COLEÇÃO

DESAFIOS DAS ENGENHARIAS:

ENGENHARIA MECÂNICA 2

HENRIQUE AJUZ HOLZMANN
JOÃO DALLAMUTA
(ORGANIZADORES)

Atena Editora
Ano 2021

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APRESENTAÇÃO

A Engenharia Mecânica pode ser definida como o ramo da engenharia que aplica os princípios de física e ciência dos materiais para a concepção, análise, fabricação e manutenção de sistemas mecânicos. O aumento no interesse por essa área se dá principalmente pela escassez de matérias primas, a necessidade de novos materiais que possuam melhores características físicas e químicas e a necessidade de reaproveitamento dos resíduos em geral. Além disso a busca pela otimização no desenvolvimento de projetos, leva cada vez mais a simulação de processos, buscando uma redução de custos e de tempo.

Neste livro são apresentados trabalho teóricos e práticos, relacionados a área de mecânica, materiais e automação, dando um panorama dos assuntos em pesquisa atualmente. A caracterização dos materiais é de extrema importância, visto que afeta diretamente aos projetos e sua execução dentro de premissas técnicas e econômicas. Pode-se ainda estabelecer que estas características levam a alterações quase que imediatas no projeto, sendo uma modificação constante na busca por melhores espostas e resultados.

De abordagem objetiva, a obra se mostra de grande relevância para graduandos, alunos de pós-graduação, docentes e profissionais, apresentando temáticas e metodologias diversificadas, em situações reais. Sendo hoje que utilizar dos conhecimentos científico de uma maneira eficaz e eficiente é um dos desafios dos novos e nheiros.

Boa leitura.

Henrique Ajuz Holzmann
João Dallamuta

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A REVIEW ON ITERATIVE AND SERIES SOLUTIONS FOR KEPLER'S EQUATION

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Paula Cristiane Pinto Mesquita Pardal
Universidade de São Paulo (EEL/USP)
Lorena, SP, Brasil

Mariana Pereira de Melo
Universidade de São Paulo (EEL/USP)
Lorena, SP, Brasil

João Francisco Nunes de Oliveira
Cia de Gás de São Paulo (COMGÁS)
São Paulo, SP, Brasil

Leonardo de Oliveira Ferreira
Universidade de São Paulo (EEL/USP)
Lorena,SP, Brasil

Pedro Novak Nishimoto
Cia de Gás de São Paulo (COMGÁS)
São Paulo, SP, Brasil

Roberta Veloso Garcia
Universidade de São Paulo (EEL/USP)
Lorena, SP, Brasil

ABSTRACT: The purpose is to review iterative and series methods applied to the solution of Kepler's equation, which is solved over the entire range of elliptic motion. The method whose results will work as a reference is the Newton-Raphson's numerical method. The results will be discussed around the number of iterations required until the convergence criterion is satisfied, that is, residual error in eccentric anomaly lower than rad (for iterative methods) or rad (for series-based methods) and the processing time. The advantages and drawbacks of each method will be presented.

KEYWORDS: *Kepler's Equation; Numerical Methods; Iterative Solutions; Series Solutions.*

1 I INTRODUCTION

Artificial satellites are employed in many activities, such as space exploration, land mapping, microgravity experiments and telecommunication. Regardless of the mission for which the satellite is designed, the knowledge of its most accurate possible orbital position is critical for the mission success. And here lies the importance of Kepler's equation: it gives a relation between the position of the satellite and time (Battin, 1999).

The elliptical form of Kepler's Equation is given by:

$$\mathcal{M} = \mathcal{E} - e \sin(\mathcal{E}) \quad (1)$$

where the three quantities are related to the orbit Keplerian elements: $\mathcal{M}$ is the mean anomaly; $\mathcal{E}$, the eccentric anomaly; and , the eccentricity. Kepler's equation is transcendental in $\mathcal{E}$; therefore, the solution for this quantity, when $\mathcal{M}$ is given cannot be expressed by a finite number of terms (Battin, 1999). The solutions for Kepler's equation can only be approximated, generally using computational methods.

Since Kepler's equation is one of the most famous transcendental equations, it has inspired many developments in mathematics during the last decades.

An algorithm based on simple initial cubic approximations and a slight generalization of the Newton-Raphson method, was presented for the solution of Kepler's equation (Ng, 1979). In Danby and Burkardt (1983), methods of iteration are discussed in relation to Kepler's equation, considering various initial "guesses", with possible strategies for their choices. Several of these iterative methods are compared; the one used in the comparisons has local convergence of fourth order. If in the first study, Danby and Burkardt considered the solution of the conventional form of Kepler's equation for elliptic orbits, in the second, they first considered hyperbolic orbits equation, then generalizations for elliptic and hyperbolic orbits (Burkardt and Danby, 1983).

Serafin (1986) analytically examined techniques for selecting the interval within which the root of the Kepler's equation of satellite motion is to be sought. In 1986, Odell and Gooding reviewed starting formulas and iteration processes for the solution of Kepler's equation, giving details of two complete procedures that operates with an iterative process. Mikkola (1987) derived a method to obtain an approximate solution for Kepler's equation that could be used for all orbit types, including hyperbolic.

Markley (1996) solves Kepler's Equation over the entire range of elliptic motion by a fifth-order refinement of the solution of a cubic equation. This method requires a square root, a cube root, and two trigonometric functions (four transcendental function evaluations). In Fukushima (1997), two approximations of the Newton-Raphson method were developed. The first is a sort of discretization, namely to search an approximate solution on pre-specified grid points. The second is a Taylor series expansion. A combination of these was applied to solving Kepler's equation for the elliptic case. Later, he developed a procedure to solve a modification of the standard form of the universal Kepler's equation, which is expressed as a nondimensional equation with respect to a nondimensional variable (Fukushima, 1999).

