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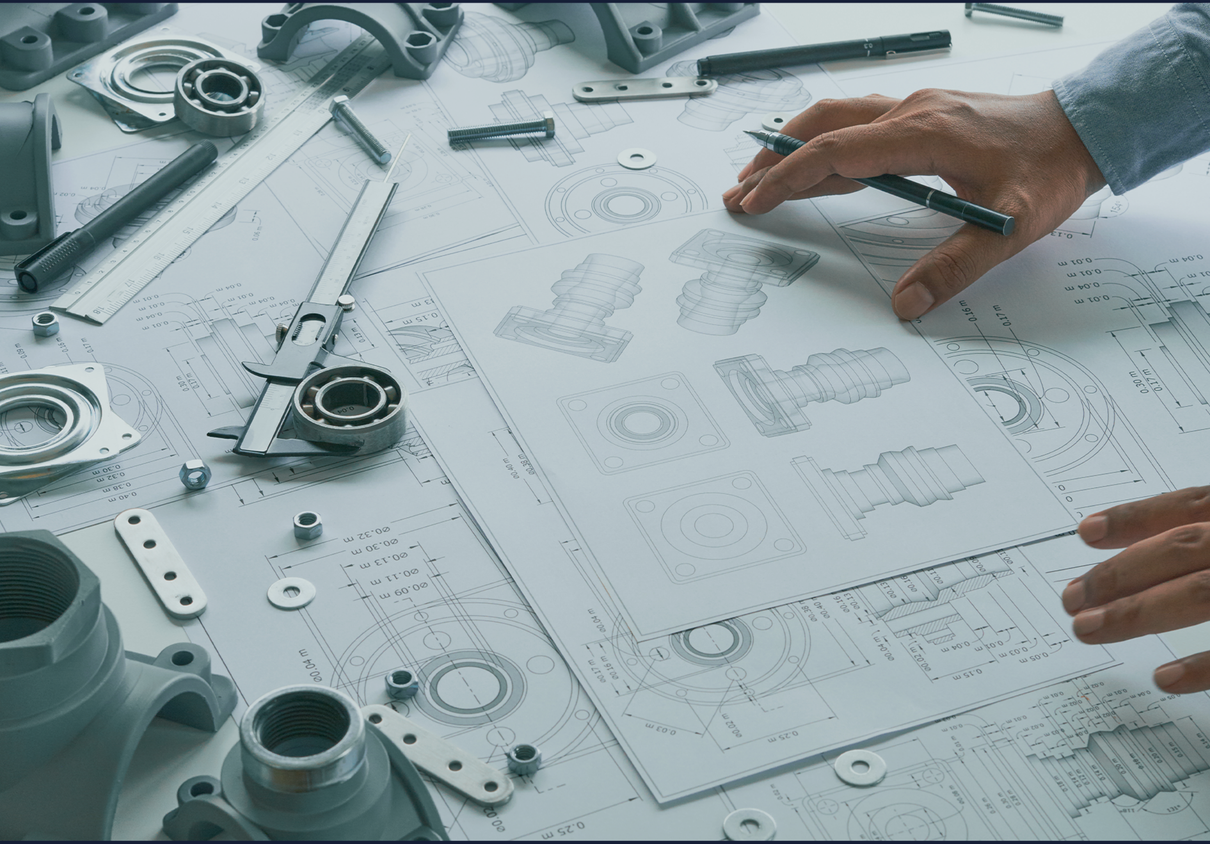

HENRIQUE AJUZ HOLZMANN
JOÃO DALLAMUTA
(ORGANIZADORES)

Atena Editora
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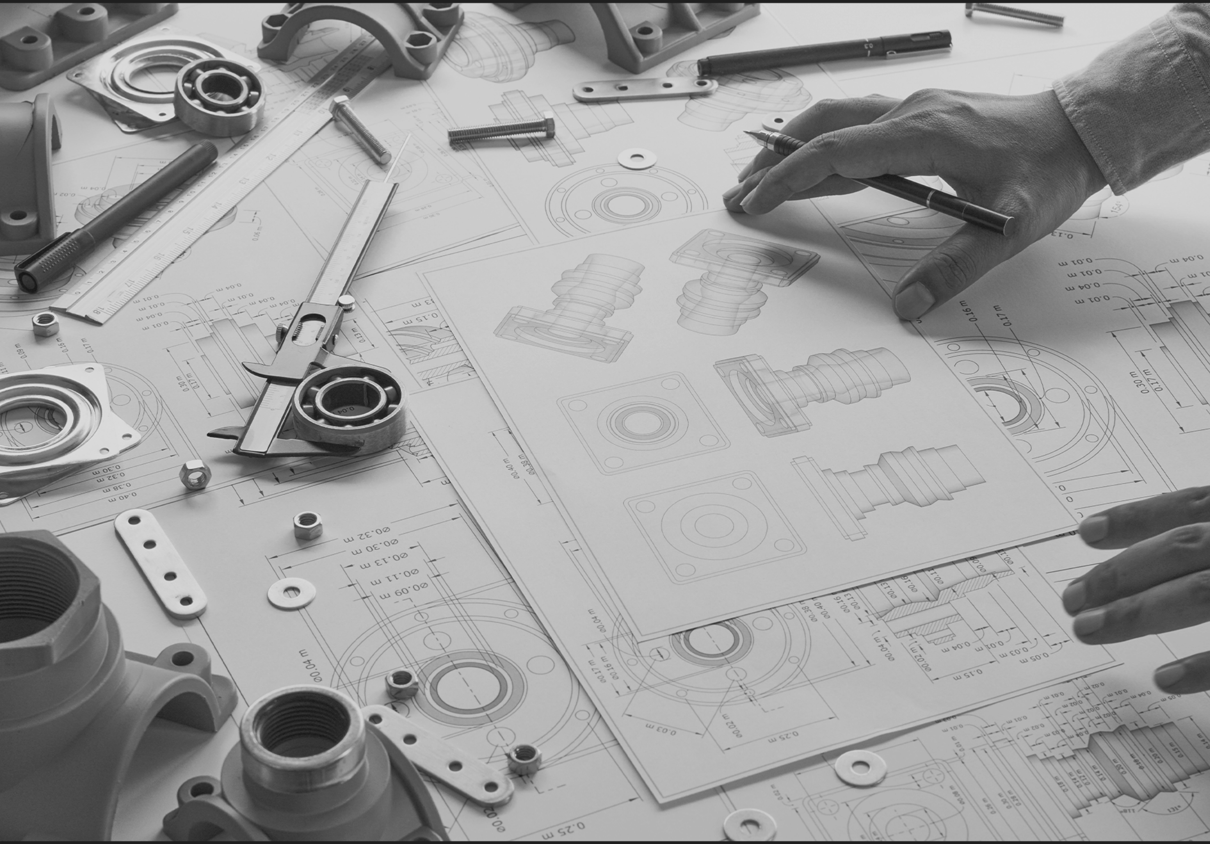

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APRESENTAÇÃO

Em um cenário cada vez mais competitivo, desenvolver novas maneiras de melhoria nos processos industriais, bem como para o próprio dia a dia da população é uma das buscas constantes das áreas de engenharia.

Desta forma buscar evitar ou prever falhas em sistemas é de vital importância, destacando-se o desenvolvimento de novos materiais, bem como de métodos analíticos e práticos para detecção. Entre os materiais os compósitos veem ganhado cada vez mais espaço devido a sua versatilidade, aliando resistência e peso.

Já para detecção de falhas os métodos de análise de vibrações é quase que unanimidade quando se quer um pleno funcionamento dos equipamentos. O estudo das análises de vibrações em sistemas vem ganhando cada vez mais espaço nos projetos, pois a redução dessas na maioria dos casos acarreta em uma maior vida útil ou um melhor funcionamento dos conjuntos.

Neste livro são apresentados trabalhos relacionados a engenharia mecânica, dentro de uma vertente teórico/prática onde busca-se retratar assuntos atuais e de grande importância para estudante, docentes e profissionais.

Boa leitura!

