v. 2, n. 1, 2022

Journal of
Engineering
Research

THE ORBITAL DYNAMICS OF ARTIFICIAL SATELLITES IN 14:1 RESONANCE

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Abstract: In this work, the orbital dynamics of synchronous satellites is studied. The considered resonance is 14:1; in other words, the satellite completes fourteen revolutions while the Earth completes one. In the development of the geopotential, the zonal harmonic J_{20} and the tesseral harmonic J_{1414} are considered. The order of the dynamical system is reduced through successive Mathieu transformations and the final system is solved by numerical integration. Numerical results show the phase space with the libration and circulation regions, the time behavior of the semi-major axis and the φ_2 angle.
Keywords: Synchronous Satellites, Orbital Dynamics, Resonance.

INTRODUCTION

Recent applications of artificial satellites require the description of the orbital motion under a precision of centimeters. In order to attain such actual expected level, several perturbations must be considered simultaneously, as well as resonant effects. Also, for the analysis of the coupled effects it is mandatory the knowledge of each effect take separately.

The space between the Earth and the Moon has several artificial satellites and objects in some resonance. Synchronous satellites in circular or elliptical orbits have been used for navigation, communication and military missions. This fact justifies the great attention that has been given in literature to the study of resonant orbits characterizing the dynamics of these satellites (LANE, 1988; ELY and HOWELL, 1996; SAMPAIO et al., 2012a, 20212b, 2014; ROSSI, 2008).

These studies are done using orbital perturbations such as due to the geopotential, lunisolar perturbations, solar radiation pressure, tide effect, spin-orbit coupling, considering commensurability between the artificial satellites mean motion and the Earths rotational motion (SAMPAIO et al., 2012c). The influence of resonances in the orbital motion problem perturbed by the geopotential has been observed by several authors, among them are Lane (1988), Ely and Howell (1996), Sampaio (2013) and Neto (2006).

The objects orbiting the Earth are classified, basically, in Low Earth Orbit (LEO), Medium Earth Orbit (MEO) and Geostationary Orbit (GEO). Most of the objects are found in the LEO region, because this region has a big quantity of space debris (OSIADER and OSTDICK, 2009; SAMPAIO, 2014; SAMPAIO and SANTOS, 2021). The Fig. 1 shows the semi-major axis versus eccentricity of the catalogued objects, (SPACE TRACK, 2016), and the exacts resonances 1:1, 2:1 and 14:1, considering the commensurability between the frequencies of the artificial satellites and space debris mean motion with the Earth's rotation motion, this last resonance is studied in the present work.

The Fig. 1 shows a big quantity of objects, including artificial satellites and space debris, around the 1:1 and 2:1 resonance in the GEO and MEO regions, respectively. But, the increasing number of objects in the LEO region and the largest quantity of this objects in the 14:1 resonance compared with the other regions, motivated the study of this resonance in the present work.

In this paper, the commensurability between the frequencies of the artificial satellite mean motion with the Earth's rotation motion is studied. The considered resonance is 14:1; in other words, the satellite completes fourteen revolutions while the Earth carries one. In the development of the geopotential, the zonal harmonic J_{20} and the tesseral harmonic J_{1414} are considered. The order of the dynamical system is reduced through successive Mathieu transformations and the final system is solved by numerical integration. In the dynamical model, the critical angle $\varphi_{14146-1}$, associated to the tesseral harmonic J_{1414}, is studied. Numerical results show the phase space with the libration and circulation regions and the time behavior of the semi-major axis and φ_2 angle.

RESONANT HAMILTONIAN AND EQUATIONS OF MOTION

In this section, a simplified Hamiltonian describing the resonant problem is derived.

Consider Eq. (2.1) to the Earth gravitational potential written in classical orbital elements (OSORIO, 1973; KAULA, 1966)

$$V = \frac{\mu}{2a} + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q=-\infty}^{-\infty} \frac{\mu}{a}\left(\frac{a_e}{a}\right)^l J_{lm}F_{lmp}(I)G_{lpq}(e)$$

$$X\cos\left(\varphi_{lmpq}(M, \omega, \Omega, \theta)\right), \qquad (2.1)$$

where μ is the Earth gravitational parameter, $\mu=3.986009 \times 10^{14}$ m^3/s^2, a, e, I, Ω, ω, M are the classical keplerian elements: a is the semi-major axis, e is the eccentricity, I is the inclination of the orbit plane with the equator, Ω is the longitude of the ascending node, ω is the argument of pericentre and M is the mean anomaly, respectively; ae is the Earth mean equatorial radius, a_e=6378.140 km, J_{lm} is the spherical harmonic coefficient of degree l and order m, $F_{lmp}(I)$ and $G_{lpq}(e)$ are Kaula's inclination and eccentricity functions, respectively. The argument $\varphi_{lmpq}(M, \omega, \Omega, \theta)$ is defined by the Eq. (2.2)

$$\varphi_{lmpq}(M, \omega, \Omega, \theta) = (l - 2p + q)M + (l - 2p)\,\omega + m(\Omega - \theta - \lambda_{lm}) + (l - m)\frac{\pi}{2}. \qquad (2.2)$$

where θ is the Greenwich sidereal time and λ_{lm} is the corresponding reference longitude along the equator.