Condurache and Martinuşi (2007) present an exact vectorial solution to the Kepler problem. A vectorial regularization linearizes Kepler's equation, using a Sundman transformation. A unified approach to the classic Kepler problem is offered, by studying both rectilinear and non-rectilinear Keplerian motions with the same instrument. In Davis et al. (2010) seven sequential starter values for solving Kepler's equation for fast orbit propagation are proposed. These methods have constant complexity (not iterative), do not require pre-computed data, and can be implemented in a few lines of code.

More recently, Reza and Ghadiri (2014) focused on Newton-Raphson's method for solving Kepler's equation. In order to increase the stability of Newton's method, various guesses were studied. Based on time of implementation, an appropriate choice is presented: first guesses that increase the isotropy and decrease the solution time of implementation. Starting algorithms for the iterative solution of elliptic Kepler's equation are also considered in Calvo et al. (2013), where new global efficiency measures are introduced and several wellknown starters with minimum computational cost are analyzed on the light of these efficiency measures. And Avendano et al. (2015) used Smale's α-theory to prove that Newton's method starting at the defined approximate zero produces a sequence that converges to the actual solution at quadratic speed.

2 I ITERATIVE AND SERIES-BASED METHODS

In this paper, the approximations of six different methods are compared: four are iterative methods, and two, methods based on series approach. The iterative methods are: Newton-Raphson (the solutions it produced were used as a reference), Halley, Regula-Falsi and Successive Approximations, and were computationally implemented. The methods based on series solutions comprise Lagrange Expansion Theorem and Fourier-Bessel Series Expansion (Battin, 1999).

2.1 Newton-Raphson's method

Newton-Raphson's method is an iterative method usually applied to numerical solution of equations of the form , where is differentiable. The iteration function of this method is (Franco, 2006):

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2}$$

Kepler's Equation can be conveniently written as:

$$\mathcal{E} - e\, sin\,(\mathcal{E}) - \mathcal{M} = f\,(\mathcal{E}) \tag{3}$$

and solved by Newton-Raphson's Method via:

$$E_{n+1} = E_n - \frac{E_n - e\sin(E_n) - M}{1 - e\cos(E_n)} \tag{4}$$

The iteration process in Eq. (4) stops when the root accuracy reaches a specific value, determined by each problem.

2.2 Halley's method

Halley's Method is a generalization of Newton's method that aims at finding the root of a nonlinear equation and requires analytical and numerical computation of higher-order derivatives of the function. The algorithm adapted to Kepler's equation for any fixed value of to iterate for is given by (Gander, 1985):

$$E_{i+1} = E_i + (n+1)\frac{\left(\frac{1}{f(E_i)}\right)^{(n)}}{\left(\frac{1}{f(E_i)}\right)^{(n+1)}} \tag{5}$$

For this application, it has been considered , then:

$$E_{i+1} = E_i + 2\frac{f(E)f'(E_i)}{[f'(E_i)]^2 - f(E_i)f''(E_i)} \tag{6}$$

2.3 Regula-falsi method

The Regula-Falsi method is a simple iterative technique, which consists in considering two initial approximations x_1 and x_2, such that $f(x_1)$ and (x_2) have opposite signs, i.e.:

$$f(x_1)\cdot(x_2) < 0 \tag{7}$$

so that can be determined by considering the equation of secant line as the function in the interval , as follow:

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} \tag{8}$$

$$\text{if } \left|\frac{x_3 - x_1}{x_3}\right| < \varepsilon \text{ or } \left|\frac{x_3 - x_2}{x_3}\right| < \varepsilon \tag{9}$$

for a given ε, then x_3 is the root searched. Otherwise, $f(\cdot)$ is calculated from the choice of a value x_1, between x_1 and x_2, so that $f(x)$ has the opposite sign of $f(x_3)$. From this point, x_3 is calculated, then x_4, and so on. The process should be repeated until a root with a specific accuracy is obtained. The formulation for the iterative process can be summarized as:

$$E_{i+1} = \frac{E_{i-1}(E_i - e\sin(E_i)) - E_i(E_{i-1} - e\sin(E_{i-1}))}{E_i - E_{i-1} + e(\sin(E_{i-1}) - \sin(E_i))} \tag{10}$$

2.4 Successive approximations method

In this iterative method, a function , continuous in the range where the root must be found, is chosen. Let be rewritten as (Franco, 2006):

$$f(x) = \varphi(x) - x \tag{11}$$

Considering:

$$\varphi(x) = x + \mathcal{A}(x) * f(x) \quad (12)$$

and that when x is the root of f (x), i.e., f (x) = 0, follows that x = φ (x), for all A (x) ≠ 0. Considering φ (x) as defined in Eq. (12), if x is the root of f (x), then:

$$\varphi(x) = x \quad (13)$$

It means that, on the point where x is the root of $f(x)$, replacing the value of x in the function $\varphi(x)$ will return the very x value.
Therefore, this method consists in finding the numerical value that, when placed in $\varphi(x)$, returns the x value. The iterative function is:

$$x_{n+1} = \varphi(x_n) \quad (14)$$

in which n is the actual n^{th} iteration.