Henrique Ajuz Holzmann

João Dallamuta

SUMÁRIO

CAPÍTULO 8

A GENERALIZED INTEGRAL TRANSFORMED TECHNIQUE: LITERATURE REVIEW AND COMPARATIVE RESULTS WITH FINITE VOLUME METHOD

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ABSTRACT: This work aims to carry out a brief review of the literature on the use of the Generalized Integral Transform Technique (GITT) in diffusion-advection problems in engineering and to analyze two works that compare the solution of a problem via GITT and the Finite Element Method (FVM). A bibliographic survey was carried out in the Google Scholar and Science Direct databases, in conference proceedings and periodicals. During the bibliographic survey phase, care was taken to research whether there was any work in the literature like the one proposed in this article, but it was not found. GITT's step by step will be presented, through the mathematical development of an engineering problem present in one of the selected articles. A brief review of the literature on the technique over the past 25 years, with a summary of 24 papers, shows the range of applications of the technique in different areas. After reviewing the literature, comparative results are presented between the solutions via GITT and FVM, two studies analyzed, showing the effectiveness of GITT and a higher rate of convergence. It is concluded that GITT has been shown to be a powerful tool in solving diffusion-advection problems, in addition to providing low computational cost, when compared to FVM.

KEYWORDS: Generalized Integral Transform Technique, Diffusion-advection, Convergence Rate, Literature Review.

1 I INTRODUCTION

The use of GITT in diffusion-advection problems in engineering has grown in importance over the last two decades, it is understood that there is a gap in the literature in presenting a work that, in a clear and objective way, makes a compilation of works that demonstrate the importance and effectiveness of this hybrid analytical-numerical methodology for resolving EDP. Thus, this work was organized to initially search the literature for a GITT review paper, which was not found. Then, a search was made for jobs in the engineering area that used GITT to solve the problem addressed. After a careful choice of works, which will be detailed in the methodology used, two works were chosen to show the development of GITT and to analyze the solutions obtained through GITT and FVM. After the Methods section, a summary of twenty-five articles is presented, whose solutions to the engineering problems addressed were implemented via GITT. To compare the techniques of GITT and FVM, in the results section, the data obtained in the two studies whose techniques were compared are presented. The main contribution of this article is to provide a summary of a collection of works, in different areas of engineering, whose solutions were implemented via GITT, demonstrating its importance in the analytical-numerical solution of applications in diffusion-advection problems. Corroborating this contribution, the results of two comparative studies between GITT and FVM are demonstrated, in which the solution via GITT is in line with the results found in the literature.

2 | METHODS

The main objective of this work is to provide the academic engineering community with a brief compendium on the application of GITT in the solution of diffusion-convection problems, mainly in the areas of Fluid Dynamics and Heat and Mass Transfer. As a way of showing the effectiveness of GITT, two papers are presented that compare the solutions to their problems via GITT and FVM. To fulfill the objective, a rigorous bibliographic survey on the topic was carried out, as shown below.

2.1 Literature Review

The bibliographical survey was carried out in the Science Direct and Google Scholar databases, in the period between 1993 and 2019. Articles that did not meet this period were excluded. The keyword used in the research was “Generalized Integral Transform Technique”. In the Science Direct database, 253 results were found and in Google Scholar 220 results. The first exclusion criterion was the year of publication of the article, as already mentioned. The second criterion was the preference for works related to mechanical engineering. However, articles in the fields of environmental engineering and oil were also included. Globally, the problems addressed are related to the areas of fluid mechanics, heat transfer, dispersion of pollutants and contaminants and flow in porous media (petroleum reservoir). At the end, twenty-nine papers were selected, with nineteen articles published in journals, two master’s dissertations and eight papers published in conference proceedings.

2.2 The Generalized Integral Transformed Technique

Generalized Integral Transform Technique (GITT) is a hybrid analytical-numerical technique for solving Partial Differential Equations (PDE) applied to convective-diffusive problems and is an extension of the classic transform, where it is used in purely diffusive problems of heat and mass, according to Cotta [9]. The technique of the classical integral transform, in general, transforms the PDE into an infinite system decoupled from Ordinary Differential Equations (ODE) that can be easily solved. However, when the classical approach is applied to convective-diffusive problems, the infinite system of transformed ODE is coupled, making the classical integral transform not to be used for this class of problems.

Miyagawa [18] mentions in his work that GITT differs from other numerical methods used to solve strongly non-linear and coupled problems, because there is no need to discretize the domain for meshing, which requires a lower computational cost. In the same work, the author summarized the steps for the application of GITT, as shown below.

i. Definition of the auxiliary problem, based, for example, on the diffusive terms of the original formulation.

ii. Solving the auxiliary problem and obtaining the self-functions, eigenvalues, norms, and orthogonality properties.

iii. Development of the transformed-inverse pair.

iv. Integral transformation of the partial differential problem into a system of coupled algebraic or ordinary differential equations or yet another partial differential equation.

v. Truncation of the infinite system and numerical solution of the resulting differential system to obtain the transformed potentials.

vi. Obtaining the original potential, using the inversion formula.

2.3 Comparative Results GITT x FVM

Next, an analysis will be made of two studies in which the central problem was solved via GITT and the Finite Volume Method (FVM). One of the works used a commercial CFD software called FLUENT, which is based on FVM. In this part of the work, the mathematical development of the models used by the authors will be presented, as well as the results obtained by the applied methodologies, verifying agreement with the model validated in the literature and comparing the convergence rates.

2.3.1 Immiscible displacement of oil by water injection in the reservoir - Petroleum Engineering (Dias et al. [13])

The first work used to compare GITT with FVM (using the CFD Fluent computational package) is by Dias et al. [13], which deals with the problem of water injection in oil reservoirs (porous medium) to promote immiscible displacement of the oil, allowing the reservoir's recovery factor to increase, and maintaining its static pressure. The work uses two formulations used in reservoir engineering, the differential saturation equation, obtained through the application of Darcy's Law and the continuity equation for each phase, and the mixture theory equation applied to fluid flow in porous media. GITT is used to solve the saturation equation, while the solution of the mixing theory is reached numerically using the computational code Fluent, whose basis is the Finite Volume Method. Only the development of the mathematical formulation related to GITT will be presented and, however the results of the formulation via FVM (mixing theory) will be shown for comparison purposes.

The following nomenclature was used by Dias et al. [13]:

x	Coordinates
S_W, S_o	Water and oil saturations
$\boldsymbol{v}_{sw}$, $\boldsymbol{v}_{so}$	Water and oil superficial velocity
$\boldsymbol{v}_{st}$	Total superficial velocity
K	Absolute permeability
k_w, k_o	Water and oil relative permeability
p_w, p_o	Water and oil mass densities
μ_w, μ_o	Water and oil viscosities
p_w, p_o	Water and oil pressures
f_w	Fractional flow function
Φ	Porosity
$\boldsymbol{v}_i$	Constituent velocity vector
S_F	Filter function
S_H	Filtered potential
S_{wi}	Connate water saturation
S_{wo}	Residual oil saturation
H	Diffusion parameter
F(x)	Source term of the equation
Ψ, ħ	Eigenfunctions and eigenvalues
N	Norm
a, b	Linearization coefficients
A, *B*, *C*, *D*	Transformed equation coefficients

2.3.1.1. Problem Formulation

a) Differential Saturation Equation

Darcy's Law for multiphase flow in the porous medium is given, for each phase and disregarding the gravitational effects, by equation 1.

$$\boldsymbol{v}_{sw} = -\frac{K\, k_w}{\mu_w}\nabla p_w; \qquad \boldsymbol{v}_{so} = -\frac{K\, k_o}{\mu_o}\nabla p_o \tag{1}$$

here v_{sw}, and v_{so} are the superficial velocities for the water and oil, respectively, μ_w and μ_o are the respective viscosities. k_w and k_o are the relative permeabilities for flow for each of the two fluids and K is the absolute permeability of the medium.

Conservation of the mass for the water and oil phases in the porous medium is given by equation 2.

$$\frac{\partial(\phi\rho_w S_w)}{\partial t} + \nabla\cdot(\rho_w \boldsymbol{v}_{sw}) = 0; \quad \frac{\partial(\phi\rho_o S_o)}{\partial t} + \nabla\cdot(\rho_o v_{so}) = 0 \tag{2}$$

here S_w and S_o are the water and oil saturations, p_w and p_o are water and oil mass densities.