In order to describe the problem in Hamiltonian form, Delaunay canonical variables are introduced.

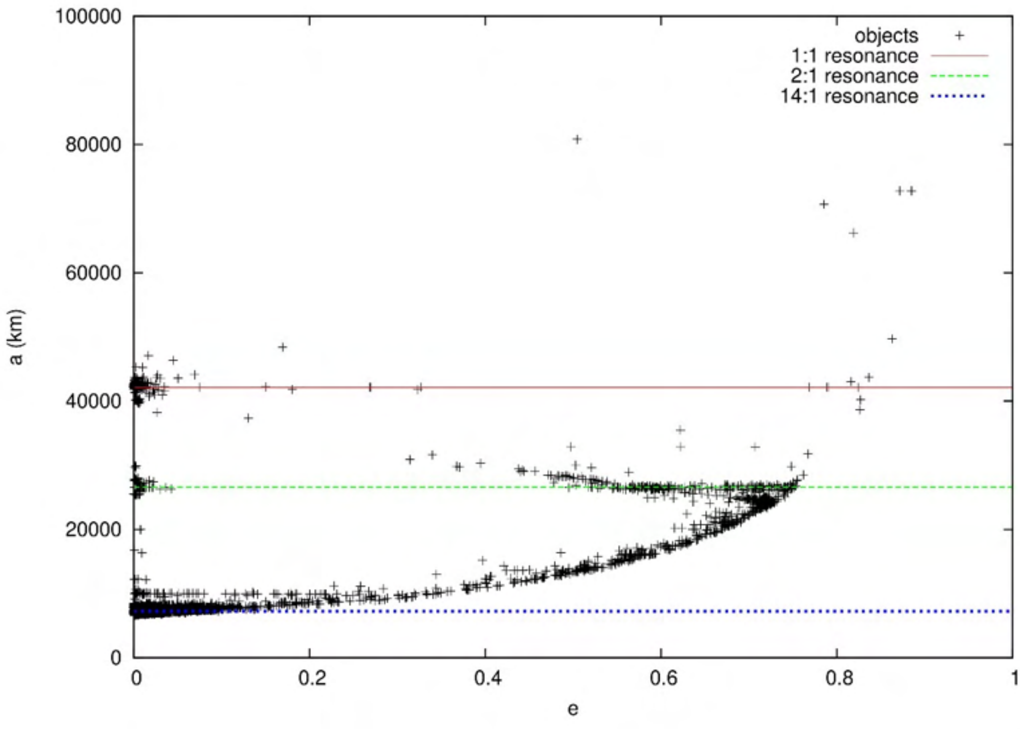

Figure 1. Semi-major axis versus eccentricity of the catalogued objects orbiting the Earth. The exacts resonances 1:1, 2:1 and 14:1 are indicated.

Journal of Engineering Research ISSN 2764-1317 DOI 10.22533/at.ed.317212210011

$$L = \sqrt{\mu a} \qquad G = \sqrt{\mu a(1-e^2)} \qquad H = \sqrt{\mu a(1-e^2)}\cos(I)$$

$$\ell = M \qquad g = \omega \qquad h = \Omega\,. \tag{2.3}$$

Using the canonical variables, one gets the Hamiltonian $\hat{F}$,

$$\hat{F} = \frac{\mu^2}{2L^2} + \sum_{l=2}^{\infty}\sum_{m=0}^{l} R_{lm}\,, \tag{2.4}$$

with the disturbing potential R_{lm} given by

$$R_{lm} = \sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty} B_{lmpq}\,(L,G,H)\cos\left(\varphi_{lmpq}(\ell,g,h,\theta)\right) \tag{2.5}$$

The argument $\varphi_{lmpq}(\ell, g, h, \theta)$ is defined by

$$\varphi_{lmpq}(\ell,g,h,\theta) = (l-2p+q)\ell + (l-2p)g + m(h-\theta-\lambda_{lm}) + (l-m)\frac{\pi}{2}\,, \tag{2.6}$$

and the coefficient $B_{lmpq}(L, G, H)$ by

$$B_{lmpq} = \sum_{l=2}^{\infty}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q=+\infty}^{-\infty}\frac{\mu^2}{L^2}\left(\frac{\mu a_e}{L^2}\right)^{l} J_{lm}F_{lmp}(L,G,H)G_{lpq}(L,G). \tag{2.7}$$

