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GENERATION LOSSES DUE TO THE TEMPORAL FACTOR INFLUENCE IN LOAD LOSS: A STUDY CASE OF SHP REPI

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Abstract: In Brazil, hydroelectricity generation is responsible for 63.50% of the energy supply, which corresponds to 397,877 GWh.year^{-1}. Being such a representative source to the electrical system, small percentage losses in the generation process may represent significant values. An increase of 1% in the intake system losses of the power plants results a reduction of 3,979 GWh.year^{-1}. This value corresponds to the annual consumption of the metropolitan region of Ribeirão Preto, São Paulo. Over the years, the hydraulic system of the power plants increases its roughness, which has a direct impact on the system load loss. This work aimed to estimate the generation losses due to the increase in roughness of the REPI Small Hydroelectric Power Plant hydraulic system, in Wenceslau Braz, MG. The study started from the methodological proposal of Lima (2018), focusing on the critical period of the National Electric Sector (dry season), and concluded an increase in the load loss that ranged between 3% and 15%. The hydraulic system under study is complex and presents an intrinsic head loss of 14.06 m (13,7%), and could reach 29.86 m (29,2%) in the 32-year (1998 to 2020) time span. The energy analysis shows a reduction of 7.8% in the generation capacity in the same time frame, which significantly impacts the power plant's cash flow.
Keywords: Load loss, Roughness, Generation loss, Hydroelectric power plant, SHP.

INTRODUCTION

In Brazil, the hydroelectric generation matrix is responsible for 62.21% of the entire national installed capacity. Despite environmental restrictions and the continuous crisis that the country has been going through since the mid-2010s, hydraulic generation grew by 31.93% in this period (ANEEL, 2021; EPE, 2020). In 2019, 397,877 GWh.year-1 of hydroelectricity was made available to the market, corresponding to 63.50% of the entire national electricity supply. The Source hydroelectric plant is divided into i) 103.39 GW of installed capacity in large plants (above 30MW); ii) 7.14 GW of capacity in small plants (SHP up to 30 MW) and; iii) 0.85 GW of capacity in mini plants (CGH up to 5MW) (ANEEL, 2021; EPE, 2020). The dimension of hydroelectric generation in Brazil shows that even small percentage losses in the system can represent significant values. Thus, a 1% increase in loss in the UHEs' adduction system means a reduction of 3,979 GWh.year-1. This value can be compared to the annual consumption of the Ribeirão Preto region in the interior of the State of São Paulo, which aggregates more than 34 cities and a population of 1,738,000 inhabitants (IBGE, 2021). The process of converting hydraulic potential energy into electrical energy contains systemic losses. A hydroelectric plant operating at its optimal operating point is capable of using up to 90% of all available gross energy (Doland, 1954; Eletrobras, 2007; Encinas, 1975; Mataix, 2009). These losses are due to the efficiency of the electric generator (95%), efficiency of the hydraulic turbine (95 to 97%) and around 5% of pressure drop in the water adduction/conduction system. The yields of the HPP systems and equipment have an increase in losses throughout their useful life and must be continuously monitored. With regard to hydraulic losses, they evolve throughout life. According to Fox, Pritchard and McDonald (2010), hydraulic head losses can be subdivided into larger losses, due to the flow fully developed and distributed along the adduction circuit, and smaller losses, due to localized losses such as inlets, accessories, variations. of areas and others. In the Eletrobras Hydroelectric Power Plants Manuals (2000) and (2007), hydraulic losses are classified as due to adduction, gratings, channels, tunnels and losses in forced piping (entrance, bends, reductions and bifurcations). These losses can suffer a considerable increase with time evolution, causing a decrease in the global income and consequently a loss of generation capacity.

