Andrei Strickler
(Organizador)

Ciência, Tecnologia e Inovação: Desafio para um Mundo Global 2

Atena Editora
Ano 2019

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(Organizador)

Ciência, Tecnologia e Inovação: Desafio para um Mundo Global
2

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APRESENTAÇÃO

As obras “Ciência, Tecnologia e Inovação: Desafio para um mundo Global” Volume 2 e 3, consistem de um acervo de artigos de publicação da Atena Editora, a qual apresenta contribuições originais e inovadoras para a pesquisa e aplicação de técnicas da área de ciência e tecnologia na atualidade.

O Volume 2 está disposto em 26 capítulos, com assuntos voltados ao ensino-aprendizagem e aplicação de procedimentos das engenharias em geral, computação, química e estatística. São apresentadas inúmeras abordagens de aplicação dos procedimentos, e além disso, estão dispostos trabalhos que apresentam as percepções dos professores quando em aulas práticas e lúdicas.

O Volume 3, está organizado em 30 capítulos e apresenta uma outra vertente ligada ao estudo da ciência e suas inovações. Tratando pontualmente sobre áreas de doenças relacionadas ao trabalho e sanitarismo. Além disso, expõe pesquisas sobre aplicações laboratoriais, como: estudo das características moleculares e celulares. Ainda, são analisados estudos sobre procedimentos no campo da agricultura. E por fim, algumas pesquisas abordam precisamente sobre empreendedorismo, economia, custos e globalização na atualidade.

Desta forma, estas obras têm a síntese de temas e abordagens que facilitam as relações entre ensino-aprendizado e são apresentados, a fim de se levantar dados e propostas para novas discussões em relação ao ensino e aplicação de métodos da ciência e tecnologia, cito: engenharias, computação, biologia, estatística, entre outras; de maneira atual. Sem esquecer da criação de novos produtos e processos levando a aplicação das tecnologias hoje disponíveis, vindo a tornar-se um produto ou processo de inovação.

Desejo uma boa leitura a todos.

Andrei Strickler

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IRRATIONALITY IN THEORETICAL MUSIC IN THE RENASSAINCE

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ABSTRACT : This presentation covers questions of how the relationship between mathematics and theoretical music throughout western history shaped modern comprehension of critical notions such as "ratio" and "proportion"; exploring the educational potentiality of such a comprehension. In order to do that, it will be consider a procedure taken by Erasmus of Höritz, a Bohemian mathematician and music theorist who emerged in the early 16th century as a German humanists very articulate with musical matters. In order to divide the tone, Erasmus preferred to use a numerical method to approach the geometrical mean, although his procedure did not recognize itself as an approximation of the true real number value of the geometric mean. The Early Modern Period saw the growing use of geometry as an instrument for solving structural problems in theoretical music, a change not independently from those occurred in the conception of ratio/number in the context of theoretical music. In the context of recovery of interest in Greek sources, Erasmus communicated to musical readers an important fruit of such a revival and was likely the first in the Renaissance to apply explicitly Euclidean geometry to solve problems in theoretical music. Although Erasmus also considered the tradition of De institutione musica of Boethius, he was based strongly on Euclid's The Elements, using geometry in his De musica in different ways in order to solve musical problems. It is this comprehensive geometrical work rather than the summary arithmetical and musical books of Boethius that serves Erasmus as his starting-point. However, Erasmus proposed a proportional numerical division of the whole tone interval sounding between strings with length ratio of 9:8, since it was a primary arithmetical problem. This presentation aims at showing the educational potentiality of the implications of such a procedure of Erasmus on the transformation of conception of ratio and on the emergence of the idea of modern number in theoretical music contexts. Under a broader perspective, it aims at show the implications on education of a historical/epistemological and interdisciplinary appraisal of theoretical music and mathematics.