Equation 3 is the differential saturation equation.

$$\boldsymbol{\phi}\frac{\partial \boldsymbol{S}_w(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}} + \boldsymbol{\nabla}\cdot\left(\boldsymbol{v}_{st}\boldsymbol{f}_w(\boldsymbol{S}_w)\right) = \boldsymbol{0} \tag{3}$$

where v_{st} is the total speed, as equation 4.

$$v_{st} = v_{sw} + v_{so} \tag{4}$$

The one-dimensional differential saturation equation as in equation 5 is called the Buckley-Leverett equation, which is a non-linear hyperbolic transport equation. The Buckley-Leverett Equation represents a simple mathematical model for simulating fluid displacement in porous media, according to Araújo and Márquez [3].

$$\boldsymbol{\phi}\frac{\partial \boldsymbol{S}_w(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{t}} + \boldsymbol{v}_{st}\frac{d\boldsymbol{f}_w(\boldsymbol{S}_w)}{d\boldsymbol{S}_w}\frac{\partial \boldsymbol{S}_w(\boldsymbol{x},\boldsymbol{t})}{\partial \boldsymbol{x}} = \boldsymbol{0}; \tag{5}$$

b) Relative Permeability

The relative permeability curve is responsible for the interaction between the water/oil phases. Then, the following relative permeability curves were considered, as seen in equation 6. S_{ro} is the oil residual saturation. K_{romax} and K_{rwmax} are the maximum relative permeability to oil and water, respectively.

$$k_w(S_w) = \frac{K_{rwmax}(S_w - S_{wi})^{nw}}{(1 - S_{or} - S_{wi})}, \qquad k_o(S_w) = \frac{K_{romax}(S_{or} - S_w)^{no}}{(1 - S_{or} - S_{wi})} \tag{6}$$

Another important function, as seen in equation 7, is the fractional flow, responsible for the non-linearity of the Buckley-Leverett equation.

$$f_w(S_w) = \frac{\dfrac{k_w(S_w)}{\mu_w}}{\dfrac{k_w(S_w)}{\mu_w} + \dfrac{k_o(S_w)}{\mu_o}} \tag{7}$$

The solutions were selected for linear and non-linear relative permeability curves. The linear and non-linear permeability curves are shown below.

The relative permeability curves and the fractional flow function for linear case is showed in Fig. 1.

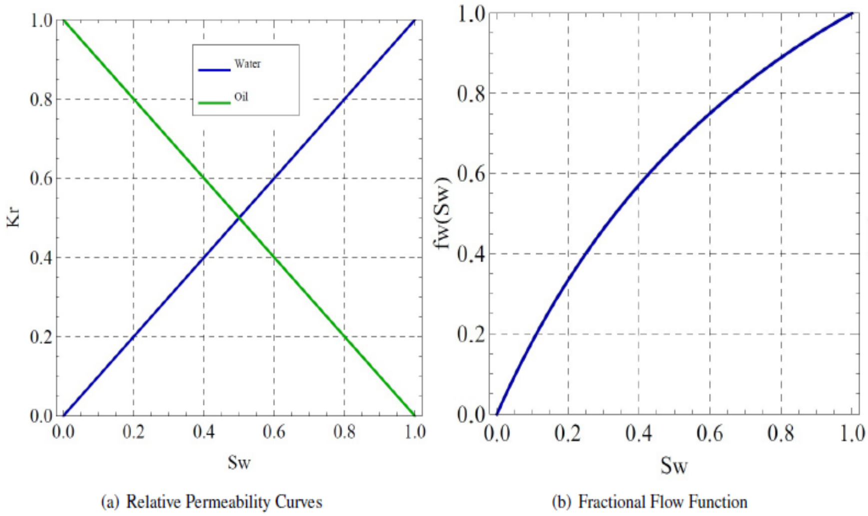


Figure 1. Linear Permeability Curves.

The relative permeability curves and the fractional flow function for nonlinear case is showed in Fig. 2. This case of relative permeability curves is more common in reservoir simulation.

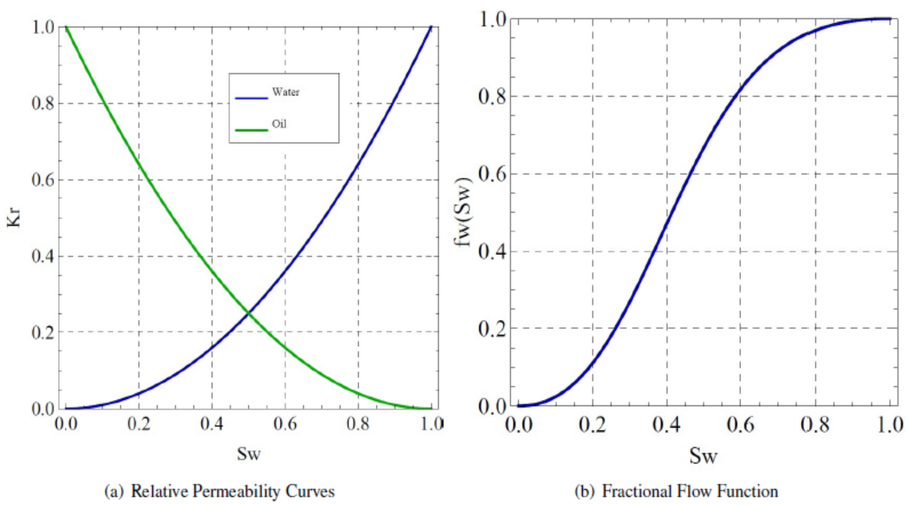


Figure 2. Non-Linear Permeability Curves.

2.3.1.2. Integral Transform Solution

To proceed with the integral transformation and obtain an expansion of eigenfunction, a diffusion term is introduced to the Buckley-Leverett equation, in which the diffusive parameter, *H*, will be considered as having a large value, as seen equations in 8 and 9.

$$\phi \frac{\partial S_w(x,t)}{\partial t} + v_{st} \frac{df_w(S_w)}{dS_w} \frac{\partial S_w(x,t)}{\partial x} - \frac{1}{H} \frac{\partial^2 S_w(x,t)}{\partial x^2} = 0; \quad (8)$$

$$S_w(0,t) = 1 - S_{or},\ S_w(1,t) = S_{wirr},\ S_w(x,0) = S_{wirr} \quad (9)$$

To eliminate non-homogeneous boundary conditions, a filter function is introduced (equations 10).

$$S_w(x,t) = S_H(x,t) + S_F(x) \text{ wich } S_F(x) = (S_{wirr} - S_{wo})x + S_{wo} \quad (10)$$

Applying equation 10 to equations 8 and 9, we obtain equation 11.

$$\frac{\partial S_H}{\partial t} + f_w'(S_w)\left(\frac{\partial S_H}{\partial x} + \frac{\partial S_F}{\partial x}\right) - \frac{1}{H}\frac{\partial^2 S_H}{\partial x^2} = \frac{1}{H}\frac{\partial^2 S_F}{\partial x^2}; \quad (11)$$

$$S_w(0,t) = 0 - S_H(1,t) = S_H(x,0) = S_{wirr} - S_F(x) \quad (12)$$

The eigenfunction expansion solution is based the following transformed-inverse pair:

$$\text{Inversion} \longrightarrow S_H(x,t) = \sum_{k=1}^{\infty} \frac{\bar{S}_{Hk}(t)\Psi_k(x)}{N_k}, \quad (13)$$

$$\text{Transform} \longrightarrow \overline{S}_{Hk}(x,t) = \int_0^1 S_H(x,t)\,\Psi_k(x)\mathrm{d}x \quad (14)$$

Where $\Psi_k(x)$ are orthogonal solutions of a Sturm-Liouville eigenvalue problem. Dias et al. [13] chose the one-dimensional Helmholtz problem as an auxiliary problem, according to equation 15.

$$\Psi''(x) + \lambda^2\,\Psi(x) = 0 \qquad \Psi(0) = 0, \qquad \Psi(1) = 0 \quad (15)$$

The norms of the eigenfunction, $\Psi_k(x)$, are given by:

$$N_k = \int_0^1 \Psi_k^2(x)dx \quad (16)$$

To obtain the transformed system, the transformation operator (equation 14) is applied in equation 11. After the development of the transformation, we arrive at the transformed system represented by equation 17.

$$\frac{d\bar{S}_{Hk}}{dt} + \frac{\lambda_k^2}{H}\,\bar{S}_{Hk} - \bar{F}_k = -\int_0^1 \Psi_k\, f_w'\, \frac{\partial S_F}{\partial x}\mathrm{d}x - \int_0^1 \Psi_k\, f_w'\, \frac{\partial S_H}{\partial x}\mathrm{d}x \tag{17}$$