MATERIALS AND METHODS

The head loss consists of the reduction of the dynamic energy of the fluid due to the friction of the particles of the same among themselves and/or against the walls of the pipe, and it depends on the diameter (D), the length of the duct (L), the roughness of the wall (ε), and also of the fluid properties, such as the specific mass (ρ), the viscosity (μ) and the flow velocity (V) (White, 1962). Equation 1, known as the Universal Pressure Loss Formula, was proposed by Darcy and Weisbach in 1845 (Weisbach, 1845).

$$hf = f\frac{L}{D}\frac{V^2}{2g} \qquad (1)$$

Where: hf is the head loss along the length of the pipe (m), f is the head loss factor, L is the length of the pipe (m), V is the flow velocity (m/s), D is the inner diameter of the pipe (m) and g the acceleration due to gravity (m/s²).

In 1933, the concept of relative roughness was established, characterized between absolute roughness and pipe diameter (ε/D). This relationship was obtained through experiments carried out by Nikuradse (1933) (apud Porto, 2006). In 1939, Colebrook-White established the equation for determining the friction factor (f), given by Equation 2 (Porto, 2006).

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3{,}7D} + \frac{2{,}51}{Re\sqrt{f}}\right) \qquad (2)$$

Where: f is the head loss factor, k is the equivalent roughness of the pipe wall (m), D is the inner diameter of the pipe (m), and Re is the Reynolds number.

According to Azevedo Netto (1998), after several attempts, Hazen and Williams, in 1903, arrived at a practical equation to determine the head loss, described below by Equation 3.

$$J = 10{,}65\frac{Q^{1{,}85}}{C^{1{,}85}D^{4{,}87}} \qquad (3)$$

Where: Q is the flow rate (m3/s), C is the head loss coefficient, D is the internal diameter (m), and J is the unit head loss (m/m).

Several studies were carried out focusing on the analysis and calculation of the friction factor for the solution of the Darcy-Weisbach Equation. Andrade and Carvalho (2001) studied the behavior of the Swamee-Jain equation in the most varied situations of water conduction in pressurized systems, proposing a correction factor. Rao and Kumar (2006) also analyzed solution equations and proposed a new equation. McKeon, Zagarola and Smits (2005) analyzed the equations at the Superpiper Laboratory at Princeton University in the USA, whose purpose is to test experiments with very high Reynolds numbers (38x106), proposing adjustments to the equations for these high Reynolds values. Recent studies still try to improve the accuracy of factor determination, as in Zanoun et al. (2021), who developed their methodology for the Prandtl-von Kármán relationship, or as Kadivar, Tormey and McGranaghan, (2021) who present a detailed review of several studies in the area and their contributions in various equations for determining the friction factor.

In order to evaluate the head loss distributed in water conducting systems, it is essential to know the values of C (Hazen–Williams equation) and of f (Darcy-Weisbach equation), which represent the coefficients that allow estimating the influence of tube surface over the flow (Lima and Martinez, 2014). However, these coefficients may vary as a function of time due to the action of aging of the material, caused by corrosion and/or deposition of material inside the pipes throughout their operation (Lima, 2018). The aging effect of the pipes and the adduction system represents a significant reduction factor in the net height available in a hydroelectric plant. This effect tends to be greater in small diameter pipes, such as those used in Small Hydroelectric Power Plants (SHPs) (Leite, 2020). In fact, some works show that the effect of aging of pipes must be considered in the hydraulic path calculations. Azevedo Netto (1998) shows that there is a significant increase in the Hazen-Willians "C" coefficient for pipes with several years of use. Doland (1954) also cites a way to correct the effect of age in pipes, multiplying the experimental coefficient "Ka" of Scobey's equation by a logarithmic correction coefficient. More recently, Lima (2018) presented an equation (Equation 4) with which it is possible to estimate the evolution of surface roughness as a function of the age of cast iron and carbon steel pipes.

$$e_{est} = \alpha \cdot t_{op}^2 + \beta \cdot t_{op} + \theta \qquad (4)$$

And e_{est} id the thickness of the surface roughness (mm), for pipes with diameters varying between 250 and 1.500 mm, t_{op} ,the pipe operating time in years, and α, β, and θ coefficients calculated by Equations 5, 6 and 7 below.

