Journal of Engineering Research

ISSN 2764-1317 vol. 5, n. 8, 2025

ARTICLE 10
Acceptance date: 09/12/2025

EVALUATION OF THE RELIABILITY OF A TEMPERATURE CONTROL SYSTEM USING THE MARKOV PROCESS

Gabriel Antonio Pérez Castañeda
Dr.
A professor at the National Technological Institute of Mexico, Tehuacán Technological Institute campus, in the Mechatronics Engineering program.

Jesús Raymundo Flores Cabrera
M.I.
A professor at the National Technological Institute of Mexico, Tehuacán Institute of Technology campus, in the Mechatronics Engineering program.

Miguel Villano Arellano
M.C.
A professor at the National Technological Institute of Mexico, Tehuacán Technological Institute campus, in the Mechatronics Engineering program.

Víctor Sandoval Adán
A student of Mechatronics Engineering at the Tehuacán Institute of Technology.

Nicolae Brinzei
Dr.
A lecturer at the National Polytechnic Institute of Lorraine, France.

Jean-François Aubry
Dr.
Professor Emeritus at the National Polytechnic Institute of Lorraine, France.

Abstract: This article presents the assessment of the operational safety of a dynamic system using a Markov process. Characteristic parameters such as MTTF, MTTR, and system availability are obtained. Based on the hybrid stochastic automaton that models the temperature control system of an oven, its normal operating states and failure states are merged to obtain the Markov graph. Failure and repair rates are used, as well as the probabilities of being in a good operating state or a failure state. The results were compared with those obtained by Monte Carlo simulation of the hybrid stochastic automaton and were satisfactory.

Keywords — Markov process, hybrid stochastic automaton, exponential law, reliability, operational safety.

Introduction

Since humans invented the first instruments, they have become more dependent on their functioning. In this sense, the concept of reliability was born. Likewise, with the advent of electronics, reliability entered a new era. However, reliability as a subject of systematic study began in the 1960s. Reliability is a popular concept that has been used for years as a commendable attribute of a person or an object. The Oxford English Dictionary defines reliability as the quality of an entity to be dependable, on which one can count at a given moment, in which trust can be placed. In English, "reliability" comes from "to rely on," meaning "to count on, to have confidence in..." while reliability in French effectively comes from the word "fiable," meaning "one who can be trusted. In 1962, the Academy of Sciences defined it as follows: a parameter that characterizes the safety of operation, or a measure of the probability of operation of equipment according to prescribed standards. Later, in the 1970s, the International Electrotechnical Committee proposed the following definition: characteristic of a device, expressed by reliability, which fulfills a required function, under given conditions, for a given duration (Pagès and Gondran, 1980). (Laprie *et al.,* 1995) define reliability as a measure of the continuity of the delivery of a correct service or, equivalently, a measure of the time to failure. (CEI 50 (191), 1990) and (Villemeur, 1988) state that reliability is the ability of an entity to perform a required function under given conditions for a given time. This ability is measured (Smith, 2001) by the probability that an entity will perform a required function under given conditions for a given period of time. Reliability can be paraphrased as the probability of the entity not failing in a given period of time. Below are some of the methods that have been developed to evaluate the reliability of an entity or system.

Methods for evaluating predictive reliability

The first type of methods used in predictive reliability theory brings together combinatorial methods. These are used to identify and evaluate the combinations of component failures that cause the system to fail. This group includes fault trees, event trees, reliability diagrams, and structure functions (Kaufman *et al.,* 1975). A second type of method is based on a representation of the system's state, in which transitions correspond to a component failure or repair. Under certain conditions, these models are Markovian (or semi-Markovian) and allow access to the probability of staying in each of the states. The probability of being in one of the operating states is the availability of the system. To evaluate reliability, it is advisable to modify the model by making the shutdown states absorbing (in which there is no repair).

Markov methods and processes

Markov models represent a class of stochastic processes. A stochastic process describes the evolution of a system by the probabilities that it will be in a given state (or subset of states) at a given moment. A Markov process is also a stochastic process in which the future state does not depend on the past trajectory. It is homogeneous when the transition rates between states do not depend on time. When the process is defined continuously in time, it is represented by a state graph called a Markov graph. When the process only describes certain discrete moments, it is referred to as a Markov chain. Through misuse of language, the latter term is sometimes used for continuous-time models. The latter are used to quantitatively evaluate the operational safety of systems, especially when the transition rates are constant, i.e., the failure and repair times of components are distributed according to exponential laws (Cocozza-Thivent, 1997).