KEYWORDS: ratio, irrationality, mathematics education

1 I INTRODUCTION

The Early Modern Period saw the growing use of geometry as an instrument for solving

structural problems in theoretical music, a change not independent from those that occurred in the conception of ratio/number in the context of theoretical music. The implications of these shifts on education are quite relevant. In the context of the recovery of interest in Greek sources, Erasmus communicated to musical readers an important fruit of such a revival. Indeed, he was likely the first in the Renaissance to apply explicitly Euclidean geometry to solve problems in theoretical music. Although Erasmus also considered *the tradition of De institutione musica* of Boethius, he was based strongly on Euclid's *The Elements*, using geometry in his *De musica* in different ways in order to solve musical problems. It is this comprehensive geometrical work, rather than the summary arithmetical and musical books of Boethius, that serves as his starting point. However, Erasmus proposed a proportional numerical division of the whole tone interval sounding between strings with length ratio of 9:8, since it was a primary arithmetical problem.

This study tries to show the educational potentiality of Erasmus' procedure as it relates to the transformation of the conception of ratio and to the emergence of the idea of modern number in theoretical music contexts. Its approach serves as an example of the implications for education of an epistemological perspective of the history of interdisciplinary relationships between theoretical music and mathematics. The importance of the problem approached lies in the historic/educational relationships between the concept of ratio and of modern number, the relationship between discrete and continuous in mathematics, as well as the idea of comparison between numbers/ratios confining irrational numbers by using only rational ones. Erasmus' method can be seen also as an arithmetical version of the Eudoxus criterion of comparison between ratios, inasmuch as while the former tries to confine numbers finding proportional ratios with bigger terms, the latter tries to confine numbers exhausting all the rational ones that are bigger and smaller than the numbers compared.

We consider here a passage from Chapter 17 of Book VI of Erasmus *De musica*, entitled *"Propositio decimaseptima Toni proportionem scilicet sesquioctavam in duas proportiones equales artificialiter et geometrice dividere*[1]*".*

There is little research on the work of Erasmus Horicius and in particular on his book De musica, whose seventeenth chapter is the reference considered in this text with regard to its educational potential. Apart from the first three chapters, in which there is an edition made by Kroyer (1918), there is no edition of this work particularly for the chapter in focus. Palisca (1994) commented on the proposal for an arithmetical division of tone presented in Chapter 17, however, there is no mention of the meaning of such a procedure for the concept of the modern number or regarding the potential that such a procedure represents.

It concerns the equal and proportional numerical division of the whole tone interval sounding between strings with length ratio of 9:8, a problem which confused musical theorists from Antiquity to the Renaissance and that played an important part in the historical process leading to the emergence of equal temperament. The mathematical

solution of such a problem would imply irrational numbers, anachronistically speaking, or incommensurable magnitudes, which would not be allowed in theoretic musical contexts. Thus, Erasmus`s solution is an attempt to make use only of arithmetic, which is allowed in theoretical music contexts, to solve the problem of the division of tone. In this passage Erasmus seemed to be in a position to solve such a problem.

2 I DIVISIONOFTHETONE

The problem of the division of the tone arose from the Pythagorean discovery of numerical indivisibility of a superparticular or epimoric ratio, i.e., n : n+1, by its geometrical mean, in particular applicable to the division of the ratio 9:8. Given p < x < q, where p and q are integers and the ratio p:q is superparticular, x cannot be both an integer and at the same time fulfill the condition p:x=x:q, that is, be the geometric mean of p and q. Mathematically, the equal division of the tone 8:9 provides ratios involving surds or incommensurable ratios underlying musical intervals. These procedures were considered impossible by Pythagoreans in theoretical music, since these intervals could be determined only by ratios of integer numbers. The constraint of not

[1] Proposition decimal septima on how to divide the sesquioctave ratio (that is, 9:8) of the whole tone into two equal ratios, artificially and geometrically.

using geometry and only using arithmetic to solve this problem brought about the use of new ways to solve the problem.

Attempts to divide the tone had existed, however, since Antiquity. Aristoxenus (fourth century B.C.) conceived the theoretical nature of music as essentially geometric and understood pitches, musical intervals and distances as continuous quantities that should follow the rules of Euclidean geometry and should be capable of being divided continuously. This inevitably raises questions concerning the nature of ratio in this context. Traditionally, it is considered that Aristoxenian music theory rejected the position of the Pythagoreans that musical intervals should properly be expressed only as mathematical ratios involving whole numbers, asserting instead that the ear was the sole guide for musical phenomena (Winnington-Ingram, 1995, 592). This did not mean, however, that Aristoxenus' theory could not be put on a mathematical base related to the developments in Greek mathematics of his time. Aristoxenus preferred geometry to arithmetic when solving problems involving relations between musical pitches and believed in the possibility of dividing the tone into two equal parts, conceiving musical intervals and ratios as continuous magnitudes.