To transform the 2 terms on the right side of the equation, a linear transformation of the function f'_w is used in the subdomain ($x_{q-1}<x<x_q$), so that we got equation 18.

$$\int_0^1 f_w'\, g(S_H, x)\, dx = \sum_{q=1}^{q_{max}} \int_{x_{q-1}}^{x_q} \left[a_q(t)\, x + b_q(t)\right] g(S_H, x)\, \mathrm{d}x \tag{18}$$

Where the coefficients of the linearization are:

$$a_q(t) = \frac{f'_{w_q} - f'_{w_{q-1}}}{x_q - x_{q-1}}, \qquad b_q(t) = \frac{x_q\, f'_{w_{q-1}} - x_{q-1}\, f'_{w_q}}{x_q - x_{q-1}} \tag{19}$$

Applying the linearization (equation 18) in equation 17, we obtain the transformed system represented in equation 20.

$$\begin{aligned}\frac{d\bar{S}_{Hk}}{dt} + \frac{\lambda_k^2}{H}\,\bar{S}_{Hk} - \bar{F}_k &= -\sum_{q=1}^{q_{max}} (f'_{w_q} - f'_{w_{q-1}})\, A_{qk} - \sum_{q=1}^{q_{max}} (x_q\, f'_{w_{q-1}} - x_{q-1}\, f'_{w_q})\, B_{qk} \\ &\quad - \sum_{q=1}^{q_{max}} \sum_{i=1}^{\infty} (f'_{w_q} - f'_{w_{q-1}})\, C_{iqk}\ \bar{S}_{Hi} \\ &\quad - \sum_{q=1}^{q_{max}} \sum_{i=1}^{\infty} (x_q\, f'_{w_{q-1}} - x_{q-1}\, f'_{w_q})\, D_{iqk}\ \bar{S}_{Hi}\end{aligned} \tag{20}$$

Where

$$f'_{w_q} = f'_{w_q}(S_w, x_q) = f'_{w_q}(S_H + S_F, x_q) = f'_{w_q}\left(\sum_{k=1}^{\infty} \frac{\bar{S}_{Hk}\Psi_k}{N_k} + S_F, x_q\right) \tag{21}$$

$$f'_{w_{q-1}} = f'_{w_{q-1}}(S_w, x_{q-1}) = f'_{w_{q-1}}(S_H + S_F, x_{q-1}) = f'_{w_{q-1}}\left(\sum_{k=1}^{\infty} \frac{\bar{S}_{Hk}\Psi_k}{N_k} + S_F, x_{q-1}\right) \tag{22}$$

$$A_{qk} = \frac{1}{x_q - x_{q-1}} \int_{x_{q-1}}^{x_q} x\, \frac{d\bar{S}_F}{dx} \Psi_k\, \mathrm{d}x, \quad B_{qk} = \frac{1}{x_q - x_{q-1}} \int_{x_{q-1}}^{x_q} \frac{d\bar{S}_F}{dx} \Psi_k\, \mathrm{d}x \tag{23}$$

$$C_{iqk} = \frac{1}{N_i(x_q - x_{q-1})} \int_{x_{q-1}}^{x_q} x\, \Psi_k \Psi_i'\, \mathrm{d}x, \quad C_{iqk} = \frac{1}{N_i(x_q - x_{q-1})} \int_{x_{q-1}}^{x_q} \Psi_k\, \Psi_i'\, \mathrm{d}x \tag{24}$$

The transformation of the initial condition leads to:

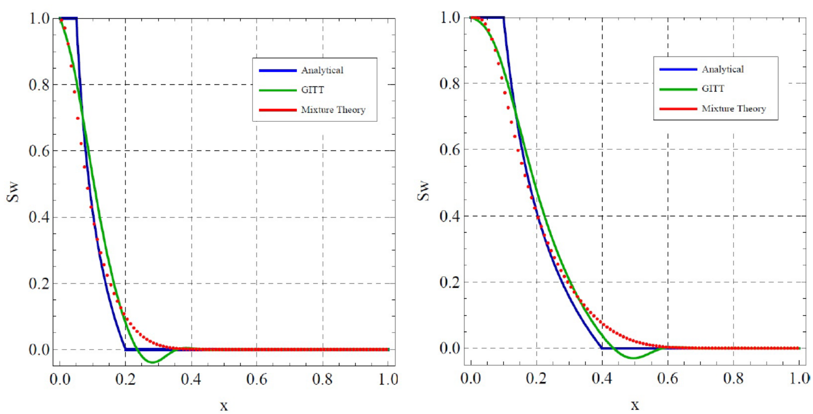
$$\bar{S}_{Hk}(0) = \int_0^1 (S_{wirr} - S_F)\, \Psi_k(x)\mathrm{d}x \tag{25}$$

The solution of the transformed potentials is obtained through the solution of the infinite ODE system, equation 20. Dias et al. [13] used the Mathematica software, using the NDSolve routine, and to arrive at the solution of the original potential, the inversion equation, equation 13, is used.

2.3.1.3. Comparative results

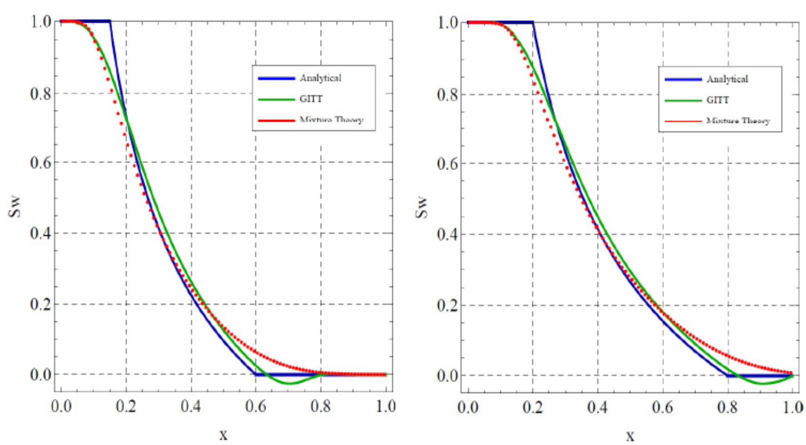
a) Results for Linear Permeability Curve

Figures 3 and 4 show comparisons of the results obtained by GITT, Fluent and the analytical solution.



(a) Dimensionless Time = 0.1 (b) Dimensionless Time = 0.2

Figure 3. Results for the Linear Permeability Curve.



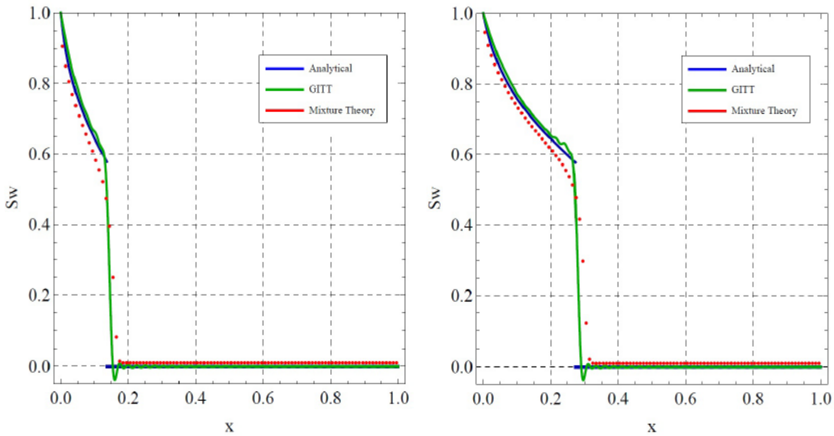
(a) Dimensionless Time = 0.3 (b) Dimensionless Time = 0.4

Figure 4. Results for the Linear Permeability Curve.

It can be seen from the above comparative curves that the numerical solutions converge to the analytical solution, despite some small detachments at certain points and regions of the analysis.