It is assumed that the transition from one state of the system to another occurs randomly due to the failure of one component or the repair of another element. Knowing the initial state of the system, it is possible to deduce either the probability of being in a given state after a certain duration or the average probability of being in a given state throughout its useful life.

Markovian processes are frequently used, as already mentioned, to study the

DOI https://doi.org/10.22533/at.ed.3175825011010

operational safety of systems, especially when dealing with repairable systems. A stochastic process is defined as the set of random variables Z(*t*), defined in the given probability space and indexed by a parameter *t* belonging to a set T:

$$\{Z(t), t\ T\} \qquad (1)$$

In practice, T represents space-time. It can be discrete or continuous. The variables Z(*t*) take their values from a set X consisting of all possible states of the system. This is the state space, which can be discrete or continuous independently of T. A stochastic process is perfectly defined by the following data:

The domain Z of the random variables,

The domain T of the parameter *t*,

the statistical relationships between Z(*t*) for different values of *t* defined by:

$$Fz(t) = Pr\ [Z(t_1)\ z_1; \ldots; \ Z(t_n)\ Z_n]\ Z = (z_1, \ldots, z_n), \quad t = (t_1, \ldots, t_n), n \qquad (2)$$

Markov processes are memoryless processes (the transition probability depends only on the current state), i.e., at each instant, the time remaining to be spent in the current state is independent of the time already elapsed. The only continuous distribution that verifies this hypothesis is the exponential distribution. On the other hand, semi-Markovian processes are processes that follow general distributions (Brinzei, 2003), (Cocozza-Thivent, 1997) and (Niel and Craye, 2002).

Markov processes

A stochastic process is said to be Markovian if:

$$\forall\ t = (t_1, \ldots, t_n, t_{n+1});$$

$$_{such} \text{ that } t(_{1)} < t(_{2)} < \ldots < t(_{n)} < t(_{n+1)}$$

thus

$$Pr[Z(t_{n+1}) = z_{n+1}/Z(t_n) = z_n; Z(t_{n-1}) = z_{n-1}; \ldots ;$$

$$Z(t(1_{)\ )} = z(1_{)\ ]} = Pr\ [Z(t(_{n+1)\ )} = z(_{n+1)\ /X}(t) = z(n_{)\ ]} \qquad (3)$$

Thus, a Markovian process is a memoryless process. Knowledge of the state at times $t(_{1)} < t(_{2)} < \ldots < t(_{n)} < t(_{n+1)}$ is information that is completely contained in knowledge of the state at time $t(_{n)} + 1$. In other words, the future evolution of the process depends only on the state at the present moment, and not on past evolution.

The probability vector P of being in a state at time *t* is the solution to the Chapman-Kolmogorov equation:

$$\dot{P}(t) = P(t).A \qquad (4)$$

where *A* is the transition matrix between the states of the system:

$$[A] = \begin{bmatrix} -\sum_{j=2}^{n} a_{1j} & a_{12} & . & a_{1i} & . & a_{1n} \\ a_{21} & -\sum_{j=1, j'2}^{n} a_{2j} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{i1} & . & . & -\sum_{j=1, j'i}^{n} a_{ij} & . & a_{in} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & -\sum_{j=1}^{n-1} a_{nj} \end{bmatrix} \qquad (5)$$

and $a_{ij}\ .dt$ is the probability of moving from state Z_i to state Z_j between *t* and *t+dt*, knowing that at time *t* the system is in state Z_i.

A Markov process can be represented graphically by a state-transition model called a Markov graph (Schoenig, 2004).