Such an idea unchained many reactions, expressed for instance in the *Sectio Canonis* (Barbera, 1991, 125) and much later in the *De institutione musica* (Bower and Palisca, 1989, 88) of Boethius in the early Middle Ages, which stood for a strong Pythagorean tradition in theoretical music. Following the Pythagorean tradition, many

medieval musical theorists maintained the impossibility of the equal division of the tone, which would mathematically lead to incommensurable ratios underlying musical intervals. Such a position began to change in the 15th century and was eventually systematically overcome in the early Renaissance by scholars like Nicholas of Cusa, Erasmus of Höritz, Faber Stapulensis, Henricus Grammateus, Pedro Ciruelo, Juan Bermudo and others, who proposed the equal division of tone, mostly by means of geometry. In his *Musica,* Erasmus of Höritz made use of an abstract numerical procedure to propose a solution for the problem of the equal division of the tone, expressing rather as a number the geometrical mean between the terms of the ratio 9:8 underlying the tone.

3 I THE *DE MUSICA* SPECULATIVA FROM ERASMUS HORICIUS

Erasmus' *De musica* emerged in a time when the rediscovery, translation and publication of sources from Antiquity, such as the works of Euclid, Archimedes and Ptolemy, stimulated increased interest and development of number theory. Gaps in the Pythagorean numerical system were quite disturbing, resulting in a crisis and conceptual changes in the demarcation of the disciplines arithmetic and geometry. Thus, ratios involving surds, i.e., incommensurable quantities, could only be discussed in the domain of continuous quantity and would request the unification of two such disciplines, as well as the conquest of a number continuum on mathematical activity.

Particularly for Erasmus, Arabic and Hindu concepts were highly influential, since they promoted the development of Greek mathematics and handled entities such as negative and irrational numbers. Furthermore, they allowed, with the introduction of Hindu numerals by Fibonacci, computation of unprecedented complexity, as well as the development of extremely large numbers, the latter being an important component in Erasmus' division of the whole tone ratio.

In Chapter 17 of Book VI, Erasmus refers specifically to the division of the 9:8 ratio, which represents the musical interval of a whole tone. In the 4 previous chapters of book VI, Erasmus demonstrated incompletely the divisibility of other superparticular ratios into equal and proportional halves, like the octave (2:1), fourth (4:3), fifth (3:2) and minor third (6:5).

Erasmus proposed an abstract numerical procedure to find the geometrical mean between the terms of ratio 9:8 underlying the tone, expressing it as a number. This is a historical interesting example of solving a problem under a constraint, which was in this time not a constraint, but the common way to solve problems in that context. In other words, in the original case, the constraint was natural or taken for granted, and from a certain point of view, not a constraint, since arithmetic was the instrument to solve problems in theoretical music contexts. However, its approach in didactics contexts clarifies situations in which one must follow a specific way or find a solution

given a particular constraint. On one hand, this can be seen as a restriction, but on the other hand, it can also represent liberty and freedom in the sense of thought flexibility.

Erasmus did not use the geometrical construction of mean proportional to two given straight lines from proposition 13 of Book VI of Euclid, as did for instance Jacques Lefèvre d'Etaples in 1496, using exclusively non-numerical Euclidean methods capable of being carried out with straightedge and compass. He attempted rather to reach an expression of the ratio for the supposedly equally proportional halves of the whole tone interval using very large integer numbers. He did it first using proposition 15 of Book V of the Elements, which asserts that a:b :: am:bm. Following his method, the half of the 9:8 ratio of a tone could be obtained by the geometric mean of its expansion into the term 34828517376: 30958682112. This ratio was derived directly from 9:8 by multiplying numerator and denominator by the factor 3869835264, a procedure guaranteed by proposition 15 of Book V for a = 9, b = 8 and m = 3869835264.

