8			
20			

a)What do you observe when comparing the number of faces with the number of edges of each polyhedron?

b)What do you observe when comparing the number of faces with the number of vertices?

c)What can you say when comparing the number of edges and vertices?

In tasks one, two and three (1, 2 and 3), using the ruler and compass, students draw parallel lines, horizontal and inclined lines, line segments, circles and arcs, identify intersections between two or more segments of lines, determining common points and which become vertices of the polyhedron, as well as whether the intersection between two polygons will become the edge of the constructed polyhedron. In this way, this activity allows the mathematics teacher, in the classroom, to promote reflections on the action of drawing and enlarging a figure, establishing relations of measures, promoting visualization and imagination between what was drawn in the plan and what will become a three-dimensional object in space.

In task 4, students need to identify with which plane figures formed by the juxtaposition of eight triangles it is possible to form an octahedron. This is a way of stimulating visual perception, relating a graphic representation to a three-dimensional object or the images to each other.

In task 5, students need to count the elements (sides, vertices and edges) of polyhedra and compare their quantities, analyze regularity features, recognize that some properties of a polyhedron are independent of both physical characteristics (color, size, texture and , thickness) and the number of its elements.

III SKELETONS OF PLATO'S POLYHEDRA WITH TRIANGULAR FACES

In this sequence of tasks, the student will be invited to assemble the polyhedra (tetrahedron, octahedron and the icosahedron), following the assembly scheme of the structures (skeletons) using colored straws and string.

Before starting the tasks, the teacher can invite the students to reflect on the conditions of existence for the assembly of solids so that they are regular. That is, discuss the length of each of the straws and the length of the string to be cut depending on the size of the straws.

Em todos os esquemas que seguem, indicamos por → o sentido em que a linha deve ser inserida num canudo vazio e por ⇒ the sense in which it should be inserted into a straw already occupied by some piece of thread.

Task 1: Assembling the tetrahedron skeleton with string and straw

a) Follow the tetrahedron step-by-step as shown in the diagram, but first check how many straws are needed to assemble it, and what line size should be for you to assemble it? Note: This measurement does not have to be exact. How did you manage to find the thread measurement?

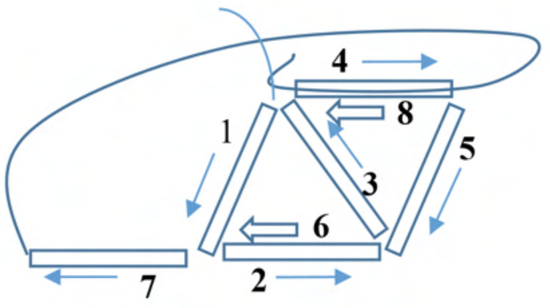

Figure : Scheme for assembling the tetrahedron skeleton

Source: Extracted and adapted from KALEFF, 1998, p. 134.

b) Can we say that each straw represents an edge of the tetrahedron?

If so, how many edges does the tetrahedron have?

c) Looking at the tetrahedral skeleton, how many hollow faces does it have?

Task 2: Assembling the octahedron skeleton with thread and straw

Now, we are going to assemble an octahedron. Take one of the octahedrons you built using the flat pattern .

How many edges does the octahedron have?

How many straws will it take to assemble your skeleton?

To assemble the octahedron, you need:

1)Assemble four triangles with the straws.

2)Join the triangles two at a time.

3)After that, follow the diagram below to continue assembling the octahedron.

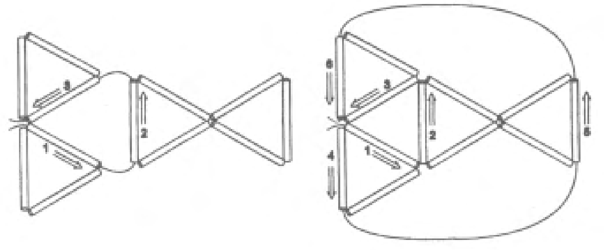

Figure 3: Scheme for assembling the skeleton of the octahedron

Source: Extracted from KALEFF, 1998, p. 134.

Task 3: Assembling the skeleton of the icosahedron

For this task, instead of using a straw and string , we will use styrofoam balls or modeling clay or clay and barbecue sticks to assemble the icosahedron. For this, we suggest that you take the icosahedron made of paper and represent the edges by the sticks and the vertices by the balls. So, how many barbecue sticks are needed and how many styrofoam balls?