DOI https://doi.org/10.22533/at.ed.3175825011010

Oven temperature control system

As can be seen in Figure 1, the system consists of two loops. The first contains a PI (proportional and integral) controller whose role is to maintain the oven temperature at a reference temperature value. The second loop is of the All-or-Nothing (TON) type. This allows the oven temperature to be maintained around the reference temperature by switching from full thermal power to zero power. These two loops cannot operate simultaneously. For this purpose, a relay switches the two contacts, activating either the PI or the TON. The command to switch from one to the other is given by the detection system, whose role is to identify faults and repairs and react by switching from one regulator to the other. Initially, the temperature is controlled by the PI controller. After a random period of time, the controller fails and the oven temperature rises rapidly. The detection system detects that the temperature has reached a dangerous value and deduces that the oven is out of control. The detection system gives the switch command to the relay to the TON controller loop. The oven temperature is now controlled by this controller. As soon as the detection system has detected that the temperature is out of control, it also initiates the PI controller repair process (the repair is a random time). However, the possibility of TON failure exists.

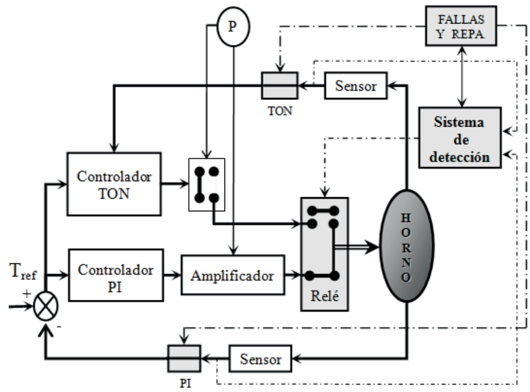

Figure 1. Oven temperature control system.

Once the PI controller is repaired, the detection system switches the relay to the PI loop, which now controls the oven temperature. The oven is considered to be functioning properly.

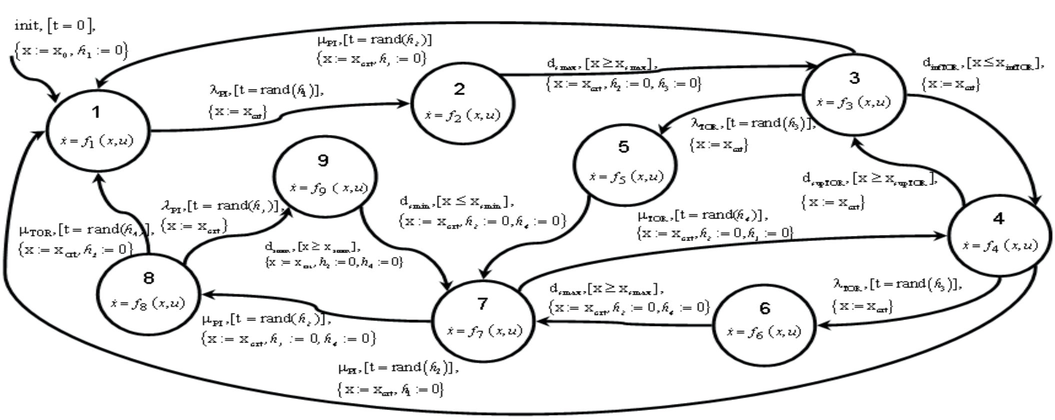

Figure 2. Hybrid stochastic automaton of the behavior of the temperature control system.

Evaluation of the operational safety of a system using the Markov process

Figure 2 shows the Hybrid Stochastic Automaton (Pérez et al., 2011) of the system presented above. It consists of nine discrete states and one continuous state variable: temperature. The automaton takes into account the deterministic behavior of the system described by the controllers and the stochastic behavior due to its failures and repairs. This automaton is constructed from embryonic finite state automata in order to take into account all the behaviors and transitions of the system. Synchronization is applied to these automata, thus obtaining the hybrid stochastic automaton.

The hybrid stochastic automaton in Figure 2, of the temperature control system of an oven, does not correspond to a Markov process. This is because the hybrid stochastic automaton models a dynamic system which has deterministic transitions that do not depend solely on an exponential distribution or on the time elapsed since arrival in the respective state. However, the hybrid stochastic automaton of the system has been approximated to a Markov process and a semi-Markovian process in order to compare and estimate the results obtained numerically by simulation with the analytical results provided by these methods. For this modification, the deterministic transitions have been removed from the hybrid stochastic automaton by merging the final discrete states with the source states of these transitions. Figure 3 shows the resulting state automaton, also known as a Markov graph.