The Hamiltonian $\hat{F}$ depends explicitly on the time through the Greenwich sidereal time θ, where $\theta = \omega_e t$ (ω_e is the Earth's angular velocity and t is the time). A new variable Θ, conjugated to θ, is introduced in order to extend the phase space. In the extended phase space, the extended Hamiltonian $\hat{H}$ is given by

$$\hat{H} = \hat{F} - \omega_e\Theta. \tag{2.8}$$

For resonant orbits, it is convenient to use a new set of canonical variables. Consider the canonical transformation of variables defined by the following relations

$$X = L \qquad Y = G - L \qquad Z = H - G \qquad \Theta = \Theta$$

$$x = l + g + h \qquad y = g + h \qquad z = h \qquad \theta = \theta\,, \tag{2.9}$$

where X, Y, Z, Θ, x, y, z, θ are the modified Delaunay variables.

The new Hamiltonian $\widehat{H'}$, resulting from the canonical transformation defined by Eq. (2.9), is given by

$$\widehat{H'} = \frac{\mu^2}{2X^2} - \omega_e\Theta + \sum_{l=2}^{\infty}\sum_{m=0}^{l} R'_{lm}\,, \tag{2.10}$$

where the disturbing potential R'_{lm} is given by

$$R'_{lm} = \sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty} B'_{lmpq}(X,Y,Z)\cos\left(\phi_{lmpq}(x,y,z,\theta)\right). \tag{2.11}$$

Consider the resonance to be studied in this work; that is, the commensurability between the Earth rotation angular velocity ω_e and the mean motion n. This commensurability can be expressed as

$$(l-2p+q)n - m\omega_e \cong 0, \tag{2.12}$$

considering l, p, q and m as integers. The commensurability of the resonance studied, (l − 2p + q)/m, is defined by α. When this commensurability occurs, small divisors, associated to the tesseral harmonics, arise in the integration of the equations of motion (LANE, 1988). These terms are called resonant.

The short and long period terms can be eliminated from the Hamiltonian $\widehat{H'}$ by applying an averaging method. A reduced Hamiltonian is obtained from the Hamiltonian $\widehat{H'}$ when only secular and resonant terms are considered. Several authors, (ELY and HOWELL, 1996; NETO, 2006), also use this simplified Hamiltonian to study the resonance. Successive canonical transformations are done with the goal to write the final Hamiltonian, $\widehat{H'}_{2,f}$ (SAMPAIO, 2013).

A new set of canonical variables is defined

$$(X, Y, Z, \Theta, x, y, z, \theta) \rightarrow (X_1, Y_1, Z_1, \Theta_1, x_1, y_1, z_1, \theta_1)$$

And this transformation is given by the following equations

$$X_1 = X \quad Y_1 = Y \quad Z_1 = \left(1 - \frac{1}{\alpha}\right)X + Y + Z \quad \Theta_1 = \Theta$$
$$x_1 = x - \left(1 - \frac{1}{\alpha}\right)z \quad y_1 = y - z \quad z_1 = z \quad \theta_1 = \theta.$$

(2.13)

Note that Z_1=C_1 and the z_1 is an ignorable variable. So, the order of the dynamical system is reduced in one degree of freedom.

The new set of canonical variables, X_1, Y_1, Z_1, Θ_1, x_1, y_1, z_1, θ_1, in the reduced Hamiltonian, one gets the resonant Hamiltonian. The word "resonant" is used to denote the which is valid for any resonance.

The resonant Hamiltonian has all resonant frequencies, relative to the commensurability α. Now, by simplification, the term αm-l +2p will be represented by α'm'. $\varphi_{1,lmp(\alpha'm')}$ argument is given by

$$\phi_{1,lmp(\alpha'm')} = m(\alpha x_1 - \theta_1) + (\alpha'm')y_1 - \phi_{1,lmp(\alpha'm')0},$$

(2.14)

with

$$\phi_{1,lmp(\alpha'm')0} = m\lambda_{lm} - (l-m)\frac{\pi}{2}. \quad (2.15)$$

Now, consider a single frequency among the several resonant frequencies that can be obtained from the expression

$$\frac{\phi_{1,lmp(\alpha'm')}}{dt} = m\left(\alpha\frac{dx_1}{dt} - \frac{d\theta_1}{dt}\right) + (l - 2p - m\alpha)\frac{dy_1}{dt}.$$

(2.16)

The frequency $\varphi_{1,lmp(\alpha'm')}$ for the fixed coefficients m and (l – 2p – mα) will be the unique resonant frequency considered in the resonant Hamiltonian. This frequency will be called "critical frequency".