2.5 Lagrange inversion theorem

The Lagrange Inversion Theorem is an exact analytical method that not relies on numerical manipulations. This method provides an analytical solution for non-linear equations in terms of an infinite series (Rathie et al., 2013). Consider the functional equation, for which the Kepler's equation is a special case (Battin, 1999):

$$y = x + \alpha * \Phi(y) \quad (15)$$

in which α is considered a small parameter (identified as the orbit eccentricity in the Keple 's equation). It follows that y, as a function of x, can be expanded as a Taylor series with $\alpha = 0$:

$$y(x,\alpha) = y(x,0) + \alpha \frac{\partial y}{\partial \alpha} + \frac{\alpha^2}{2!}\frac{\partial^2 y}{\partial \alpha^2} + \cdots \quad (16)$$

which can be turned into the following power series, given by the Lagrange Inversion Theorem:

$$y = x + \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} \frac{d^{n-1}}{dx^{n-1}} \Phi(x)^n \tag{17}$$

2.6 Fourier-bessel series expansion

Another approach to determine an approximate solution to Eq. (1) is a representation in a power series (no iterative approach). This expansion, called Lagrange Expansion, presents the eccentric anomaly in terms of a power series of the eccentricity . However, this series does not converge for all values of the eccentricity. The Fourier-Bessel Series Expansion solves this problem, because it is convergent for all eccentricity values. The expansion is defined below and details can be found in Battin (1999) and Colwell (1992)

$$E = M + 2\sum_{k=1}^{\infty} \frac{1}{k}\left(\frac{1}{\pi}\int_{0}^{\infty} \cos(kE - ke\sin(E))\, dE\right)\sin(kM) \tag{18}$$

in which $\mathcal{K}$ is the number of terms required for the expansion to provide the value of $\mathcal{E}$ with the desired accuracy.

3 I RESULTS

In this paper, four iterative methods were investigated regarding the calculation of the eccentric anomaly $\mathcal{E}$. The reference solution was obtained via Newton-Raphson's Method, and its results were compared with three methods: Halley, Regula-Falsi, and Successive Approximations. This study also aims at evaluating two series-based methods: Lagrange Expansion Theorem and Fourier-Bessel Series Expansion. Since their approaches are different from the iterative methods, these results were compared directly to each other.

There is no uniform way in which the various authors evaluate the efficiency of methods to solve Kepler's equation (Nijenhuis, 1991). Herein, the evaluation was carried out using a convergence criterion: (i) the number of iterations required until the residual error (in $\mathcal{E}$) is lower than 10^{-12} rad for iterative methods and lower than 10^{-4} rad for series-based methods; and (ii) the processing time to achieve such tolerance. For the iterative methods, the choice of a residual error lower than 10^{-12} rad is due to the guarantee that the error in the orbit will be lower than the order of centimeters; while 10^{-4} rad is due to the series-based methods limitations.

In order to assess whether the behavior of these variables differs according to eccentricity and mean anomaly values, for each method, the calculation of the eccentric anomaly was implemented as follows:

- 21 values were taken into account for the eccentricity, over the entire elliptical interval ;

- 11 values were considered for the mean anomaly, over the interval [0,π].

Therefore, Kepler's equation was solved for a grid of points in the $(e, \mathcal{M})$-plane. The increments for the grid size were $\Delta e = \frac{1}{20}$ and $\Delta\mathcal{M} = \frac{\pi}{10}$ rad, which leads to 231 pairs of points $(e, \mathcal{M})$. The calculations covered the value $e = 1$, which is orbitally, but not mathematically pointless.

It is important to explain that the results could have been discussed around the error (the absolute difference between each method's solution and Newton-Raphson's solution). However, as the admitted errors are low, especially for the iterative methods, such analysis was not significant in this application

Regarding the processing time, Tab. (1) carries the information from the computer used to perform the simulations.

Processor	Intel ® Core ™ i7-7500U CPU @ 2.70 GHz 2.90 GHz
Installed memory (RAM)	8.00 GB
Cash memory	4.00 GB
System type	64-bit operating system, x64-based processor
Programming language	MATLAB ®

Table 1. Specifications of the computer used for the simulations

3.1 Number of Iterations Assessment

Iterative Methods

The first analysis will be around the number of iterations that each method needed to reach the specified accuracy (according to the method's nature: iterative or in series), for each pair of points used as the initial guess. In all graphs, the grid of 231 pairs of previously established points and its corresponding performance are mapped.

For better analysis and understanding of the graphs, two elements were used to rank the results:

- Size of the circles that represent each pair results: directly proportional to the number of iterations, that is, the smaller its size, the smaller the number of iterations, and vice versa.
- Color of the circles that represent each pair results: the color gradient goes from blue (fewer number of iterations) to yellow (greater number of iterations), and the green color portrays the intermediate values.

Figure (1) shows the analysis related to the Halley's method. For all initial guesses, the Halley's method requires none, 3 or 4 iterations to achieve accuracy lower than 10^{-12} rad. The mean anomaly $\mathcal{M}$ represents the conversion to angle of the time elapsed since the body passed through the orbit perigee ($\mathcal{M} = 0$). If $\mathcal{M} = 0$ or $\mathcal{M} = \pi$, it is known that $\mathcal{E} = 0$ or $\mathcal{E} = \pi$, respectively, and if $e = 0$, automatically $\mathcal{M} = \mathcal{E}$; in these cases, the numerical solution becomes unnecessary (the method does not require iteration), as shown in Fig. (1). For all other regions, 3 or 4 iterations were sufficient for Halley's method to achieve accuracy, including $e = 1$, which does not represent elliptical motion (parabolic orbit). The results for this method were very similar to those of the reference (Newton-Raphson method) and, for this reason, the reference was withdrawn from the graphs and its results will be presented in Tab. (2).