$$\alpha = 0{,}0006\, ln(D_{tub}) - 0{,}0006 \qquad (5)$$
$$\beta = 0{,}0076 ln(D_{tub}) + 0{,}0355 \qquad (6)$$
$$\theta = -(0{,}0612\, ln(D_{tub}) - 0{,}1264) \qquad (7)$$

The head loss coefficient "f" mentioned above can be obtained through several equations and are presented in the literature, such as Buzzelli (2008), Eck (1941), Serghides (1984), Vatankhah, Kouchakzadeh (2008), Zigrang, Sylvester (1982) or Papaevangelou, Evangelides and Tzimopoulos (2010).

The localized head losses were obtained through Equations 8 to 12, presented in Table 1.

Adduction channel
$h_c = \sqrt{\frac{n * Q}{A_a * (RH_c)^{2/3}} * L_c}$
(8)

Grid
$h_g = \left(\frac{e_1}{e_2}\right)^{\frac{4}{3}} . sen(\theta_1) . \frac{v^2}{2g}$
(9)

Entry
$\square_e = k . \frac{v^2}{2g}$
(10)

Reductions
$h_r = k . \frac{v_2^2 - v_1^2}{2g}$
(11)

Located
$h_L = k . \frac{v^2}{2g}$
(12)

Table 1 – Localized head loss equations.

Where: n is the Manning coefficient, Q

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is the adduction flow, Aa is the adduction flow, RH is the hydraulic radius, L is the length of the adduction channel, e1 thickness of the bars, e2 spacing between the bars, θ the slope of the grid, v a flow velocity, g the acceleration due to gravity, k the head loss coefficient and $h_{(c,g,e,r,L)}$ the pressure losses in the channel, grid, inlet, reductions and located, respectively.

The installed power of a hydroelectric plant (Pinst) is directly proportional to the specific weight of the water (γ), the design flow (Q), the existing unevenness at the site (H), the acceleration of gravity (g) and the global yield. of the installation (η), the product of these quantities results in the gross power of the installation (Doland, 1954). Being obtained by Equation 13.

$$P_{inst} = \gamma \cdot g \cdot Q \cdot H \cdot \eta \quad [W] \tag{13}$$

Firm Energy (FE) is a concept that was introduced at the end of the 19th century, during studies of reservoirs that supplied the water systems of cities. At that time, the concept of firm flow was introduced, which would make it possible to supply the site even in times of severe drought recorded at the determined time of the study (Kelman, Kelman and Pereira, 2004). By definition, Firm Energy corresponds to the maximum production that a plant can provide, considering the driest period recorded in the flow history of the river where it is located without the occurrence of deficits, considering the entire historical record of inflows (Hicks et al, 1974 apud Oliveira et al., 2009). According to Kelman, Kelman and Pereira (2004), the critical period recommended for Brazil must follow the one presented in Table 2.

Setting	Tolerance %	Critical period
Long term	1.5	June/1948 to November/1956
Mid-term	1.5	May/1949 to November/1956
Short term	1.5	May/1951 to November/1955

Table 2 – Critical Period of the Brazilian Electric Sector.

Source: (Kelman, Kelman and Pereira, 2004).

The firm energy available in the system will depend on the swallowing flow (Qe) of the difference in level existing at the location (H), discounting the head losses, which results in the net height "HLiq", of the efficiency of the hydro-electro-mechanical equipment (η), the acceleration of gravity (g) and the period considered (h year-1) (Kelman, Kelman and Pereira, 2004). Resulting in Equation 14.

$$E_{firme} = \left(P_{inst} = g\ Q\ H_{liq}\ \eta\right) \cdot T_{op.med.} \tag{14}$$