To determine a critical frequency, one needs to fix all the coefficients of the variable x_1, y_1, θ_1; in other words, one fixes α, m and (l-2p-mα).

Once this critical frequency has been chosen among the possible resonant frequencies, the other periodic terms of the resonant Hamiltonian are taken as short period terms, with frequencies different from the critical frequency.

Defining a single critical frequency, or, assuming the isolated study of each frequency, a new Hamiltonian is obtained. The coefficients k = l-2p and m are fixed. This Hamiltonian contains secular and critical terms only. Since k is a fixed value, the Hamiltonian can be put in the simplified form.

Using the first integral, a new Mathieu transformation can be defined. This canonical transformation is given by the following equations

$$X_2 = X_1 \quad Y_2 = (k - m\alpha)X_1 - m\alpha Y_1 \quad \Theta_2 = \Theta_1$$
$$x_2 = x_1 + \left(\frac{k - m\alpha}{m\alpha}\right)y_1 \quad y_2 = -\frac{1}{m\alpha}y_1 \quad \theta_2 = \theta_1.$$

(2.17)

The Hamiltonian function is invariant with respect to this new Mathieu transformation. And now considering q=αm-k, one gets the final Hamiltonian, $\widehat{H}_{2,f}$

$$\widehat{H}_{2,f} = \frac{\mu^2}{2X_2^2} - \omega_e\Theta_2 + \sum_{j=1}^{\infty} B_{2,2j,0,j,0}(X_2, C_1, C_2) +$$
$$\sum_{p=s}^{\infty} B_{2,(2p+k)mp(\alpha m-k)}(X_2, C_1, C_2)\cos\left(\phi_{2,(2p+k)mp(\alpha m-k)}(x_2, \theta_2)\right),$$

(2.18)

Note that Y2 = C2 and y2 is an ignorable variable.

The new angle $\varphi_{2,(2p+k)mp(\alpha m-k)}$(x2, θ2) is given by

$$\phi_{2,(2p+k)mp(\alpha m-k)}(x_2, \theta_2) = \phi_2 - \phi_{2,(2p+k)mp(\alpha m-k),0},$$

(2.19)

where φ_2 = m(αx2 – θ2), and

$$\phi_{2,(2p+k)mp(\alpha m-k),0} = m\lambda_{(2p+k)m} - (2p + k - m)\frac{\pi}{2} = \phi_{1,lmp(\alpha m-k)0}.$$

(2.20)

The Hamiltonian, $\widehat{H}_{2,f}$ has all tesseral related to the chosen critical frequency. The dynamical system generated by Hamiltonian, $\widehat{H}_{2,f}$ is

$$\frac{dX_2}{dt} = -m\alpha \sum_{p=s}^{\infty} B_{2,(2p+k)mp(\alpha m-k)}(X_2, C_1, C_2) sen\left(\phi_{2,(2p+k)mp(\alpha m-k)}\right) \tag{2.21}$$

$$\frac{d\phi_2}{dt} = m\alpha \frac{\mu^2}{X_2^3} - m\omega_e - m\alpha \sum_{j=1}^{\infty} \frac{\partial B_{2,2j,0,j,0}(X_2,C_1,C_2)}{\partial X_2} - m\alpha \sum_{p=s}^{\infty} \frac{\partial B_{2,(2p+k)mp(\alpha m-k)}(X_2,C_1,C_2)}{\partial X_2} \times \cos\left(\phi_{2,(2p+k)mp(\alpha m-k)}(x_2, \theta_2)\right) \tag{2.22}$$

The Eqs. (2.21) and (2.22) represent the equations of motion in a resonance of commensurability α.

In the Eq. (2.21), the term $B_{2,(2p+k)mp(\alpha m-k)}$ (X2, C1, C2) is given by

$$B_{2,(2p+k)mp(\alpha m-k)}(X_2, C_1, C_2) = \frac{\mu^{2p+k+2}}{X_2^{4p+2k+2}} a_e^{2p+k} J_{(2p+k)m} \times F_{(2p+k)mp}(X_2, C_1, C_2) \times G_{(2p+k)p(\alpha m-k)}(X_2, C_2), \tag{2.23}$$

The term $B_{2j,0,j,0}(X_2, C_1, C_2)$ is

$$B_{2j,0,j,0}(X_2, C_1, C_2) = \frac{\mu^{2j+2}}{X_2^{4j+2}} a_e^{2j} J_{2j,0} F_{2j,0,j}(X_2, C_1, C_2) G_{2j,j,0}(X_2, C_2). \tag{2.24}$$