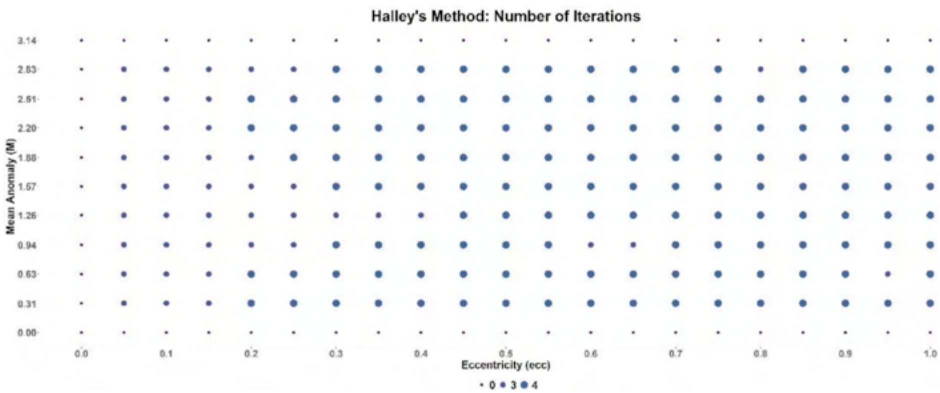

Figure 1. Number of iterations required for Halley's Method to achieve accuracy lower than rad.

Figure (2) exhibits the results regarding Regula-Falsi method. For the entire grid of initial guesses, 1 to 21 iterations were needed in order to reach accuracy lower than 10^{-12} rad. As with Halley's method, if $\mathcal{M} = 0$, $\mathcal{M} = \pi$ or $e = 0$, the numerical solution becomes unnecessary and, according to the nature of the method, iteration was necessary, according to Fig. (2). The Regula-Falsi method presents greater sensitivity in relation to the eccentricity variation, that is, as $e \to 1$, the number of iterations necessary to score accuracy increases, for any value of $\mathcal{M}$ in the interval $[0, \pi]$.

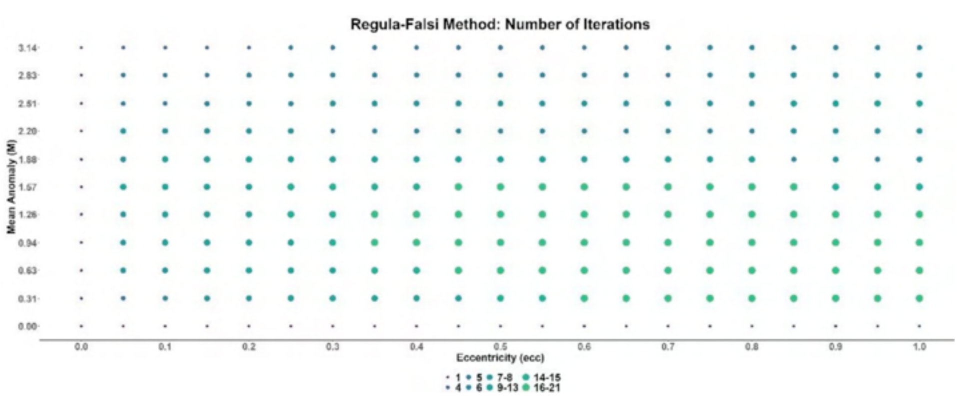

Figure 2. Number of iterations required for Regula-Falsi Method to achieve accuracy lower than 10^{-12} rad.

Figure (3) presents the method of Successive Approximations performance. For each pair $(e, \mathcal{M})$, it took from 1 to 1790 iterations so that the method reaches the required accuracy. If $\mathcal{M} = 0$, $\mathcal{M} = \pi$ or $e = 0$, the behavior is exactly the same as the Regula-Falsi method, as shown in Fig. (3). The number of iterations in the Successive Approximations method is even more sensitive to the variation of the eccentricity, and when 0.8 < 1 and $\frac{3\pi}{4} < \mathcal{M} < \pi$, simultaneously, a problematic region starts to stand out, where convergence still occurs, but it is very costly from a computational point of view, requiring more than 100 interactions to achieve accuracy smaller than 10^{-12} rad.

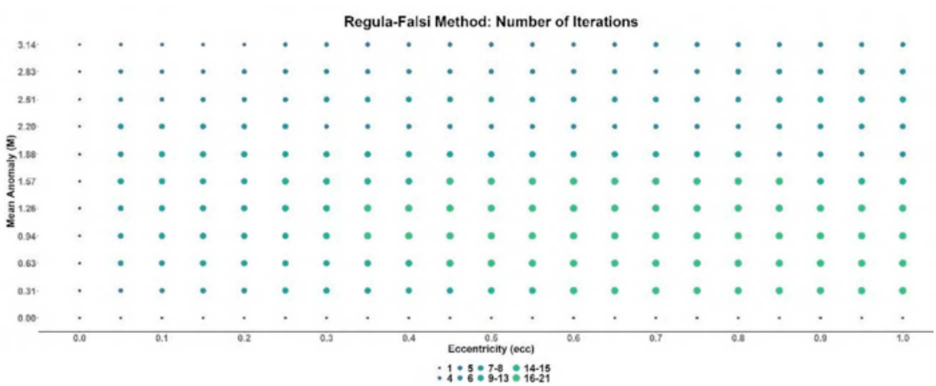

Figure 3. Number of iterations required for Successive Approximations Method to achieve accuracy lower than 10^{-12} rad.