In these tasks, the visualization of the edges and vertices of the polyhedron s is prioritized. The visualization of all edges can contribute to the understanding of the conventions of the planar representation of polyhedra, in perspective, in which all edges are represented and edges that are hidden are represented by dotted lines. In addition, students realize that triangular structures are rigid structures.

MAKING FOLDS TO ASSEMBLE THE POLYHEDRONS WITH TRIANGULAR FACES

Origami is the traditional Japanese art of folding paper, creating representations of some beings and objects using geometric folds of a piece of paper, without cutting or gluing it.

According to scholars, the origin of Origami is as old as the origin of paper, which appeared in China in 105 BC to replace the silk that was used for writing. In the Chinese empire , this technique became a secret and was kept for a long time. It only arrived in Japan in the 6th century through Chinese Buddhist monks, but only the nobility had access, as it was considered a luxury item, used in kimonos patterns and in religious festivals (Shino).

According to Rancan and Giraffa (2012),

> Three-dimensional Origami, also known as Structural Origami, develops the virtual and three-dimensional perception of objects that are built, generally based on fitted pieces (modules). They can be investigated through new methodologies and discoveries of relationships between solids, characteristics of each figure and visualization of geometric concepts (RANCAM and GIRAFFA, 2012, p. 3).

The activities involving folding have a dynamic that values the construction using the hands, the discovery, the visualization. Geometric exploration using origami is quite enriching, as it allows the use of basic concepts related to angles, parallelism, symmetries, similarity of figures, as well as notions of proportionality that are evidenced in practice. During the process of building the folds, notions of form and space are developed.

The proposal for the elaboration of solids using origami begins with the analysis of pre-existing solids (Task 1), to then build the connectors and triangular faces (Tasks 2 and 3), respectively.

Task 1: Completing the table

You already know that the tetrahedron, octahedron, and icosahedron are polyhedra whose faces are triangular. For these solids it is necessary to make two types of parts: triangular modules and connectors.

The triangular modules will be faces of the polyhedrons and the connectors, as the name implies, make the connection, that is, they join the faces to form the polyhedron. So each connector works as if it were the edge, right?

Complete the table to find out how many pieces must be made to assemble these solids.

polyhedron name	Number of faces (F)	Number of connectors (A edges)
Tetrahedron		
Octahedron		
icosahedron		
total parts		

Now that you know how many triangular modules and how many connectors you will need, get to work!

To build polyhedrons we have to learn how to make connecting pieces and faces. As they are different, let's separate them by parts.

Task 2: Making the connectors

The plug-in module is made from square paper, the size of those used to make triangular modules and polyhedron faces. The fitting module works as if it were the edge of the polyhedron because it joins two triangular faces, in this case.

- Step 1: With a square of the same size as the one used in the triangular module, side measuring 10 cm, divide it into four equal parts and cut them out. Once cut, you have four smaller squares that, following Steps 2 to 4, represented in Figures 1 to 6 of Table 2, will form the plug-in modules.
- Step 2: Take one of the pieces. Fold it in half and then unfold it. You will see a crease (Table 2 – Figure 1), dividing the square into two equal parts. Then fold it in half in the other direction (Table 2 – Figure 2), and unfold it again. The square will be divided into four equal parts.
- Step 3: Make a valley fold taking the four vertices of the square to the center (Table 2 – Figures 3 and 4).
- Step 4: Turn the obtained square (Table 2 – Figure 5) over and fold it in half (Table 2 – Figure 6). And the plug-in module is ready.

International Journal of Human Sciences Research ISSN 2764-0558 DOI 10.22533/at.ed.5582222219074

Analyzing the bends of the connectors:

a)The marks obtained by the folds made in the two directions of the square represent two line segments that meet at a point, and determine four equal parts. What can you say about the position between the two segments? And what about the angle formed between them?

b)Compare the sizes of the square in Figure 1 of Table 2 with the square in Figure 4 of Table 2. What can you say? Explain.

Task 3: Making the triangular faces.

In this task, you will see the step-by-step to make the triangular module.