In the Eq. (2.22), the terms $\frac{\partial B_{2,2j,0,j,0}(X_2,C_1,C_2)}{\partial X_2}$ and $\frac{\partial B_{2,(2p+k)mp(\alpha m-k)}(X_2,C_1,C_2)}{\partial X_2}$ are

$$\frac{\partial\left(B_{2j,0,j,0}(X_2,C_1,C_2)\right)}{\partial X_2} = -\frac{4j+2}{X_2^{4j+3}} \mu^{2j-2} a_e^{2j} J_{2j,0} F_{2j,0,j} G_{2j,j,0} - \frac{\mu^{2j+2}}{X_2^{4j+2}} a_e^{2j} J_{2j,0} \frac{\partial(F_{2j,0,j})}{\partial X_2} G_{2j,j,0} - F_{2j,0,j} \frac{\partial(G_{2j,j,0})}{\partial X_2}, \tag{2.25}$$

$$\begin{aligned} \frac{\partial\left(B_{2,(2p-k)mp(\alpha m-k)}(X_2, C_1, C_2)\right)}{\partial X_2} &= -\frac{(4p+2k+2)}{X_2^{4p+2k+3}} \mu^{2p+k+2} a_e^{2p+k} J_{(2p+k)m} F_{(2p+k)mp} G_{(2p+k)p,(\alpha m-k)} \\ &+ \frac{\mu^{2p+k+2}}{X_2^{4p+2k+2}} a_e^{2p+k} J_{(2p+k)m} \frac{\partial\left(F_{(2p+k)mp}\right)}{\partial X_2} G_{(2p+k)p,(\alpha m-k)} \\ &+ F_{(2p+k)mp} \frac{\partial\left(G_{(2p-k)p,(\alpha m-k)}\right)}{\partial X_2}. \end{aligned} \tag{2.26}$$

In the Eqs. (2.25) and (2.26), the terms related with the inclination function $\frac{\partial(F_{2j,0,j}(X_2,C_1,C_2))}{\partial X_2}$, $\frac{\partial(F_{(2p+k)mp}(X_2,C_1,C_2))}{\partial X_2}$ can be expressed as

$$\frac{\partial\left(F_{2j,0,j}(X_2, C_1, C_2)\right)}{\partial X_2} = \frac{\partial(F_{2j,0,j})}{\partial I} \frac{\partial I}{\partial X_2}$$

$$\frac{\partial\left(F_{(2p+k)mp}(X_2, C_1, C_2)\right)}{\partial X_2} = \frac{\partial(F_{(2p+k)mp})}{\partial I} \frac{\partial I}{\partial X_2}$$

The inclination $I = I(X_2, C_1, C_2)$ is

$$I = \arccos\left(\frac{m\alpha C_1 + mX_2}{kX_2 - C_2}\right),$$

And the derivative $\frac{\partial I(X_2,C_1,C_2)}{\partial X_2}$ is

$$\frac{\partial I(X_2,C_1,C_2)}{\partial X_2} = -\left(\frac{m}{kX_2 - C_2} - \frac{(m\alpha C_1 + mX_2)k}{(kX_2 - C_2)^2}\right) \frac{1}{\sqrt{1 - \frac{(m\alpha C_1 + mX_2)^2}{(kX_2 - C_2)^2}}}. \tag{2.27}$$

The inclination function is given by (KAULA, 1966; GOLEBIWSKA et al., 2010; WNUK, 1988)

$$F_{lmp}(I) = (-1)^{\delta} \frac{(l+m)!}{2^l p!(l-p)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2p)}{jj} X \binom{2p}{l-m-jj} c^{2l-\sigma} s^{\sigma}, \tag{2.28}$$

With $\delta = E\left(\frac{l-m+1}{2}\right)$, E(.) means "integer part of", jj1=max(0,l-m-2p), jj2=min(l-m, 2l-2p), c=cos(I/2), s=sin(I/2), $\sigma = m - l + 2p + 2kk$ and kk=E(l-m/2).

And $\frac{\partial(F_{lmp})}{\partial I}$ is

$$\frac{\partial(F_{lmp})}{\partial I} = -\frac{1}{2}(2l - \sigma)(-1)^{\delta} \frac{(l+m)!}{2^l p!(l-p)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2p)}{jj} \binom{2p}{l-m-jj} s^{2l-\sigma} s^{\sigma} + \frac{1}{2}\sigma(-1)^{\delta} \frac{(l+m)!}{2^l p!(l-p)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2p)}{jj} \times \binom{2p}{l-m-jj} c^{2l-\sigma} c^{\sigma}. \tag{2.29}$$