From the results presented, regarding the number of iterations necessary to achieve an accuracy lower than 10^{-12} rad, it is clear that, among the iterative methods, the Halley's Method is much more efficient than the other two methods, and the Successive Approximations methods, on average, is the one which requires the greater number of iterations to reach the established accuracy. While a maximum of 21 interactions were required in the Regula-Falsi method, the Successive Approximations method performed up to 1790, in order to achieve convergence. From the graphs, there is a difference in the critical regions in which the methods required a greater number of iterations: the Successive Approximations method needs a greater number of iterations when eccentricity is greater than 0.90 and mean anomaly is greater than 1.88, while the Regula-Falsi method reaches its maximum for eccentricity between 0.5 and , and mean anomaly between 0.63 and 1.57.

In 34 combinations of eccentricity and mean anomaly values (15% of the possible combinations), both methods obtained the same number of iterations, in 78 combinations (34%), the Regula-Falsi method required a greater number of iterations compared to the Successive Approximations method. However, the largest difference in these cases was 15 iterations. Finally, in 119 combinations (52%), the Successive Approximation method had a greater number of iterations than the Regula-Falsi, with a difference of up to 1783 iterations between them (of these, in 7 cases, the difference was greater than 50 iterations).

Series-Based Methods

Figures (4) and (5) discuss the methods based in series performance: Fourier-Bessel Series Expansion and Lagrange Inversion Theorem, respectively. Due to the distinct nature of these methods, they were compared with each other, separately from iterative methods. The difference of performance between the two is huge: for each pair $(e, \mathcal{M})$, while the Fourier-Bessel Series Expansion needs a maximum of 13 iterations to achieve an accuracy less than 10^{-4}, the Lagrange Inversion Theorem needs up to 4900 iterations (~ 376 times more than Fourier-Bessel Series Expansion). In all 231 combinations of eccentricity and mean anomaly, the Lagrange Inversion Theorem required a greater number of iterations than the Fourier-Bessel Series Expansion. For entries , $\mathcal{M} = 0$, $\mathcal{M}= \frac{\pi}{2}$ and $\mathcal{M} = \pi$ ($\forall\ e$ Є [0,1]) or $e = 0$ ($\forall\ \mathcal{M}$ Є [0, π]), both methods converge quickly. The Lagrange Inversion Theorem high number of iterations elsewhere in the grid indicates that a more accurate analysis needs to be made on this expansion.

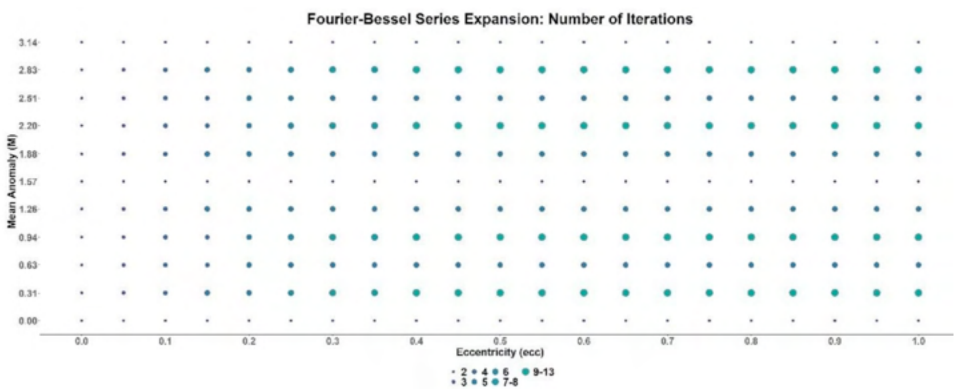

Figure 4. Number of iterations required for Fourier-Bessel Series Expansion to achieve accuracy lower than 10^{-4} rad.

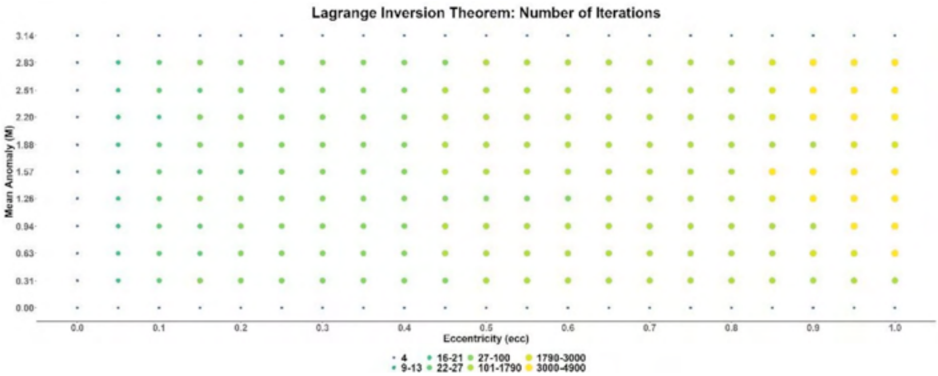

Figure 5. Number of iterations required for Lagrange Inversion Theorem to achieve accuracy lower than 10^{-4} rad.