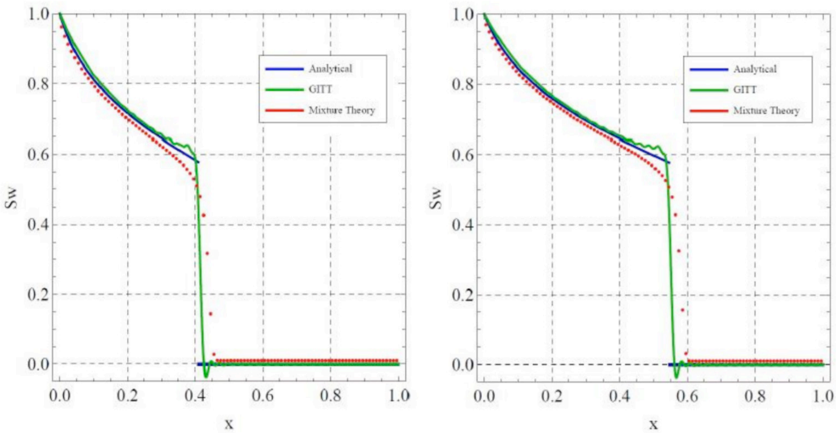
b) Results for Nonlinear Permeability Curve

Figures 5 and 6 show comparisons of the results obtained by GITT, Fluent and the analytical solution.



(a) Dimensionless Time = 0.1 (b) Dimensionless Time = 0.2

Figure 5. Results for the Nonlinear Permeability Curve.



(a) Dimensionless Time = 0.3 (b) Dimensionless Time = 0.4

Figure 6. Results for the Nonlinear Permeability Curve.

The behavior of the simulated curves is well adjusted to the analytical solution, as well as to the linear permeability curve. In the work in question, the results of GITT and the Fluent code were very well adjusted to the analytical solution, showing the good accuracy of the methodologies studied.

2.3.2 Laminar Flow between Parallel Flat Plates in Thermal Development

Queiroz et al. [25] compared two solution methodologies for convection-diffusion problems: The Generalized Integral Transform Technique and the Finite Volume Method. A problem was selected to illustrate the two approaches, laminar flow in thermal development between parallel plates. The present study evaluated the effect of the variation in the number of Péclet, based on the transverse length, on the convergence of the two techniques. The flow was considered hydrodynamically developed, with the Hagen-Poiseuille velocity profile, however, results for simplified slug-flow cases were also presented. The advantages and disadvantages of each method are presented, where the results indicate that, in general, the Integral Transform Technique presents a better convergence rate.

The following nomenclature was used by Queiroz et al. [25]:

x, y	Cartesian coordinates
N	norm of eigenfunctions
Nu_H	Nusselt Number
Pe_H	Péclet Number
T_s	surface temperature
T_0	entrance temperature
T_m	Bulk or average mixing temperature
u	velocity component in x-direction
$\overline{u}$	average velocity in x-direction
Y_n	eigenfunctions
α	thermal diffusivity
ξ, η	dimensionless coordinates
ϕ	arbitrary function
λ_n	eigenvalues
θ	dimensionless temperature
$\overline{\theta}$	transformed dimensionless temperature
ξ_{max}	dimensionless channel length

2.3.3 Problem formulation

As in the previous work, mathematical model of the studied phenomenon and the mathematical development of the GITT will also be shown. Equation 26 is the government equation with its respective boundary conditions (equation 27).

$$\frac{1}{2}u^*\frac{\partial\theta}{\partial\xi} = Pe_H^{-2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial\eta^2}, \qquad 0 \le \xi \le \infty \quad e \quad 0 \le \eta \le 1 \tag{26}$$

$$\theta(\xi, 1) = 0, \quad \left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = 0, \quad \theta(0,\eta) = 1, \quad \left(\frac{\partial\theta}{\partial\xi}\right)_{\xi\to\infty} = 0, \tag{27}$$

The dimensionless quantities are given by equations 28:

$$\theta = \frac{T - T_0}{T_{in} - T_0}, \quad \eta = \frac{y}{H/2}, \quad \xi = \frac{x}{L}, \quad \mathrm{Pe}_H = \bar{u}\frac{H}{\alpha}, \tag{28}$$

The dimensionless speed profile is given by the Hagen-Poiseuille profile, as equation 21.

$$u^* = \frac{u}{\bar{u}} = \frac{3}{2}(1 - \eta^2) \tag{29}$$

The Nusselt number in terms of the dimensionless variables is given by equation 30.

$$Nu_{D_H} = \frac{-4\,(\partial\theta/\partial\eta)_{\eta=1}}{\int_0^1 u^*\theta \mathrm{d}\eta} \tag{30}$$

The solution of the problem is started by defining the transformation pair (equations 31 and 32):

Inversion ⟶ $$\theta(\xi,\eta) = \sum_{n=1}^{\infty}\frac{\bar{\theta}_n(\xi)\mathrm{Y}_n(\eta)}{N(\lambda_n)}, \tag{31}$$

Transform ⟶ $$\bar{\theta}_n(\xi) = \int_0^1 \theta(\xi,\eta)\,\mathrm{Y}_n(\eta)\mathrm{d}\eta \tag{32}$$

where Y_n are orthogonal solutions to a Sturm-Liouville eigenvalue problem. For the convection-diffusion problem considered in the work of Queiroz et al. [25], the following eigenvalue problem is selected (equations 33 and 34):

$$Y''_n(\eta)+\lambda^2_n Y_n(\eta) = 0, \text{ for } 0\le \eta \le 1,\ Y'(0) = 0,\ Y(1) = 0 \tag{33}$$

The norms of the Y_n are given by:

$$N(\lambda_n) = \int_0^1 Y_n^2(\eta)\, d\eta \tag{34}$$

The transformation of the given problem is accomplished by multiplying equation 26 by Y_n, integrating within $0 \le \eta \le 1$, and applying the inversion formula (31) to the non-transformable terms. This process yields (equations 35 and 36):

$$\mathrm{Pe}_H^{-2}\,\overline{\theta}''_n(\xi) - \frac{1}{2}\sum_{m=1}^{\infty} A_{n,m}\bar{\theta}'_m(\xi) - \lambda_n^2\bar{\theta}_n(\xi) = 0, \tag{35}$$

with the boundary conditions

$$\bar{\theta}_n(0) = b_n = \int_0^1 Y_n(\eta)\mathrm{d}\eta \qquad and \qquad \lim_{\xi\to\infty} \bar{\theta}_n'(\xi) = 0, \tag{36}$$

where the $A_{n,m}$ coefficients are given by equation 37:

$$A_{n,m} = \frac{1}{N(\lambda_m)} \int_0^1 u^*(\eta)\, Y_n(\eta)\, Y_m(\eta)\, \mathrm{d}\eta \tag{37}$$

For the flow situation with Hagen-Poiseuille velocity profile, equations 35 and 36 can be written in the following matrix form (equations 38 and 39):

$$\bar{\boldsymbol{\theta}}''(\xi) - \boldsymbol{B}\bar{\boldsymbol{\theta}}'(\xi) - \boldsymbol{D}\bar{\boldsymbol{\theta}}(\xi) = \boldsymbol{0}, \qquad \bar{\boldsymbol{\theta}}(0) = \boldsymbol{b}, \quad \bar{\boldsymbol{\theta}}'(\xi_{max}) = \boldsymbol{0} \tag{38}$$

in which the coefficients of ***b*** are given by equation 38 and matrices ***B*** and ***D*** are given by equations 39:

$$B_{n,m} = \frac{1}{2}\mathrm{Pe}_H^2\, A_{n,m}, \qquad D_{n,n} = \mathrm{Pe}_H^2\, \lambda_n^2\, \delta_{n,m}, \tag{39}$$

Where $\delta_{n,m}$ is the Kronecker delta. The system above (equation 38) can be converted to a first order initial-value problem if the boundary condition at δ_{max} is replaced by an initial condition and a new variable is introduced (equation 40 and 41):

$$\bar{\theta}'(0) = p, \qquad \bar{\theta}'(\xi) = \bar{\phi}(\xi), \tag{40}$$

yielding

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\begin{Bmatrix}\bar{\phi}\\ \bar{\bar{\theta}}\end{Bmatrix} = \begin{pmatrix}\boldsymbol{B} & \boldsymbol{D}\\ \boldsymbol{I} & \boldsymbol{0}\end{pmatrix}\begin{Bmatrix}\bar{\phi}\\ \bar{\bar{\theta}}\end{Bmatrix}, \tag{41}$$

Where ***I*** is the identity matrix and **0** is a zero matrix.

2.3.4 Comparative results

The comparative results are presented in terms of the Nusselt Number, varying the number of Péclet and the position from the channel entrance.