The inclination function used in the secular terms $F_{2j,0,j}(I)$

$$F_{2j,0,j}(I) = (-1)^{\delta} \frac{(2j)!}{2^{2j} j!(j)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2j)}{jj} X \binom{2j}{2j-jj} c^{4j-\sigma} s^{\sigma}, \quad (2.30)$$

And $\frac{\partial(F_{2j,0,j})}{\partial I}$ can be expressed as

$$\frac{\partial(F_{2j,0,j})}{\partial I} = -\frac{1}{2}(4j - \sigma)(-1)^{\delta} \frac{(2j)!}{2^{2j} j!(j)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2j)}{jj} \binom{2j}{2j-jj} s^{4j-\sigma} s^{\sigma} + \frac{1}{2}\sigma(-1)^{\delta} \frac{(2j)!}{2^{2j} j!(j)!} \sum_{jj=jj^1}^{jj^2} (-1)^{jj} \binom{(2jj-2j)}{jj} \binom{2j}{2j-jj} c^{4j-\sigma} c^{\sigma}. \quad (2.31)$$

The derivatives related with the eccentricity function $\frac{\partial\left(G_{2j,j,0}(X_2,C_2)\right)}{\partial X_2}$ and $\frac{\partial\left(G_{(2p+k)p,(\alpha m-k)}(X_2,C_2)\right)}{\partial X_2}$ can be expressed as

$$\frac{\partial\left(G_{2j,j,0}(X_2,C_2)\right)}{\partial X_2} = \frac{\partial\left(G_{2j,j,0}\right)}{\partial e}\frac{\partial e}{\partial X_2}$$

$$\frac{\partial\left(G_{(2p+k)p,(\alpha m-k)}(X_2,C_2)\right)}{\partial X_2} = \frac{\partial\left(G_{(2p+k)p,(\alpha m-k)}\right)}{\partial e}\frac{\partial e}{\partial X_2}$$

The eccentricity $e=e(X_2,C_2)$ is defined as

$$e(X_2,C_2) = \frac{1}{(m\alpha)X_2}\sqrt{X_2(m\alpha+k)-C_2}\sqrt{X_2(m\alpha-k)-C_2}, \quad (2.32)$$

And the derivative $\frac{\partial e}{\partial X_2}$ is

$$\frac{\partial e}{\partial X_2} = -\frac{\sqrt{X_2(m\alpha+k)-C_2}\sqrt{X_2(m\alpha-k)+C_2}}{m\alpha X_2{}^2} + \frac{1}{2}\frac{\sqrt{X_2(m\alpha-k)+C_2}(m\alpha+k)}{m\alpha X_2\sqrt{X_2(m\alpha+k)-C_2}} + \frac{1}{2}\frac{\sqrt{X_2(m\alpha+k)-C_2}(m\alpha-k)}{m\alpha X_2\sqrt{X_2(m\alpha-k)+C_2}}. \quad (2.33)$$

The eccentricity $G_{lpq}(e)$ function is defined in Kaula (1966) and Golebiwska et al. (2010).

In the next section are shown some results of the numerical integration of the Eqs. (2.21) and (2.22) involving the 14:1 resonance.

RESULTS

Figures 2 and 3 show the phase spaces, a versus φ_2, for the critical angle $\varphi_{14146-1}$, associated to tesseral harmonic J_{1414}, according to the numerical integration of the equations of motion, (2.21) and (2.22). The initial conditions, in the Figs. 2 and 3, for inclinations are 87^0 and 95^0, respectively and eccentricities are 0.019 and 0.005, respectively. The initial values of the semi-major axis are around the critical semi-major axis.

Figures 4 and 5 show the time behavior of the semi-major axis and φ2 angle, according to the phase spaces of the Figures 2 and 3. The libration and circulation regions are differentiated in the Figs. 4 and 5.

The Figs. 2 to 5 show the phase spaces, *a* versus Φ_2 and time behavior of the semi-major axis, considering the critical angle $\Phi_{14146-1}$ associated to J_{1414}. Around the 14:1 resonance, there are several space debris orbiting the Earth, without control and risking the useful time of the artificial satellites in operation. The knowledge of regular or stable regions, in the LEO zone, can be very important to provide greater security for the orbital motion of artificial satellites and, possibly, lower fuel consumption with orbital maneuvers compared to unstable regions

CONCLUSIONS

In this work, the orbital dynamics of synchronous satellites is studied. The dynamical behavior of the critical angle $\Phi_{14146-1}$ associated to the 14:1 resonance problem in the artificial satellites motion have been investigated.