In Tab. (2) some descriptive measures of the number of iterations are presented, both for iterative and series-based methods, even though the series-based methods have been compared separately. As the average is a measure of position sensitive to outliers, it was decided to analyze its results together with the median; the standard deviation is the measure of data dispersion around its average. In the first line of Tab. (2) are the statistics obtained for the Newton-Raphson's method (reference) and the similarity of its results with those obtained via Halley's method stands out, as already discussed.

So far, from the results presented and Tab. (2) analysis, it is possible to list the methods, identifying which ones are the most efficient in relation to the number of iterations. The iterative Halley's method showed the best results, followed by the Fourier-Bessel series expansion and, later, the iterative Regula-Falsi method. These three methods presented close mean and median values and a low standard deviation value, which indicates that there are no outliers, that is, for all the values of eccentricity and mean anomaly used in this work, there was not a large number of iterations necessary for convergence. The methods that presented less satisfactory results were the iterative Successive Approximations methods and, finall , the Lagrange Inversion Theorem. Both presented mean values much greater than their respective medians, in addition to a high standard deviation value. These results indicate the presence of outliers in the data, that is, for certain values of eccentricity and average anomaly, these methods required an anomalous number of iterations, quite different from the usual one.

Number of Iterations	Mean	Standard deviation	Median
Newton-Raphson	3.446	1.921	4
Halley	2.931	1.608	4
Regula-Falsi	9.883	6.025	8
Successive Approx.	33.468	125.631	13
Fourier-Bessel	5.152	2.926	5
Lagrange Theorem	617.420	1276.703	81

Table 2. Statistics of number of iterations analysis.

In order to improve the analysis and to measure the impact of the findings related to the number of iterations in the processing time, additional results are presented in the next subsection

3.2 Processing Time Assessment

Iterative Methods

The processing time was measured posteriori and, therefore, involves the actual execution time of the algorithms. It depends on factors related to the machine, the programming language used and, sometimes, it is a function of additional aspects of a particular input (Linder, 2021).

Figure (6) shows the measured processing time for the reference method, Newton-Raphson, over the entire grid of 231 possible inputs. It is possible to perceive that one of the limit regions, in which the orbits are circular or quasi circular ($e \to 0$), has the longest processing time, for any value of $\mathcal{M}$. Thus, as the algorithm is the same for all initial guesses, the method is sensitive to this entry of eccentricity. For the other points, the processing time starts to increase if $\frac{3\pi}{4} < \mathcal{M} < \pi$, $\forall e \in (0,1]$. All the other iterative methods studied here showed shorter processing time in this specific situation

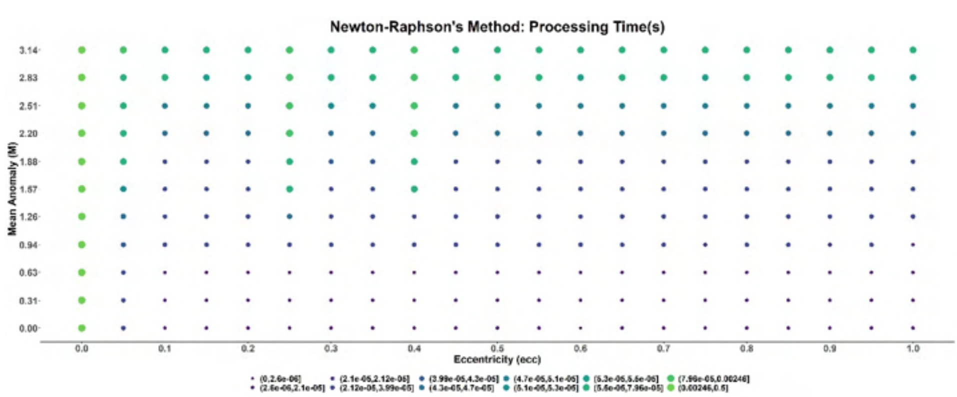

Figure 6. Processing time for Newton-Raphson's Method (iterative reference).

Figure (7) presents the processing time required by the method of Successive Approximations, taken into account the 231 pairs of . It is noticeable that the processing time in the limit region where is higher, a behavior that is also observed in Fig. (6). As for the other points, the longest processing times occurred , when:

- 1.88 < $\mathcal{M} < 3.14$ for the Successive Approximations Method.
- 2.51 < $\mathcal{M} < 3.14$ for Regula-Falsi Method.
- 1.88 < $\mathcal{M} < 3.14$ for Halley's Method.

Based on these results, it is concluded that particular aspects of the input values need to be better analyzed, which is beyond the objective of this study.

As a behavior very similar to that shown in Fig. (7) occurred for the methods of Halley and Regula-Falsi, the respective graphs will be omitted and more results related to these methods will be discussed in the statistical analysis of Tab. (3).

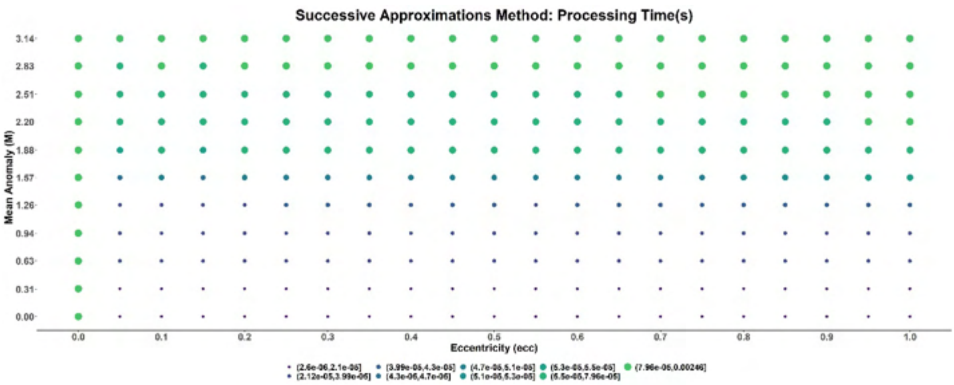

Figure 7. Processing time for Successive Approximations Method.