CASE STUDY – SHP REPI

SHP REPI is located in the municipality of Wenceslau Brás, in the south of Minas Gerais, at geographic coordinates at 22°32'8.673"S, 45°21'44.25"W. The plant started its operation on December 8, 1932 and belongs to the Ministry of Defence. Having been designed to meet the energy demand of gunpowder and war materials factories in the Serra da Mantiqueira region (Itajubá, MG and Piquete, SP) (IMBEL, 2021). Currently, the SHP has an installed capacity of 3.34 MW (ANEEL, 2021), divided between two engine rooms, called auxiliary plant and main plant (FUPAI, 2009). According to Ricardo (2005), the auxiliary plant is the result of an expansion of the plant in the 1940s, when the energy demand in the manufacturing units increased, due to the occurrence of the 2nd World War. At the time of this expansion, a dam was built, upstream of the original dam, with a reservoir for daily regularization, however, its current operation is run-of-river.

CHARACTERIZATION OF THE REPI SHP GENERATION CIRCUIT

The auxiliary plant has a gross drop of 29.15 m, a 700 kVA generator set and a horizontal axis Francis turbine. The main plant has a gross head of 102.34 m and five generator sets, three of which are 875 kVA and two of 450 kVA, all with horizontal axis Francis turbines. The hydraulic circuit consists of two segments. The first, referring to the auxiliary plant and the upstream reservoir, has two forced mild steel pipes of 0.9m in diameter and 6.4m in length, which are joined in a single pipe of 1.2m in diameter and 44.19m of lenght. The second segment, referring to the main plant and the downstream reservoir, is more complex, being divided into five sections: i) 2 conduits of 1.2m in diameter and 6m in length; ii) 2 ducts of 1.1m in diameter and 535.32m in length; iii) 2 ducts of 1.0m in diameter and 528m in length; iv) 2 ducts measuring 0.9m in diameter and 538.1m in length; and, v) 1 conduit measuring 1.1m in diameter and 3.45m in length. All in welded mild steel. The entire hydraulic circuit underwent inspection and maintenance in 1988 (FUPAI, 2009; Ricardo, 2005). The generation circuit is shown in figures 1 and 2 below.

The characteristics of the generator sets and the set of penstocks of the two plants are shown in Table 3 below.

DETERMINATION OF LOAD LOSS

In order to estimate the head loss due to the aging of the pipelines, the head loss was calculated, section by section, for year zero (start of the SHP after intervention in 1988) and for year 32 (2020). Starting from the average monthly flows of the critical period, the flow velocity in the hydraulic circuit, the Reynolds number, the friction factor "f" and the head loss were calculated by the Universal Equation of Darcy-Weisbach head loss, resulting in the obtaining the net drop height. To calculate the head loss in year 32, the equation proposed by Lima (2018), equations 4 to 7, was used to determine the temporal roughness. Through the temporal roughness, the height of net fall was obtained for the 32 years of operation. From this calculation, the generation loss was accounted for due to the increase in load loss against the operating condition in the critical period, determining the firm energy which is presented in Table 4.

DETERMINATION OF ENERGY GENERATED

The energy analysis was performed monthly, considering the long critical period (Table 2), from June 1948 to November 1956. For that, the power was calculated by equation 13, using the net fall height determined in item 3.2, both for the starting condition (year 0) and for the temporal evolution (year 32). The generated energy was calculated by equation 14, being presented in a grouped form by Semester in Table 4 (2nd Semester from 1948 to 1st Semester from 1956).

RESULTS

The generation loss calculated monthly was grouped every six months and is presented in Table 4 below, which can be better visualized in figure 3. The addition of energy losses for each generation level of the REPI SHP after 32 years of operation and with the pipe having suffered The roughness increase process can be seen in figure 4. The reduction in generation using the critical period as a benchmark was 7,835 MWh over a period of 8 years, which represents an average reduction of approximately 7.8%.