		Pe$_H = 10$						Pe$_H = 1$			
$n_{\max}$	WP	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$	$\xi = 1$	$n_{\max}$	WP	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$	$\xi = 1$
5	100	37.6242	23.5100	8.15700	7.74044	5	100	39.7508	37.5555	23.1993	8.45041
10	200	69.5288	27.7774	8.14983	7.73986	10	100	78.8666	69.6035	27.9173	8.45012
20	300	120.003	28.7805	8.14897	7.73982	20	100	155.286	120.397	29.0950	8.45011
30	500	156.839	28.8167	8.14889	7.73982	30	100	229.341	157.494	29.1452	8.45010
40	600	183.733	28.8168	8.14887	7.73982	40	100	301.107	184.589	29.1472	8.45010
50	700	203.372	28.8164	8.14886	—	50	100	370.653	204.379	29.1472	8.45010
60	800	217.714	28.8162	8.14886	—	60	100	438.048	218.833	29.1472	8.45010
70	1000	228.188	28.8161	—	—	70	100	503.358	229.390	29.1472	8.45010
80	1100	235.837	28.8160	—	—	80	200	566.649	237.101	29.1472	8.45010
90	1300	241.424	28.8160	—	—	90	200	627.982	242.733	29.1472	8.45010
100	1400	245.504	28.8160	—	—	100	200	687.418	246.847	29.1472	8.45010
110	1500	248.483	—	—	—	110	200	745.016	249.851	29.1472	8.45010
120	1700	250.660	—	—	—	120	200	800.833	252.046	—	—
130	1800	252.249	—	—	—	130	200	854.923	253.649	—	—
140	1900	253.410	—	—	—	140	200	907.341	254.820	—	—
150	2100	254.258	—	—	—	150	300	958.137	255.675	—	—
160	2200	254.877	—	—	—	200	300	1189.50	257.510	—	—
170	2300	255.329	—	—	—	250	400	1387.24	257.891	—	—
180	2500	255.659	—	—	—	300	500	1556.23	257.970	—	—

Table 1. Nusselt numbers for Hagen-Poiseuille (GITT).

		Pe$_H = 10$						Pe$_H = 1$			
I	J	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$	$\xi = 1$	I	J	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$	$\xi = 1$
12	12	142.762	107.173	-20.8444	7.73673	12	12	144.438	123.545	30.9689	8.45042
12	25	297.397	219.275	-57.1762	7.75869	12	25	298.694	233.484	4.57372	8.43917
12	50	594.589	434.854	-126.822	7.76699	12	50	594.554	439.571	-48.4931	8.43693
12	100	1188.88	866.008	-266.010	7.77077	12	100	1186.24	852.247	-151.747	8.43643
12	200	2377.44	1728.32	-544.336	7.77431	12	200	2369.72	1679.08	-355.683	8.43637
12	400	4754.48	3452.93	-1100.96	7.77433	12	400	4736.78	3333.82	-762.009	8.43634
25	12	138.154	70.5440	8.25530	7.77968	25	12	144.190	122.079	38.4667	8.46242
25	25	286.594	133.459	8.25791	7.79041	25	25	295.355	208.840	32.7172	8.45054
25	50	571.951	255.049	8.25796	7.79382	25	50	580.491	333.474	29.8808	8.44809
25	100	1142.62	498.577	8.25787	7.79521	25	100	1148.14	563.546	29.2981	8.44750
25	200	2283.95	985.823	8.25783	7.79638	25	200	2283.56	1025.35	29.1925	8.44737
25	400	4566.56	1960.41	8.25782	7.79640	25	400	4555.12	1954.67	29.1697	8.44734
50	12	131.320	33.5192	8.16864	7.77186	50	12	144.287	123.483	38.7984	8.46474
50	25	268.013	32.1138	8.17273	7.77924	50	25	294.343	206.831	32.5424	8.45274
50	50	530.943	30.9288	8.17298	7.78115	50	50	568.416	275.704	29.6815	8.45025
50	100	1056.99	30.2176	8.17295	7.78184	50	100	1096.25	296.966	29.2280	8.44964
50	200	2109.23	29.8355	8.17293	7.78225	50	200	2142.07	287.711	29.1471	8.44949
50	400	4213.78	29.6383	8.17293	7.78226	50	400	4234.42	275.353	29.1283	8.44946
100	12	124.318	26.3604	8.15113	7.75112	100	12	144.413	124.753	38.9017	8.46529
100	25	240.174	-7.20932	8.15516	7.75768	100	25	294.764	213.912	32.5456	8.45326
100	50	460.594	-74.3350	8.15541	7.75912	100	50	564.791	291.872	29.6871	8.45076
100	100	902.173	-205.750	8.15538	7.75952	100	100	1051.16	266.854	29.2510	8.45015
100	200	1786.32	-465.961	8.15536	7.75971	100	200	1947.31	37.1027	29.1711	8.44999
100	400	3555.26	-984.789	8.15535	7.75973	100	400	3702.74	-479.736	29.1525	8.44996
200	12	121.577	37.0201	8.14622	7.73862	200	12	144.495	125.246	38.9261	8.46542
200	25	213.013	30.6256	8.15028	7.74506	200	25	295.336	217.470	32.5376	8.45338
200	50	362.802	26.9930	8.15054	7.74637	200	50	566.692	310.593	29.6798	8.45088
200	100	655.213	26.2346	8.15051	7.74670	200	100	1039.82	343.126	29.2509	8.45027
200	200	1242.91	26.2734	8.15049	7.74680	200	200	1791.34	294.025	29.1718	8.45012
200	400	2421.90	26.3936	8.15049	7.74682	200	400	3021.41	247.072	29.1533	8.45008
400	12	122.538	37.7968	8.14498	7.73378	400	12	144.540	125.338	38.9321	8.46545
400	25	204.593	31.9331	8.14906	7.74022	400	25	295.707	218.081	32.5350	8.45341
400	50	283.688	29.2716	8.14932	7.74150	400	50	569.023	313.298	29.6775	8.45091
400	100	365.713	28.8534	8.14929	7.74181	400	100	1048.82	350.119	29.2505	8.45030
400	200	508.964	28.7825	8.14927	7.74189	400	200	1768.74	304.978	29.1716	8.45015
400	400	804.824	28.7659	8.14926	7.74191	400	400	2575.70	266.582	29.1532	8.45011
800	12	123.951	37.9440	8.14467	7.73227	800	12	144.562	125.361	38.9336	8.46546
800	25	210.603	31.9672	8.14875	7.73872	800	25	295.902	218.234	32.5343	8.45342
800	50	280.516	29.3177	8.14901	7.73999	800	50	570.452	313.963	29.6769	8.45092
800	100	242.552	28.9124	8.14898	7.74029	800	100	1058.09	351.632	29.2504	8.45030
800	200	14.2330	28.8384	8.14897	7.74037	800	200	1812.00	305.826	29.1716	8.45015
800	400	-475.662	28.8212	8.14896	7.74039	800	400	2627.15	266.967	29.1532	8.45011
1600	12	124.687	37.9775	8.14459	7.73185	1600	12	144.572	125.366	38.9340	8.46546
1600	25	215.905	31.9558	8.14868	7.73829	1600	25	295.984	218.272	32.5342	8.45342
1600	50	307.048	29.3076	8.14894	7.73957	1600	50	571.079	314.129	29.6767	8.45092
1600	100	338.681	28.9122	8.14891	7.73987	1600	100	1062.65	351.995	29.2504	8.45031
1600	200	298.107	28.8392	8.14889	7.73995	1600	200	1842.04	305.906	29.1716	8.45015
1600	400	261.858	28.8221	8.14888	7.73996	1600	400	2790.44	266.832	29.1532	8.45012

Tabela 2. Nusselt numbers for Hagen-Poiseuille flow (FVM).

Evaluating tables 1 and 2 above, it appears that for higher values of the number of Péclet and positions further away from the entrance of the channel, the convergence rates are better, providing lower computational cost. Regarding the results achieved by the two methods, a higher rate of convergence for the GITT methodology is shown. In the case of FVM, due to the high number of equations to obtain the same precision, it was necessary to perform discretization in the axial direction.

3 | LITERATURE REVIEW

Almeida and Cotta [1] used GITT to analytically solve a convection-diffusion problem present in petroleum engineering: the injection of tracers in injection wells, assuming the unit mobility rate hypothesis. In this application of GITT in oil reservoir problems, the plotter's two-dimensional equation is solved for a fully developed five spot well distribution pattern, that is, infinite square cells composed of an oil producing well at each vertex and an injector well in the center of the square. The following classic hypotheses were assumed: homogeneous, horizontal, and isotropic reservoir, monophasic, incompressible, and steady flow, and ideal tracer (without adsorption, chemical reaction, or radioactive decay). Comparisons were made with alternative analytical solutions and numerical solutions obtained by the finite difference method, where reference results were established.