Series-Based Methods

Here, it was used the same condition applied in the number of iterations assessment: the methods whose solution is based on the series approach were analyzed separately. Figures (8) and (9) show the measured processing time of the Fourier-Bessel Series Expansion and the Lagrange Inversion Theorem, respectively. These methods called for a considerably longer processing time, which was expected, due to the nature of the approach and the number of operations that needed to be carried out to obtain the methods results. The minimum processing times are very close to the equivalent times required by the iterative methods, and the differences emerged in the maximum processing times. When analyzing Fig. (8), it is noted that the largest order of Fourier-Bessel Series Expansion processing time is similar to the reference ones, although its occurrence is greater, that is, most of the points in the grid demanded the maximum time to reach convergence. In addition, Fig. (9) shows that the maximum processing time obtained for the Lagrange Inversion Theorem is up to times greater than the Newton-Raphson’s solution equivalent time, which designates this as the least efficient method among all. Furthermore, the same problematic region, previously observed, starts to stand out (and , simultaneously), in which convergence is detected, at the cost of a higher computational burden.

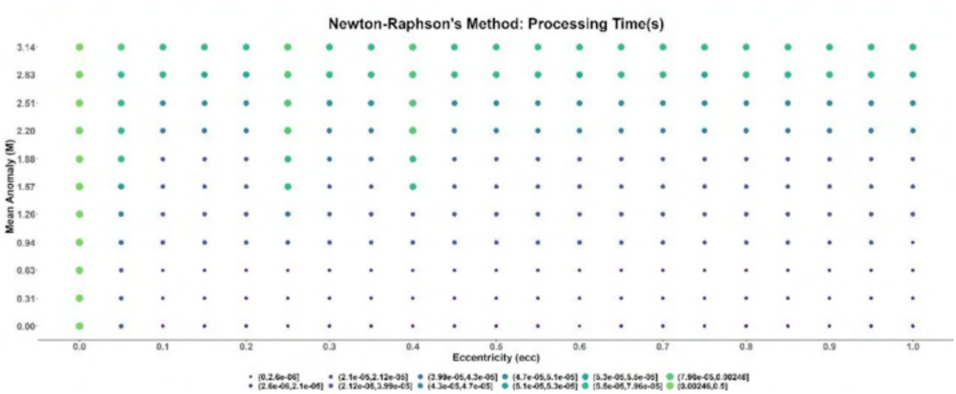

Figure 8. Processing time for Fourier-Bessel Series Expansion.

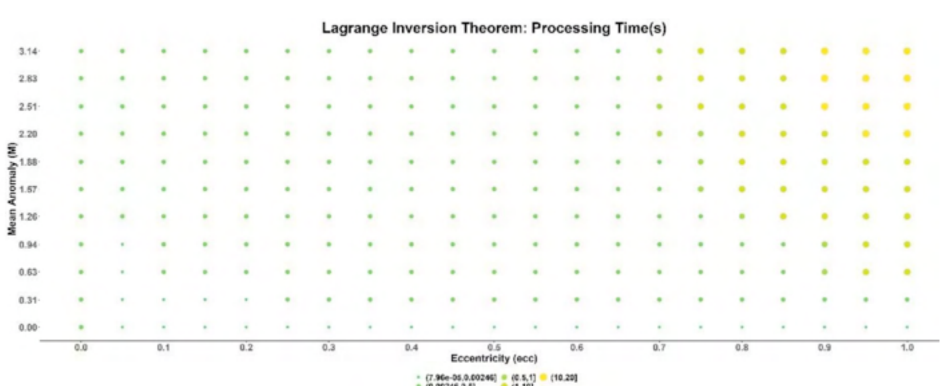

Figure 9. Processing time for Lagrange Inversion Theorem.

Table (3) contains data from the statistical analysis of the six methods, which again include mean, standard deviation and median. The first line of Tab. (3) presents the statistics obtained for the Newton-Raphson method (reference) and it is clear that, in terms of processing time, the iterative methods were competitive with each other, due to the behaviors shown in the graphs, confirmed through Tab. (3) statistics. Series-based methods were computationally more costly, which is an expected conclusion, given their approaches. Even so, Fourier-Bessel Series Expansion sustained a competitive performance, compared to the iterative methods, while Lagrange Inversion Theorem results did not, which statistically corroborated the discrepancies detected in the graphs. In theory, if more terms are added to the Lagrange Inversion Theorem series, an increase in processing time is expected (although this analysis needs to be done with more criteria).

Processing Time (s)	Mean	Standard deviation	Median
Newton-Raphson	1.65210^{-4}	5.77710^{-4}	3.78010^{-5}
Halley	1.34810^{-4}	4.26110^{-4}	3.99010^{-5}
Regula-Falsi	1.55110^{-4}	5.09510^{-4}	4.30010^{-5}
Successive Approx.	1.47510^{-4}	4.27310^{-4}	5.12010^{-5}
Fourier-Bessel	4.33910^{-3}	1.04810^{-2}	9.62010^{-4}
Lagrange Theorem	1.327	3.481	3.94010^{-2}

Table 3. Statistics of processing time analysis.