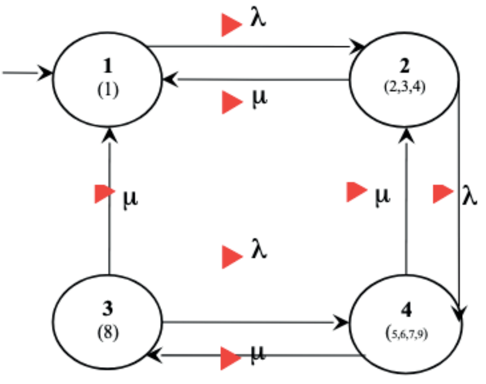

Figure 3. Markov graph equivalent to transition matrix A.

States 1, 2, and 3 are the states of proper system operation:

- state 1: the PI controller controls the oven temperature. The TON controller is inactive,

- state 2: the PI controller fails and the TON controller now controls the oven temperature. Repair of the PI controller or failure of the TON controller may occur.

- Status 3: The PI controller has been repaired and is now active, but the TON controller is still faulty.

- State 4: The PI and TON controllers are in fault and are being repaired. The system is in total failure.

The transition matrix of the Markov graph in Figure 3 is given by the following equation:

$$A = \begin{bmatrix} -\lambda_{PI} & \lambda_{PI} & 0 & 0 \\ \mu_{PI} & -(\mu_{PI}+\lambda_{TOR}) & 0 & \lambda_{TOR} \\ \mu_{TOR} & 0 & -(\mu_{TOR}+\lambda_{PI}) & \lambda_{PI} \\ 0 & \mu_{TOR} & \mu_{PI} & -(\mu_{PI}+\mu_{TOR}) \end{bmatrix} \quad (6)$$

We are interested in determining the MTTF (Mean Time to Failure), the avai-

DOI https://doi.org/10.22533/at.ed.3175825011010

lability, and the MTTR (Mean Time to Repair) of the system expressed by the Markov graph in Figure 3. By integrating the reliability R(*t*) of the system, we can deduce the MTTF (Niel and Craye, 2002). However, there is a more practical way to obtain the same parameter (Corazza, 1975), (Osaki, 2002). The transition matrix (6) is divided according to the states of good operation and the states of failure. That is, the first three columns correspond to the good functioning states 1, 2, and 3 of the Markov graph in Figure 3. The last column corresponds to the system failure state. The submatrix $G(_{11)}$represents the good functioning states, while the matrix $G(_{22)}$corresponds to the failure state.

$$G_{11} = \begin{bmatrix} -\lambda_{PI} & \lambda_{PI} & 0 \\ \mu_{PI} & -(\mu_{PI}+\lambda_{TOR}) & 0 \\ \mu_{TOR} & 0 & -(\mu_{TOR}+\lambda_{PI}) \end{bmatrix} \quad (7)$$

$$G_{22} = \left[-(\mu_{PI}+\mu_{TOR})\right] \quad (8)$$

Thus, the MTTF is determined by equation (9):

$$MTTF = P_F(0).(-G_{11})^{-1}.1_{nf} \quad (9)$$

where $P_F$is the probability vector of the good functioning states, $G_{11}$is the submatrix of the transition indices between good functioning states. $1_{nf}$is the sum vector of all functioning states, in this case nf = 3.

The MTTR is determined by equation (10):

$$MTTR = P_D(0).(-G_{22})^{-1}.1_{nd} \quad (10)$$

with P_D being the probability vector of the failure state. G_{22} is the submatrix of transition indices between failure states, and 1nd corresponds to the failure states.

Results

The values of the failure and repair indices for the PI and TON controllers are as follows:

l_{PI}= 13·10(-05) h(-1) (1);l (TO)(N) = 8·10(-05) h(-1);m (PI) = $^{21\cdot10(-03)\ h(-1);}$$m_{(TO)\ (N)}$= $14^{\cdot10(-03)\ h(-1)}$

Applying the above equations, we obtain:

$$MTTF = \frac{\lambda_{PI}+\lambda_{TOR}+\mu_{PI}}{\lambda_{PI}\lambda_{TOR}} = 2.039\cdot10(6)\ \ h$$

$$MTTR = \frac{1}{\mu_{PI}+\mu_{TOR}} = \ \ 28.57\ h$$

System availability is determined by equation (11):

$$\Pi = [0,0,0,1].A_m^{-1} \quad (11)$$

The matrix A_m^{-1} is obtained by replacing the last column of matrix *A* with $1\prod$. is the vector of asymptotic probabilities of these in each of the states.