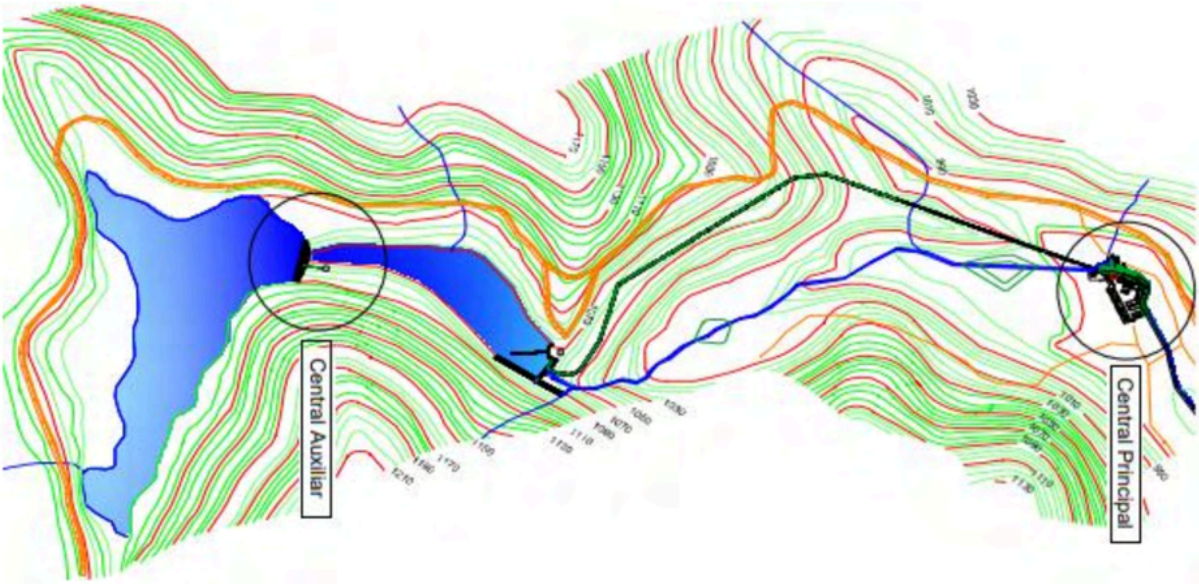

Figure 1 - Plan view of SHP REPI.

Source: (FUPAI, 2009)"title":"Projeto Básico Pequena Central Hidrelétrica REPI"","type":"report"},"uris":["http://www.mendeley.com/documents/?uuid=f4779398-e4ed-4019-aff1-ef0a7ee055c9"]}],"mendeley":{"formattedCitation":"(FUPAI, 2009.