- Step 1: Considering a square of vertices ABCD, measuring 10 cm, make a fold so that *AD* is on *BC* , determining the bisector (Table 3 – Figures 1 and 2). unfold.
- Step 2: Keeping point *A* fixed, make a bend so that vertex *B* is on the bisector.
- Step 3: Unfold. Let *E be* the end of this last fold. Then bend the angle bisector DÂE.
- Step 4: Make a fold taking point E to the first fold, that is, to the bisector, thus forming an equilateral triangle.
- Step 5: Fold as shown.
- Step 6: Fold bringing vertex B to the indicated point. Also fold the left corner flap.
- Step 7: Fold by placing vertex *A* inside the flap.

At the end of the scheme, with the steps performed, we obtain the triangular module and the faces of the polyhedra. Then, the procedure is repeated until obtaining the amount necessary to assemble each of the polyhedrons.

Note in Figure 4 that the triangle obtained contains a pocket on each of the three sides. The fitting modules will be placed on them (Figure 5), which will unite the faces of the Polyhedron.

The sequence of photos presented below were taken by a licensee as one of the techniques used to record the activities carried out in the classroom. Her exquisite care in this type of record and her interest in deepening her knowledge led her to prepare a text (monograph), in which she developed a series of tasks on Plato's polyhedra, among them tasks involving the use of origami.

Follow the octahedron assembly script by looking at the step-by-step photos in the table.

Finally, we present the assembly of the icosahedron, for which 30 triangular modules and 60 connectors will be needed. In Table 8, we show the assembly of five of these modules whose process must be executed twice.

To give it more firmness, if you prefer, you can glue the connectors so that the triangles are very firm.

And in Figure 1 of Table 9, the remaining ten triangular modules are connected using the connectors to join them together to form a strip.

Then, join the ends of the band (Table 9 – Figure 1) with a connector forming a circular band. Once this is done, fit the two previous pieces, one on each side of the strip. And the icosahedron is ready (Table 9 – Figure 2).

A variation on the proposal made earlier can be found by assembling the tetrahedron on a single square sheet of paper. To carry out this task, the teacher can suggest that students watch videos on Youtube [1].

In task 1, the analysis of solids of triangular faces was proposed to identify the number of faces and edges. The idea is to propose a forecast of the material needed to make the connectors and triangular faces.

In task 2, priority was given to making the connectors and reflecting on the folds, associating them with the relative positions of

1 The video available at < https://www.youtube.com/watch?v=n7kC5corx0Y >

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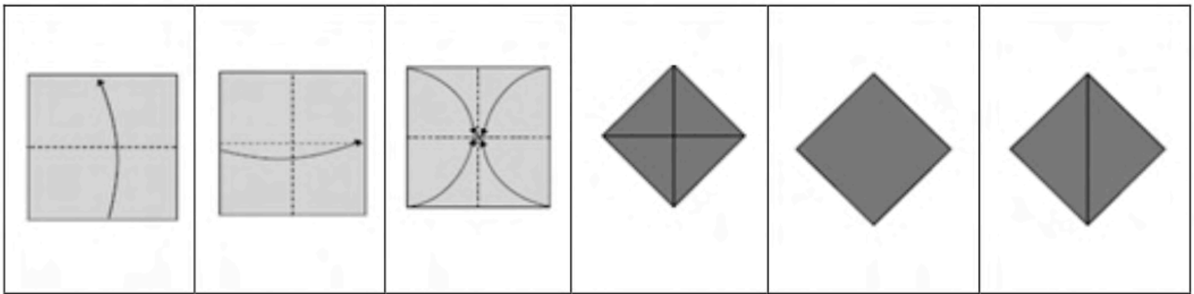


Figures 1 and Figure 2	Figure 3 and Figure 4	Figure 5 and Figure 6

Table 2: Roadmap for preparing the connectors

Source: Lucas, 2013, p. 48.