Results show the phase spaces, *a* versus Φ_2, and the time behavior of the semi-major axis for the critical angle $\Phi_{14146-1}$, associated to tesseral harmonic J_{1414}. The initial conditions used for inclinations are 87° and 95° and eccentricities are 0.019 and 0.005, respectively. The initial values of the semi-major axis are

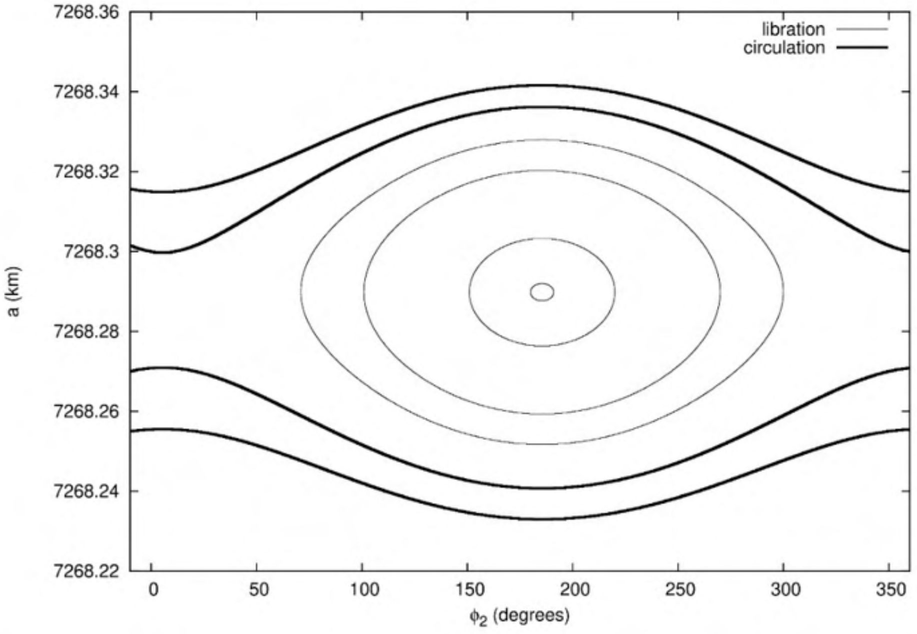

Figure 2. Semi-major axis versus Φ_2 angle, considering the critical angle $\Phi_{14146\text{-}1}$ associated to J_{1414}. The initial conditions for inclination and eccentricity are I=87° and e=0.019, respectively.

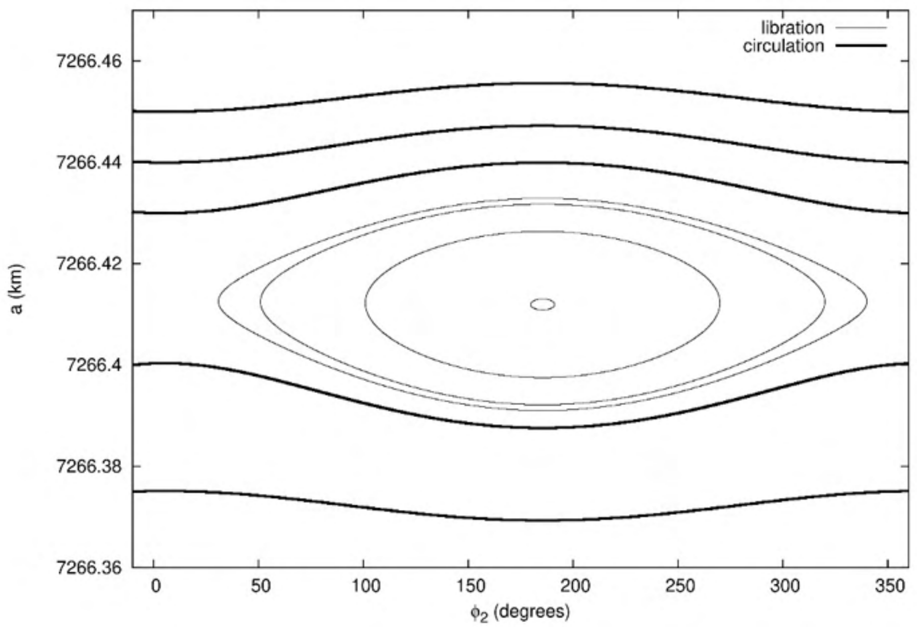

Figure 3. Semi-major axis versus Φ_2 angle, considering the critical angle $\Phi_{14146\text{-}1}$ associated to J_{1414}. The initial conditions for inclination and eccentricity are I=95° and e=0.005, respectively.