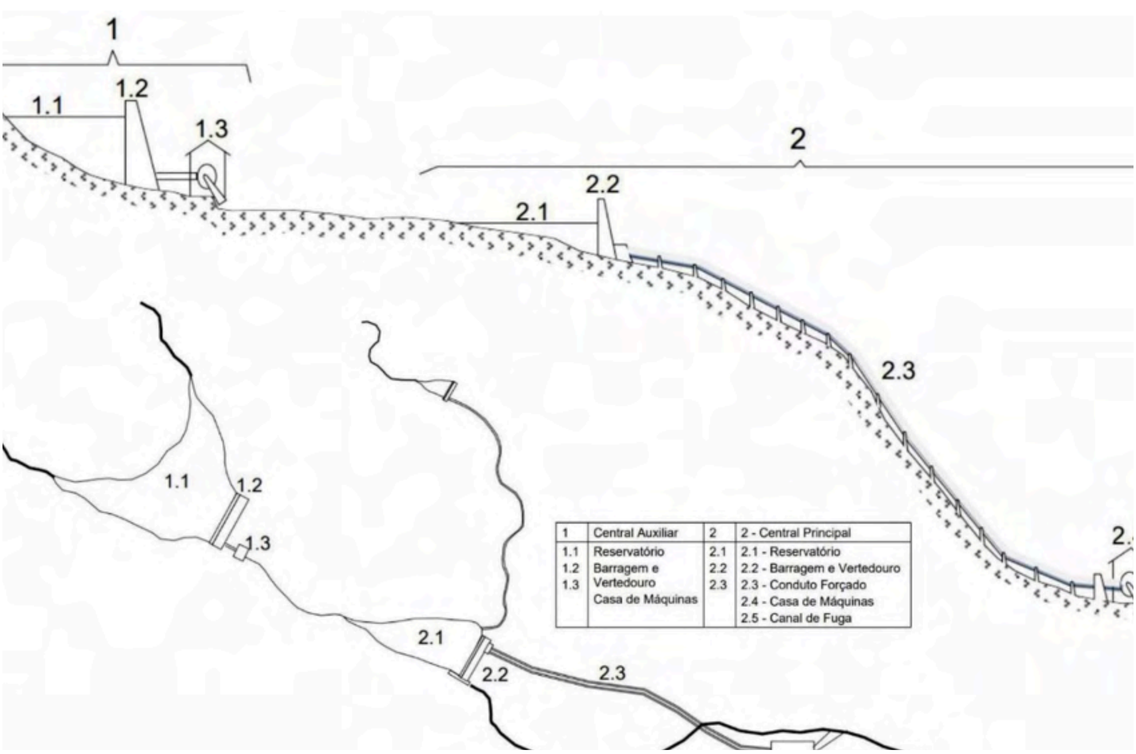

Figure 2 - Scheme of SHP REPI.

Source: (FUPAI, 2009)"title":"Projeto Básico Pequena Central Hidrelétrica REPI"","type":"report"},"uris":["http://www.mendeley.com/documents/?uuid=f4779398-e4ed-4019-aff1-ef0a7ee055c9"]}],"mendeley":{"formattedCitation":"(FUPAI, 2009.

	Main central					Central Auxiliar	
	Group 1	Group 2	Group 3	Group 4	Group 5	Group 7	
Operating time [years]	88					78	
Unit power [kW]	700	700	360	360	700	560	
	Adduction circuit sections						
	Main central					Auxiliary Central	
	1	2	3	4	5	1	2
Diameter [mm]	1.200	1.100	1.000	900	1.100	900	1.200
Length [m]	6,0	535,3	528,0	538,1	8,4	6,4	44,2
Number of duct lines	2	2	2	2	1	2	1
Gross fall [m]	102,34					29,15	
Turbine	Francis horizontal axis						
Turbine efficiency [%]	$\eta_{Francis} = 5,7.10^{-5} (\%Q)^3 - 0.01877 (\%Q)^2 + 1.84128 (\%Q)$						
Generating income [%]	95%						
Unit power [kVA]	875	875	450	450	875	700	
Maintenance time [years]	32 years						
Individual swallowing flow [m³/s]	0,83	0,83	0,43	0,43	0,83	2,37	
Design flow [m³/s]	3,34					2,37	
Load loss per year:0 [m]	Up to 14,06m or 13,73%, for the flow of					0,40 m	
Load loss per year:32 [m]	Up to 29,86m or 29,17%, for the flow of					0,59 m	

Table 3 – Characteristics of penstocks and generator sets of the SHP REPI complex.

Source: (FUPAI, 2009; Ricardo, 2005)"title":"Projeto Básico Pequena Central Hidrelétrica REPI","type":"report"},"uris":["http://www.mendeley.com/documents/?uuid=f4779398-e4ed-4019-aff1-ef0a7ee055c9"]}],"mendeley":{"formattedCitation":"(FUPAI, 2009; RICARDO, 2005.