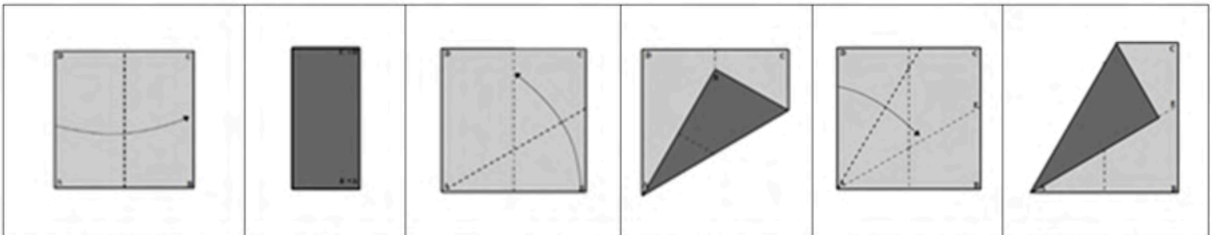
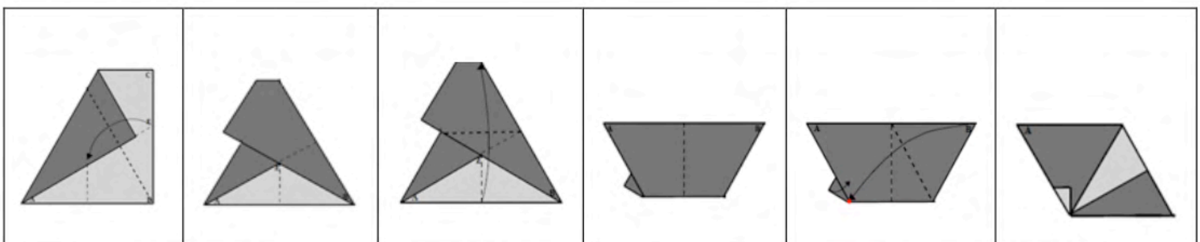


Figure 1 and Figure 2 Step 1	Figure 3 and Figure 4 step 2	Figure 5 and Figure 6 step 3

Table 3: Steps 1 to 3 of the Roadmap for triangular face.

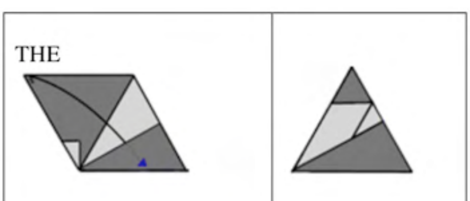
Source: Lucas, 2013, p. 44-45.



Figures 7 and 8. step 4	Figures 9 and 10 step 5	Figures 11 and 12 step 6

Table 4: Steps 4 to 6 of the Roadmap for Triangular Face.

Source: Source: Lucas, 2013, p. 45-46.



Figures 13 and 14. Step 7

Table 5: Step 7 of the Roadmap for Triangular Face

Source: Source: Lucas, 2013, p. 46.

International Journal of Human Sciences Research ISSN 2764-0558 DOI 10.22533/at.ed.5582222219074

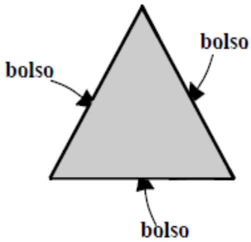


Figure 15

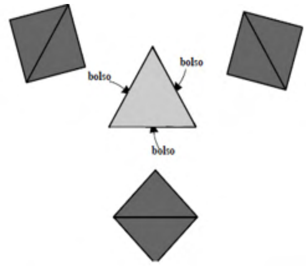


Figure 16

Source: Lucas, 2013, p. 47.

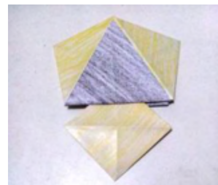
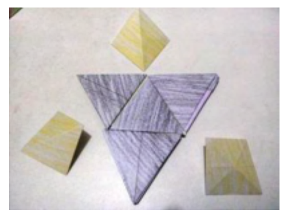
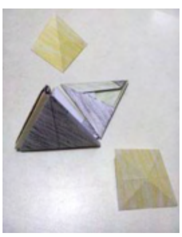
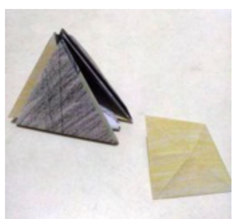
		
Step 1	step 2	step 3
		
step 4	step 5	step 6
		
Step 7	step 8	Step 9

Table 6: Assembly of the tetrahedron with triangular faces and connectors (Continued).

Photos: Barbosa, 2015.