Cheroto, Dos Santos and Kakaç [7] presented a theoretical study of the transient laminar forced convection for a flow between parallel plates in thermal development. The flow is subjected to periodic variation of the inlet temperature and GITT is used to provide a hybrid solution (analytical-numerical) and the periodic analysis is performed using two similar coupled problems. The problem is analyzed by solving the thermal boundary layer equations for forced laminar convection using a complementary formulation that consists of dividing the problem into two parts, one real and the other imaginary. In the end, the results obtained were compared with others already existing in the literature to provide validation.

Barbuto and Cotta [4] applied GITT to elliptical problems in irregular domains, obtaining an analytical-numerical solution. The authors initially used a general formulation, which includes pipelines with irregular geometry as a special case, where recent developments in computational implementation are highlighted, including automatic global error control. Basically, the original partial differential equation is transformed into an infinite system coupled with ordinary 2nd order differential equations for the transformed potentials, in the non-eliminated direction, through the integral transformation process. For the solution of the transformed system commercial packages are used that offer a robust error control. An adaptive procedure is implemented, which automatically controls the order of the truncated system until the required accuracy is achieved. To complement the study and show the application of the methods, three configurations of ducts were analyzed, namely, triangular isosceles, circular, and elliptical. Numerical results were obtained for the field of longitudinal speed and friction factor, but due to the lack of reference results in the literature, it was not possible to validate them.

Almeida and Cotta [2] investigated the solution of diffusion-convection problems within a domain without borders using the Generalized Integral Transform Technique (GITT). GITT was tested in a borderless domain using two schemes: simple domain truncation procedure and a coordinate transformation method. A classic one-dimensional problem based on the Burgers Equation was used, where, despite its simplicity, it is provided with important characteristics that allow an efficient exploratory analysis, in addition to having several practical applications.

Chongxuan et al. [8] applied GITT to solve the advection-dispersion equation (ADE) in heterogeneous porous medium coupled with linear and non-linear adsorption and decomposition. When both are linear, analytical solutions are obtained using the GITT for one-dimensional ADE’s with temporal and spatial variations in flow, dispersion coefficient, initial conditions (spatial variation) and boundary conditions (temporal variation). When adsorption or decomposition is non-linear, the solution via GITT is an analytical-numerical hybrid. In both linear and both nonlinear cases, the transform-inverse pair for the problems addressed in this work is relatively simple. The authors present some illustrative examples with linear sorption and decomposition to demonstrate the problem approach via GITT and check the accuracy of the analytical solutions found.

Guerrero, Quaresma and Cotta [15] used GITT to find a hybrid analytical-numerical solution of the two-dimensional and steady-state Navier-Stokes equations, defined within domains arbitrarily, for an internal, laminar, and incompressible flow. The mathematical formalism is illustrated in a classic laminar flow test case in a gradual expansion duct. Numerical results with automatic global error control are obtained for Reynolds number values suggested in the literature. These results are compared with reference solutions published in the literature for the same problem. The authors conclude the work by stating that the solution’s convergence via GITT proved to be excellent and completely in agreement with the reference results. They also concluded that automatic error control and computational cost reduction are hallmarks of GITT for this class of problems.

Sphaier and Cotta [27] used GITT to obtain solutions of Sturm-Liouville eigenvalue problems, described by multidimensional partial differential models within irregularly shaped domains. Through integral transformations, the successive elimination of independent variables generates a problem of associated algebraic eigenvalue, solved by algorithms from scientific computational libraries. A diffusion problem with an exact known solution is presented, to validate the methodology used. The approach used proved to be extremely efficient and computationally robust. The work also shows that the approach used is also applicable to non-linear formulations, providing an adequate base of self-functions.

Tunc and Bayazitoglu [29] investigated the heat transfer in rectangular micro-channels. The Integral Transform Technique is applied twice, once for the speed field and once for the temperature field. The flow is assumed to be thermally and hydrodynamically developed with heat flow applied to all walls of the channel. The momentum equation was solved to obtain the velocity profile, and then to be replaced in the energy equation, obtaining the temperature profile. Values of the Nusselt number were calculated for various values of the aspect ratio. The results show a behavior like the studies with circular microtubes present in the literature.

Sphaier and Cotta [28] used the integral transformation to obtain an exact solution to linear diffusion problems within irregular domains. The authors used an approximate contour representation strategy for cases in which the boundary limits are points in space, as well as they can be used as an alternative to reduce computational effort for cases in which the contour has a functional representation of shape. closed and accurate. The strategy considered in this work, according to the authors, has not yet been used in previous works with the same type of approach, where it provides a significant increase in the flexibility and applicability of the proposed method. The analysis was carried out using mixed symbolic-numerical computation using the Mathematica software, where a computational code was developed that includes analytical derivations, numerical evaluations, and graphs. A test case, with an exact known solution, of heat conduction in a portion of a cylindrical region is used to verify and validate the solution methodology and computational implementation.

Neto, Quaresma and Cotta [22] studied a three-dimensional transient Darcy model of natural convection in cavities filled with porous medium, using a potential vorticity vector formulation and the generalized integral transform technique. A general formulation and solution methodology for vertical cavities are developed. Results are presented for cubic cavities, evaluating the effects of the Rayleigh number, observing the transient evolution of the heat transfer process. The behavior of the convergence of the solution for the expansion of the proposed self-functions is investigated and comparisons are made with solutions in permanent regime present in the literature.

Silva, Quaresma and Dos Santos [26] used GITT to analyze the entrance region of the two-dimensional laminar flow in parallel plate ducts. A formulation in terms of the original potentials was adopted. Expansions for the velocity field have been proposed so that the continuity equation is satisfied. After applying the integral transformation process, a coupled system of ordinary differential equations is generated, whose formulation is like the formulation by the current function. The results for the speed field and for the product of the friction factor by the Reynolds number (f Re) are analyzed and compared with the results in the literature.

Diniz et al. [14] investigated the transient temperature distribution in a uniform flow (slug-flow) in thermal development, with conduction-radiation coupling, between two parallel flat plates. The temperature distribution was obtained through the application of GITT, while the radiative contribution is determined analytically by the Galerkin Method. The results are analyzed in terms of stability, convergence, and computational cost, considering the contributions of parameters such as optical medium thickness and conduction-radiation coupling factor.

Nascimento, Macedo and Quaresma [20] applied GITT to obtain the solution of the momentum equations for a laminar flow of a non-Newtonian power-law fluid under hydrodynamic development in circular ducts. A formulation of primitive variables is adopted to avoid the singularity of the auxiliary eigenvalue problem in terms of Bessel functions in the center line of the duct when GITT is applied. Results were presented for the speed field and for the friction factor, evaluated for different power-law indices, and presented graphically as functions of the dimensionless coordinates. The results obtained were compared with results found in the literature to validate the numerical codes developed and demonstrated extreme agreement.

Barros and Cotta [5] considered a mathematical model of flow in a permanent three-dimensional regime to predict the behavior of contaminants dissolved in rivers and channels subjected to turbulent flow conditions. The proposed model considers variable speed fields and non-uniform turbulent diffusivities within channels of rectangular cross section. The authors used GITT to obtain a hybrid analytical-numerical solution for the fields studied. The solution's convergence behavior was investigated and the criteria for reordering the terms of the infinite series were discussed, with the objective of reducing the computational effort associated with the expansion of multiple self-functions. The authors present a test case illustrating the proposed methodology.

Lima et al. [17] obtained a hybrid solution using GITT for an MHD flow, between parallel plates, which consists of an electronically conducting Newtonian fluid and subjected to a perpendicular and uniform magnetic field, taking into account the heat transfer during the fluid movement. A simple mathematical formulation was adopted, considering the flow in a transient regime sustained only by the pressure gradient, as well as the flow in a permanent regime with constant pressure gradient and upper plate movement, with suction and injection perpendicular to the porous plates. Results for the temperature and speed fields are obtained according to the following governing parameters: pressure gradient, suction speed, upper plate speed and Hatmann numbers. A convergence analysis was carried out showing the consistency of the results and later they were compared with those already existing in the literature, showing an excellent agreement.