The following asymptotic probabilities are obtained.

$$\prod = [0.99382664237564 \quad 0.00613828172876 \quad 0.00002096765749 \quad 0.00001410823811]$$

(1) Availability is the sum of the probabilities for the system to be in one of the operating states.

DOI https://doi.org/10.22533/at.ed.3175825011010

Therefore, the asymptotic availability = 99.99867%. The results obtained (Table 1) show that the approximate Markov processes provide a good approximation to those obtained by the hybrid stochastic automaton (HSA) (Pérez *et al.*, 2011).

Parameter	HSA	Markov process
Availability	99.999	99.99867%
MTTF	2.056·10(6) h	2.039·10(6) h
MTTR	28.26 h	28.57 h

Table 1. Results obtained with the AEH and by Markov process.

Conclusions

The results obtained show the efficiency of the Markov process in dealing with this type of system by approximating them through the fusion of discrete states, on the one hand, of good operation and, on the other, of failure of a hybrid stochastic automaton that models dynamic systems. The results obtained by the hybrid stochastic automaton through a Monte Carlo simulation and by Markov processes show an acceptable approximation. In fact, the results obtained by Markov are more conservative. For non-complex dynamic systems, it is recommended to use Markov processes. These results will allow the analysis of preventive maintenance programs and the implementation of measures to prevent unexpected failures in the system.

Credits and acknowledgments

This article was made possible thanks to the support of the Tecnológico Nacional de México campus Instituto Tecnológico de Tehuacán and the participation of research professors from the Institut National Polytechnique de Lorraine, France.

References

Brinzei N. “Contribution à la modélisation et à la commande modulaire des systèmes de production, par une approche UML et réseaux de Pétri”. *Thèse de doctorat* présentée à l’Université de Technologie de Belfort Montbéliard, 19 septembre 2003.

CEI 50 191 “Vocabulaire Electrotechnique International”, Chapitre 191 – *Sûreté de fonctionnement et qualité des services* – 1990.

Cocozza-Thivent C. “Processus stochastiques et fiabilité des systèmes”. *Mathématiques & Applications 28*. Springer, 1997.

Kaufmman A., Gronchko G., Cruon R. “Modèles mathématiques pour l’étude de fiabilité des systèmes”. *Masson et Cie*. 1975-

Laprie J.-C., Arlat J., Blanquart J.-P., Costes A., Crouzet Y., Deswarte Y., Fabre J.-C., Guillermain H., Kaâniche M., Kanoun K., Mazet C., Powell D., Rabéjac C., Thévenod P. “Guide de la Sûreté de fonctionnement”. *Cépaduès Éditions*. 1995.

Niel E., Craye E. “Maîtrise des risques et sûreté de fonctionnement des systèmes de production. Productique : information, commande, communication”. *Lavoisier*, 2002.

Pagès A., Gondran M. “Fiabilité des systèmes”. *Edition Eyrolles*. 1980.

Schoenig R. “Définition d’une méthodologie de conception des systèmes mécatroniques surs de fonctionnement”. Thèse de doctorat présentée à l’Institut National Polytechnique de Lorraine, le 26 octobre 2004.

Smith D. J. "Reliability, maintainability and risk. Practical methods for engineers". Sixth Edition. *Butterworth Heinemann*, 2001.

Villemeur A. Sûreté de fonctionnement des systèmes industriels. Edition Eyrolles. 1988.

Pérez Castañeda G. A., Aubry J. F., Brinzei N. "Autómata estocástico híbrido y simulación de Monte Carlo para la predicción de la seguridad de funcionamiento de un sistema dinámico". Revista Dyna, Año 78, Nro. 165, pp. 276-286. Medellín, Febrero de 2011.