Period	Energy, year 0 [kWh]	Energy, year 32 [kWh]
2ºSemester 1948	4.131.354,99	4.024.176,75
1ºSemester 1949	7.259.202,83	6.483.811,93
2ºSemester 1949	4.596.507,02	4.447.186,69
1ºSemester 1950	9.363.477,15	7.998.551,73
2ºSemester 1950	6.251.682,57	5.786.425,27
1ºSemester 1951	9.383.895,72	7.992.064,06
2ºSemester 1951	5.349.202,20	5.152.439,25
1ºSemester 1952	9.236.421,52	8.041.699,07
2ºSemester 1952	5.084.206,72	4.943.333,62
1ºSemester 1953	6.139.087,49	5.863.325,92
2ºSemester 1953	4.536.324,69	4.421.582,77
1ºSemester 1954	7.084.711,87	6.619.828,84
2ºSemester 1954	4.438.677,69	4.340.644,25
1ºSemester 1955	6.691.463,19	6.155.949,19
2ºSemester 1955	4.851.783,74	4.621.393,04
1ºSemester 1956	6.240.229,61	5.910.855,46
Total	**100.638.229,01**	**92.803.267,87**

Table 4 – Loss of generation due to increased roughness.

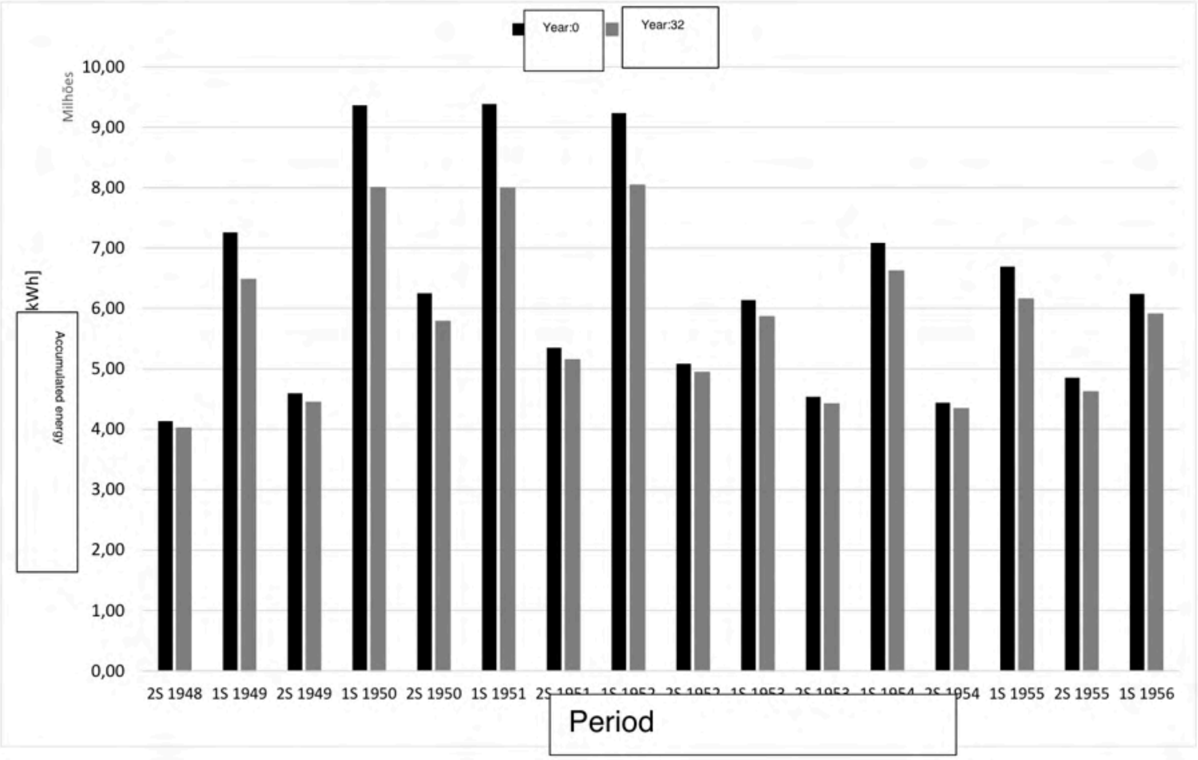

Figure 3 - Energy loss due to increased roughness.