Naveira, Lachi and Cotta [21] used GITT to obtain a hybrid solution of a transient laminar forced convection problem on flat plates, subjected to temporal variations in the applied heat flow. From the Blasius velocity distribution and making a coordinate transformation, which takes into account only the thermally affected region along the main flow direction, the transient temperature distribution is expanded in self-functions obtained from the diffusion operator in the direction transversal to the flow. The coupled system of partial differential equations, resulting from the transformed potentials, is solved numerically in terms of the dimensionless time and longitudinal coordinate variables, using the Mathematica software. Numerical solutions for the wall temperature and heat transfer coefficient were obtained, generating the temporal behavior and longitudinal distribution of these parameters for the applied heat flow functions. The GITT approach proved to be robust and accurate, in addition to being quite flexible to allow the use of different combinations of boundary conditions and different geometric configurations of external flows governed by the formulation of the boundary layer.

Chalub et al. [6] performed a comparison between the solution of a convection-diffusion problem via GITT and via the Finite Volume Method (FVM). The proposed problem was that of a laminar flow between thermally developed parallel flat plates. Both methods were focused on transforming a partial differential formulation into an ordinary differential formulation. In the case of GITT, this operation was carried out through an integral transformation and for the FVM the discretization of the independent variable transversal to the flow was made. The resulting systems of ordinary differential equations are solved analytically, and the comparison of results is presented, indicating the advantages and disadvantages of each method. The temperature fields and Nusselt numbers obtained by both methodologies are analyzed, compared, and validated using results that already exist in the literature.

Monteiro et al. [19] investigated the problem of the propagation of a thermal wave inside a finite bar using the generalized integral transform technique. The use of GITT in the analysis of the hyperbolic heat conduction equation generates a coupled system of ordinary differential equations of the 2nd order that varies over time. The ODE system is solved numerically by the GEAR method for consistent initial value problems. The numerical results are presented for local and average temperature fields for different Biot numbers and dimensionless thermal relaxation times, allowing a critical evolution of technical performance. A comparison was made with existing results in the literature for special cases and obtained through the application of the Laplace Transform and the Finite Volume Method.

Presgrave, Guedes and Neto [23] used the Integral Transform Technique as a support tool for studying a medical procedure called Endometrial Ablation. This procedure consists of removing the endometrium by inserting a latex balloon filled with a fluid solution at high temperature into the uterine cavity. To assess the efficiency of the procedure, accurate predictions of the temperature field are necessary. The thermal problem in question is discussed in this work, where Penne's bioheat transfer equation is adopted to obtain the temperature distribution in the uterus wall. The solution to the equation is obtained by applying the integral transformation. Special attention is given to situations in which the temperature of the fluid solution present inside the flask is not constant, which may impair the effectiveness of the procedure. In analyzing the results, the authors concluded that the temperature distribution in the wall of the uterus is greatly affected by the rate of fluid temperature decay inside the balloon.

Guerrero et al. [16] present an exact solution of the linear advection-diffusion transport equation with constant coefficients, in the permanent and transient regimes. Through a mathematical substitution, the original equation is transformed into a pure diffusion equation. The transformed diffusive problem is solved analytically through the classical integral transform, resulting in an explicit formal solution. The authors compare the convergence of the transformed diffusive problem, solved by the classical integral transformation, with the advective-diffusive transport problem, solved directly by GITT. After analyzing the results, the authors concluded that the solution of the transformed diffusive problem converged more quickly than the hybrid analytical-numerical solution obtained by the direct use of GITT in the original advective-diffusive transport problem.

Cotta et al [10] summarize the theory and describe the algorithm related to the construction of a mixed, symbolic-numerical, open computational code called UNIT (Unified Integral Transforms), which provides a development platform to obtain solutions of partial differential equations (EDP) linear and non-linear, via integral transforms. UNIT was developed on the Mathematica symbolic computing system, version 7.0, in conjunction with GITT's numerical-analytical methodology. The objective of the work was to illustrate a robust simulation with precision control in transient, non-linear, and multidimensional convection-diffusion problems. Test cases were selected based on non-linear multidimensional formulations of Burgers' equations, providing reference results for specific numerical values of government parameters.

Cotta et al. [11] used GITT to generate a hybrid analytical-numerical solution of the bioheat transfer model in heterogeneous media, which is reduced to an exact solution obtained by applying the classical integral transform in a linear model with constant coefficients. Pennes's equation is used to describe the process of heat transfer within living tissues. Several models are present in the literature with analytical solutions that represent the temperature distribution throughout the tissue structure, however these solutions have hypotheses, such as: constant thermophysical properties and linear blood perfusion rates. The present work performs a more complex analysis, since it formulates the environment as heterogeneous, allows a spatial variation of the parameters along the thickness of the tissues and takes into account the variation of the blood perfusion rate with the temperature. The numerical results for a set of government parameters are obtained using the UNIT source code (Unified Integral Transform). In addition, an analysis of the convergence of the expansion of the proposed self-functions and the evaluation of the importance of the spatial variation of the government parameters for the prediction of the thermal response in living tissues due to external stimulus were performed. The results obtained show that the proposed approach provides an excellent estimate of temperature distributions in bioheat transfer problems, in addition to being robust and capable of generating accurate results for abrupt variations in thermophysical properties. The nonlinear behavior of the perfusion rates was modeled and computed via the integral transform.

Da Silva and Sphaier [12] applied GITT to solve a one-dimensional problem of diffusion in a domain with irregular geometry. In general, irregular geometries are treated either by transforming coordinates or by using simpler eigenvalue problems defined in the irregular geometry itself. However, both approaches are limited, as they cannot deal with an arbitrary general domain. The authors used a new approach for treating irregular domains, which is the surrounding domain technique. This methodology provides satisfactory results to calculate eigenvalues of a Sturm-Liouville 1D problem, using an auxiliary problem defined within a surrounding domain. This work extends this technique, solving a 1D diffusion problem using expansions of the self-function in terms of a defined eigenvalue problem within a domain that involves the original domain. As a proposal for validating the methodology, results of a test case with an exact known solution were generated and compared with reference values present in the literature.

Queiroz [24] applied the generalized integral transform technique to analyze different strategies for solution of flows in thermal development, but hydrodynamically developed, and to determine which solution is more efficient for each configuration. The author used the classic boundary conditions of forced internal convection, namely: constant flow on the wall and constant temperature on the surface. Different Péclet numbers were analyzed, from high Péclet (without axial diffusion) to single Péclet (advection rate with the same order of magnitude as diffusion in the same direction). Three different eigenvalue problems were analyzed, generating three possibilities of integral transformation of the problem: the simple Helmholtz problem for the transversal direction, a variation of this, which takes the speed profile into account, and a eigenvalue problem in the flow direction. In addition to these different strategies for integral transformation, different strategies for the solution of the transformed system were analyzed, from totally numerical solutions to analytical solutions. The comparison between the different solution strategies was performed by analyzing the convergence rate of the Nusselt number. The computational time spent by different solution strategies of the transformed system was also analyzed, as well as the rate of convergence for the temperature in different positions. The computational solutions and implementations were validated by comparing the value calculated for the Nusselt number away from the channel entrance with the known value in the literature for the flow developed. Every implementation of the problem was carried out in the Mathematica program through mixed symbolic-numerical computation.

4 I CONCLUSIONS

This work aimed to perform a literature review on the Generalized Integral Transform Technique and analyze the comparison of GITT with the Finite Volume Method, through the results of two articles that used both methodologies to obtain the solution of the problem in question. and compare your results. The first conclusion that can be drawn is about the diversity of diffusion-advection problems in which GITT is used as a solution tool. These diverse problems are in areas such as: heat transfer, flow of fluids in ducts and channels, flow in porous media, pollutant dispersions, biomedical engineering, petroleum engineering, among others. GITT basically transforms a system of partial differential equations into an infinite system of ordinary differential equations transformed by eliminating spatial dependencies. This infinite transformed ODE system is solved with a lower computational cost when compared to the purely numerical solution of the original EDP system, where these can be solved in a simpler way, with the advantage of producing a more accurate and more economical solution besides allowing control over the relative error of the results. The conclusion mentioned above was verified, mainly in Queiroz et al. [25], where the convergence rates in the solution of the problem via GITT were higher than the solution via FVM, showing the accuracy of the technique.

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ÍNDICE REMISSIVO

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