The proportionality between the original ratio 9:8 and 34828517376:30958682112 allows mapping between intermediate terms of the ratio 9:8, including the mean. Numbers between the terms of its expansion may be formed into a large number ratio, considering that the interval determined by the expansion is much more subdivisible and that the greater the distance between the terms in the large number ratio, the greater the precision one can get for the intermediate terms of the ratio 9:8, represented by the large number between the two terms of the large number ratio.

Since there were not decimal fractions at this time, the proportional extended ratio is used for the purpose of extracting the square root with a high degree of precision, in this case associated with large integer numbers rather than with places after the decimal point. The bigger the distance between the terms in the large number ratio, the better the precision with which the geometrical mean is obtained. Nevertheless, Erasmus seemed not to worry carrying out any computation in the text, and he did not present his result as an approximation of the true real number. He is the first author to propose an abstract numerical procedure for the given problem, expressing it as a number and avoiding using the construction of a geometrical line. Since it was a primary arithmetical problem, it could be solved "*artificialiter*", that is, numerically.

Erasmus asserts that "... in musical demonstrations we are forced to use all kinds of ratios ... since not all shapes of consonances and also dissonances are founded in rational ratios and for that reason we must not neglect the ratios of surds..." (Erasmus Horicius, [ca. 1500], fo. 61v). Here, Erasmus considered incommensurable ratios or irrational numbers in musical contexts. For music theoretical purposes, it would seem in principle that in order to make use of Eudoxus' theory of Book V of Euclid's *Elements,* on which the theory of ratios of surds is based and which deals with abstract quantities with continuous nature, he established a link between continuous and discrete quantity[2]. Erasmus realized that the search for a geometrical mean to the ratio

[2] It is possible to identify similar ideas between the numerical division of the tone

proposed by Erasmus and the Eudoxuś definition V of Book 5 of the Elements. Whereas Erasmus confined a searched irrational number by using only integers, Eudoxus' definition corresponded, arithmetically speaking, to establish a proportionality of ratios through the confinement of ratios with integer terms. In these analogous procedures, Erasmus and Eudoxus got precision, to find an irrational number and to establish a proportionality between underlying the whole tone could not result in a rational number; so, instead of changing the domain at this point from a discrete quantity of numbers to a continuous quantity of geometrical lines, he established a link between continuous and discrete quantities, proposing a number continuum, although not explicitly, and creating a very dense, discrete point space between the original terms 9 and 8 by their expansion.

4 I CONCLUDING REMARKS

It is plausible that if Erasmus really thought that he could divide the sesquioctave ratio in terms of a purely numerical operation, he must have possessed an at least rudimentary concept of the number continuum. Such an assumption is corroborated by a passage appearing later in Chapter 17, where he seems to refer directly to the idea of such a continuum, mentioning Boethius as a prisoner of the Pythagorean doctrine of discrete integer numbers not accessing all ratios of numbers (Erasmus Horicius, [ca. 1500], fo. 67v). Just before this passage, Erasmus asserts that the exact half of the whole tone interval would be provided by extracting the square root of the product of its terms 8 and 9, which would be sqrt (72) (Erasmus Horicius, [ca. 1500], fo. 67v). He did not, however, relate explicitly this result with the computations he presented.

He formulated the large number ratio, but he still needed to find the geometrical mean between the two terms. Since he presented the way to do this by extracting the square roots of 9:8, one might ask why he did not do it from the ratio 9:8. Moreover, if he produced the proportional large number ratio, how could he use this representation to the extraction mentioned above and/or to approach the geometrical mean. It might be assumed that he left it to the reader. Such a method is structurally analogous to that used by Eudoxus to establish a criterion to compare ratios, including incommensurable ones. In such an analogy, it is especially interesting that both procedures made use only of commensurable ratios in geometric and arithmetic contexts. Perhaps it is this attribute that makes this analogy structurally strong.

This feature has educational potential, because it is an example of using history for epistemological purposes in learning/teaching dynamics. It is also an example of searching for two given ratios, respectively, through ratios with big terms. Erasmus made use of "The Elements", nevertheless his source was the Campanus' translation, which had an arithmetical terminology that was not derived from the geometrical ratio theory of Book V of Euclid, but instead from a number of different sources including

very likely the Arithmetic of Jordanus de Nemore from 13th century. Such a fact makes, on the one hand, not plausible that Erasmus would have had access to Eudoxus definition in the original sense and, on the other hand, very instigating the strong and curious structural analogy between both procedures.