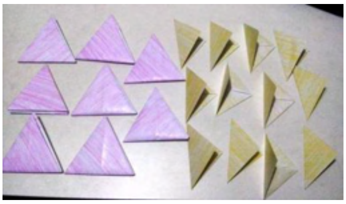
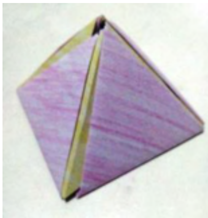
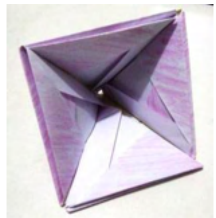
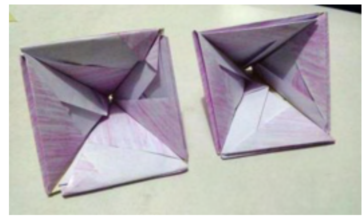
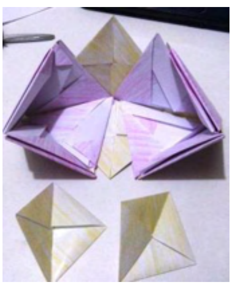
		
Step 1	step 2	step 3
		
step 4	step 5	step 6
With 4 triangular faces assemble a pyramid without a base as described in the previous steps. repeat the process to make another pyramid that will be connected as explained in the next steps.		
		
Step 7	step 8	Step 9

Table 7: Assembly of the octahedron.

Photos: Barbosa, 2015.

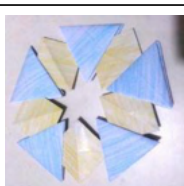
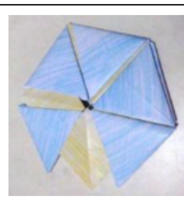
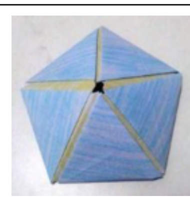
		
Step 1	step 2	step 3

Table 8: Assembly of five triangular faces

Source: Barbosa, 2015 .

Figure 1	Figure 2
	

Table 9: Assembly of the side faces and the assembled icosahedron .

Source: Barbosa, 2015.

straight lines. This step is important so that a connection is established between doing and mathematical thinking. While in task 3, the focus was on assembling the solids.

In these tasks, the visualization of the faces and edges represented by the connectors is prioritized. The connectors show the condition of definition of polyhedra, in which at the intersection of two faces, we have an edge. In addition, the experience of inserting folding (origami) as an alternative in geometry classes, provides opportunities for learning geometric concepts and interaction between students, while they help each other to assemble the solids, a task not always easy for all students. , as it requires visualization and motor control.

FINAL CONSIDERATIONS

This proposal was based on the presentation of tasks that stimulate exploration and reflection on action. Thus, although the proposal does not intend to exhaust the theme and expose conclusive aspects, we emphasize that the considerations are intended to, in addition to proposing didactic situations supported by the visualization and analysis of the folds, contribute to the structuring, both of other works involving the other solids, as well as in the approach of other geometric contents with the use of assembling the plans, assembling the skeletons, origami and technological tools (rulers and compasses), leaving the teacher, as far as possible, to insert other resources, such as GeoGebra, for example.

The elaboration of the tasks developed in this work aimed to systematize a work already carried out at different times as an active teacher at different levels of education, and to propose a way of approach in which both prioritize the doing and the analysis of what was done, differently from of the proposals published in textbooks, where it is possible to verify, through an analysis of them, a predominantly algebraic and algorithmic treatment in the treatment of the theme. We also understand that such aspects promote considerable distance in the way these concepts were developed over time, considering that aspects inherent to Euler's relation and the characterization of elements, faces, sides, edges, angles, and polyhedral angles , have been defined. from the analysis of concrete objects that can be physically manipulated.

In this way, we believe that the proposal of different tasks can instigate students to develop autonomy in their knowledge construction process, provided by the exploratory experience, in line with the change in the teacher's posture in the classroom, contributing to learning. effectiveness of the concepts involved.

We believe that such improvements in the teaching process are possible thanks to the shift in the focus of attention in the classroom, which shifts from the content itself to the teacher's pedagogical practice, which is characterized by allowing the student to experience the experience of mathematical investigation, and for demanding that the teacher place himself as a mediator in the process. As a result, they can listen to what students have to say, observe the progress of their work, motivating them and intervening when necessary.

The proposal included tasks in which we articulate actions of construction, assembly, visualization and manipulations in order to obtain the understanding of concepts related to the differentiation of elements of plane and spatial geometric figures and the adaptations carried out to make them more investigative were provided by the methodology adopted and that values an active and autonomous attitude of the students in their learning process, hence the detailing of the step by step to be followed and the care in presenting the tasks with a language aimed at stimulating the doing and thus making the learning most significant.

We hope that teachers interested in working with this proposal can find ways to adapt it to their reality, although it is not easy to introduce new teaching methods and strategies. However, those who are apprehensive may find an alternative in this work.

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