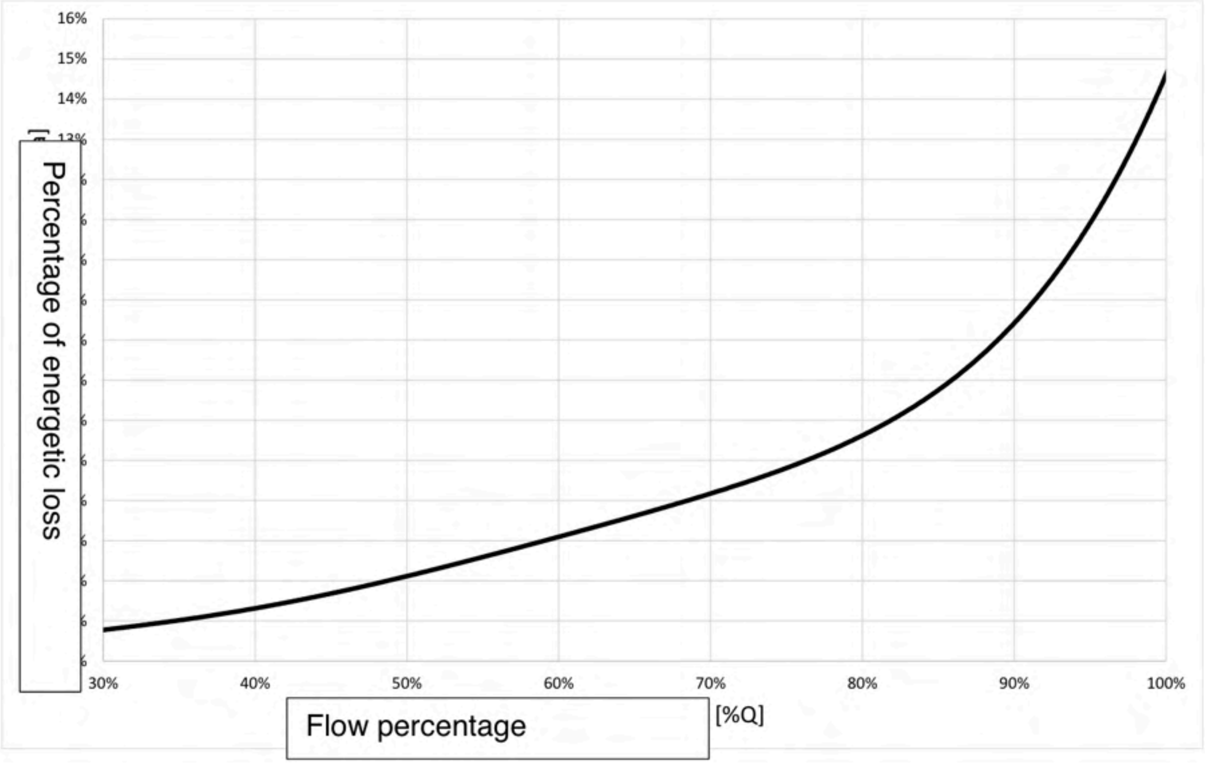

Figure 4 - Energy loss curve.

CONCLUSIONS AND RECOMMENDATIONS

The pressure drop is dynamic, being a function dependent on the instantaneous flow and the relative roughness of the internal surface of the pipe. For the critical period of the electrical sector, the study showed a greater variation in energy losses due to the temporal evolution of roughness, which ranged between 3% and 15%. The hydraulic circuit of the study site is extensive, which adds greater losses distributed to the system. A total head loss intrinsic to the system was calculated, considering it in year zero, of 14.06 m. When analyzed to the temporal evolution and applying the equations proposed by Lima (2018), with 32 years of operation (since its maintenance in 1988), the hydraulic circuit accounts for an internal loss due to the increase in roughness of 29.86 m, doubling the loss value in the system. This implies a 7.8% reduction in generation, which significantly impacts the facility's cash flow. It is recommended to carry out field campaigns in order to assess the adherence of the equations of Lima (2018), to point measurements of pressure drop in the generation circuit in order to validate these results.

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