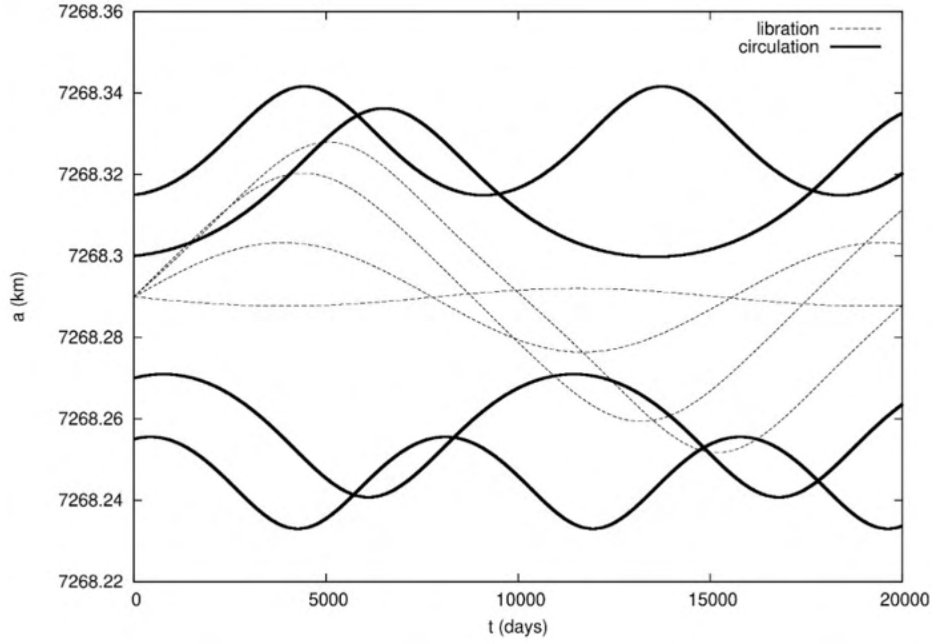

Figure 4. Time behavior of the semi-major axis, considering the critical angle $\Phi_{14146\text{-}1}$ associated to J_{1414}. The initial conditions for inclination and eccentricity are I=87° and e=0.019, respectively.

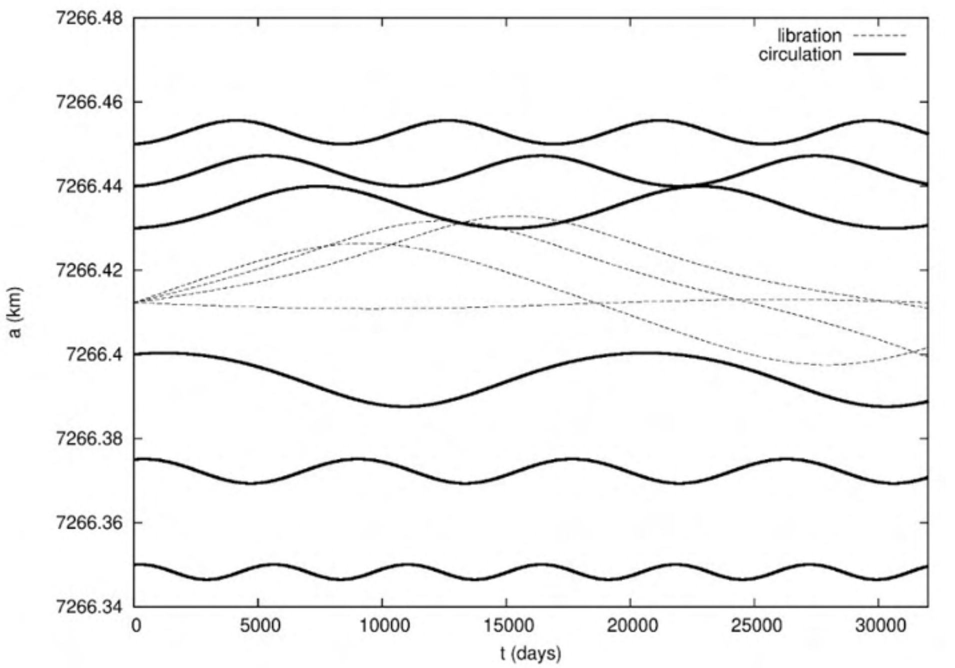

Figure 5. Time behavior of the semi-major axis, considering the critical angle $\Phi_{14146\text{-}1}$ associated to J_{1414}. The initial conditions for inclination and eccentricity are I=95° and e=0.005, respectively.

around the critical semi-major axis.

The appearance of the phase spaces found by numerical integration, resembles the phase space of the simple pendulum, with two different regions, separated by the separatrix.

The theory developed for the resonant Hamiltonian and the equations of motion can be applied for any resonance.

ACKNOWLEDGEMENTS

This work was accomplished with support of the PROPEX / IFS, CNPq: Nº 305981-2022-0 and 303102/2019-5 and Fapesp: Nº 2016/24561-0.

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