New ways to solve a given problem, which is submitted to a constraint in cases deviating from tradition. This case illustrates the importance of introducing historic or cultural conditions to the approach of a given concept, since it is, for instance, an example that shows the methods used in music/mathematical thought in a certain way given by the cultural context and traditions in which it was inserted. On the other hand, it illustrates how Erasmus was an innovator. Inasmuch as he showed he was aware of all geometrical tolls required to solve the problem of division of the tone, as noted by many of his successors like Faber Stapulensis or Pedro Ciruelo, he chose the traditional context to do it.

Both procedures established by Eudoxus and Erasmus use only commensurable or rational ratios and numbers to introduce incommensurable or irrational ones, but with a geometrical approach and an arithmetical one, respectively. These exemplify ways for introducing irrational numbers making use only of integers. Such historical analogous examples also allow us to introduce a broader sense for the crisis of the incommensurable, and now present it in parallel to its musical version, for which Erasmus created a criterion to deal with such magnitudes making use only of commensurable ones just as Eudoxus.

Theoretically based on many geometrical propositions and unusually modeled on Euclidean style, *Musica* dealt with ratio as a continuous quantity, announcing perhaps what would emerge as an arithmetical treatment of ratios in theoretical music contexts during the sixteenth century and approaching ratio to a real number. Under an educational perspective, these two historical approaches for theories of ratio make music a favorable context for the differentiation between ratios, fractions and numbers, insofar as the semantic distinction between two such approaches stands out in this context. In music contexts, two musical intervals produced by two proportional ratios are clearly different, although somewhat similar, whilst such a difference disappears in an arithmetical context, in which these ratios are identified with numbers. For instance, the ratios 2:3 and 4:6 produce musically two fifths with an octave difference. They are proportional and similar, but not the same, whereas the difference between such ratios disappears in an arithmetical context, since 2/3 is equal to 4/6 arithmetically speaking.

Interestingly, Erasmus could have easily solved the equal division of the tone making use of the proposition of Euclid's *Elements,* which provide the geometrical mean as the height of a right angled triangle. Nevertheless, missing the concept of infinity, he preferred to use a numerical method to approach such a mean, although his procedure did not recognize itself as an approximation of the real number value of the geometric mean. Thus, Erasmus provided a mathematical theoretical structure for a virtual pitch relation space and a continuum of rational numbers, which can be seen

as an important step towards laying the foundations for the real number system.

REFERENCES

Barbera, A. 1991. The Euclidean Division of the Canon. Lincoln: Univ. Nebraska Press.

Bower, Calvin M. and Palisca, Claude V. eds. 1989. Fundamentals of Music. Anicius Manlius Severinus Boethius. New Haven & London: Yale University Press.

Erasmus Horitius. 1500 "De Musica" [ca. 1500], Reg. Lat. 1245, Biblioteca Apostolica Vaticana Theodor Kroyer, "Die Musica speculativa des Magister Erasmus Heritius," in Festschrift zum 50.

Geburtstag Adolf Sandberger (München: Ferdinand Zierfuss, 1918), 72-91.

Palisca, Claude V. 1994 “The Musica of Erasmus of Höritz,” C. Palisca, Studies in the History of Italian Music and Music Theory. Oxford: Clarendon Press, pp. 146-67.

Rommevaux, Sabine; Vendrix, Philippe; Zara, Vasco. 2011. Proportions: science, musique, peinture & architecture. Collection – études Renaissantes. Brepols: Turnhout.

Winnington-Ingram, R. P. 1995. “Aristonexus” In: *The new Grove dictionary of music and musicians,* edited by Stanley Sadie, 592. London: Macmillan.

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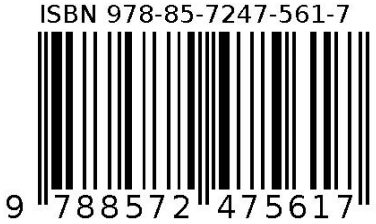
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