4 I CONCLUSIONS

The goal of this paper was to review an application of six methods, iterative and in series, for the solution of the Kepler's equation, over a grid of points in the elliptical motion interval. The analysis is based on the number of iterations that each method requires to achieve a stipulated accuracy and on the post-processing time (execution time). The comparison was separated according to the nature of the methods. The solution obtained using the Newton-Raphson's method worked as reference for the iterative methods.

Regarding the number of iterations, Halley's method performed as competitively as the reference solution. Regula-Falsi and Successive Approximations methods, on the other hand, were more sensitive to the variation of the eccentricity input value, in the interval (0,1]. For the latter method, even, a problematic region began to be laid out: $e \in (0.81)$ (quasi-parabolic orbits) and $\mathcal{M} \in \left(\frac{3\pi}{4}, \pi\right)$, simultaneously, where convergence still occurs, but it is very computationally costly, in terms of processing time and/or number of iterations. It implies that the method of Successive Approximations is not stable for very eccentric orbits ($e \to 1$), considering that $e = 1$ does not configure orbital elliptic motion

The processing time analysis indicated competitiveness among the four iterative methods and the Fourier-Bessel Series Expansion, observed both in the graphical and in the descriptive measures' analysis. The only method that presented results that were very distant from the others, with maximum processing time of the order of 100 times greater than the others, was the Lagrange Inversion Theorem. Consistently, it had already exhibited signs of convergence trouble in the number of iterations analysis for the same region, because the same region stood out in the analysis of the iterative Successive Approximations methods.

The difference presented by the Lagrange Inversion Theorem results for both number of iterations to reach the required accuracy and processing time indicates that a more accurate analysis needs to be made on this expansion and the impact of the number of terms in the series and the truncation order should be studied further.

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REFERENCES

Avendano, M., Martín-Molina, V., Ortigas-Galindo. J.: A new method for obtaining approximate solutions of the hyperbolic Kepler's equation. Celestial Mech Dyn Astr, 123, 435-451 (2015). https://doi.org/10.1007/s10569-015-9645-0.

Battin, R.H.: An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition. AIAA Education Series, Reston (1999).

Burkardt, T.M., Danby, J.M.A.: The solution of Kepler's equation, II. Celestial Mechanics 31, 317-328 (1983). https://doi.org/10.1007/BF01844230.

Calvo, M., Elipe, A., Montijano, J.I., Rández, L.: Optimal starters for solving the elliptic Kepler's equation. Celestial Mech Dyn Astr, 115, 143-160 (2013). https://doi.org/10.1007/s10569-012-9456-5.

Colwell, P.: Bessel Functions and Kepler's equation. The American Mathematical Monthly, vol. 99, n. 1, 1992, p. 45-48.

Condurache, D., Martinuşi, V.: A complete closed form vectorial solution to the Kepler problem. Meccanica, 42, 465-476 (2007). https://doi.org/10.1007/s11012-007-9065-7.

Danby, J.M.A.; Burkardt, T.M.: The solution of Kepler's equation, I. Celestial Mechanics 31, 95-107 (1983). https://doi.org/10.1007/BF01686811.

Davis, J.J., Mortari, D.; Bruccoleri, C.: Sequential solution to Kepler's equation. Celestial Mech Dyn Astr, 108 (1), 59-72 (2010). https://doi.org/10.1007/s10569-010-9292-4.

Franco, N.M.B.: Cálculo Numérico. Prentice Hall, 2006.

Fukushima, T.: A method solving Kepler's equation without transcendental function evaluations. Celestial Mech Dyn Astr, 66, 309-319 (1996). https://doi.org/10.1007/BF00049384.

Fukushima T.: Fast procedure solving universal Kepler's equation. Celestial Mech Dyn Astr, 75, 201-226 (1999). https://doi.org/10.1023/A:1008368820433.

Gander, W.: On Halley's iteration method. The American Mathematical Monthly, vol. 92, n. 2, 1985, p. 131-134.

Linder, M.: Noções de Complexidade de Algoritmo. Notas de Aula. Available at: http://www.univasf.edu.br/~marcelo.linder/arquivos_ed1/aulas/aula21.pdf. Accessed on Feb. 21st 2021.

Markley, F.L.: Kepler equation solver. Celestial Mech Dyn Astr, 63, 101-111 (1995). https://doi.org/10.1007/BF00691917.

Mikkola, S.: A cubic approximation for Kepler's equation. Celestial Mechanics, vol. 40, n. 3-4, 1987, p. 329-334. https://doi.org/10.1007/BF01235850.

Ng, E.W.: A general algorithm for the solution of Kepler's equation for elliptic orbits. Celestial Mechanics (20) 1979, p. 243-249. https://doi.org/10.1007/BF01371365.

Nijenhuis, A.: Solving Kepler's equation with high efficiency and accurac . Celestial Mech Dyn Astr, 51, 319-330 (1991). https://doi.org/10.1007/BF00052925.

Odell, A.W.; Gooding, R.H.: Procedures for solving Kepler's equation. Celestial Mechanics, vol. 38, n. 4, 1986, p. 307-334. https://doi.org/10.1007/BF01238923.

Reza, E., Ghadiri, H.: Appropriate starter for solving the Kepler's equation. International Journal of Computer Applications, 89 (7), 31-38, 2014.

Serafin, R.A.: Bounds on the solution to Keple 's equation. Celestial Mechanics (38) 1986, p. 111-121. https://doi.org/10.1007/BF01230424.

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