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NUMERICAL METHODS FOR ANALYSING PROBLEMS IN RAILWAY ENGINEERING

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Abstract: The article addresses the importance of linking the content of Numerical Methods courses with practical applications in Railway Engineering. It proposes a more realistic and contextualized approach that can motivate students and introduce them to topics such as the design and analysis of forces in railway bridges or catenary elements, using systems of linear equations, matrices, and polynomial extrapolation. It highlights that Numerical Methods courses in most higher education institutions are usually unique throughout the degree program and often lack practical examples, which demotivates students and hinders their learning. This contributes to high failure and dropout rates, especially in the early years of higher education, as observed in engineering degree programs at the National Technological Institute of Mexico, Cancún campus. The text emphasizes that basic skills and mathematical tools at intermediate levels of study are essential for addressing real problems in railway engineering. Providing practical examples, such as calculating forces in lattice structures or Warren bridges, helps students apply numerical methods using the knowledge they have acquired up to that point. In addition, application programs are mentioned, which are also useful for advanced applications when related subjects are taken in later semesters. In conclusion, the document advocates an approach that links numerical methods with real-world applications in the field, seeking to improve learning and reduce dropout and failure rates.

Keywords: Engineering; Railway; methods; numerical; TecNM.

INTRODUCTION

Professors who teach the Numerical Methods course in the Railway Engineering specialization are concerned because real-world applications are not usually incorporated to solve problems, or they are presented in a theoretical or very abstract way, which has no meaning or real-world applications for engineering students. Furthermore, most universities only offer one course on Numerical Methods throughout the entire degree program. Therefore, it is recommended that the time allocated to the course be used to the maximum to understand and give real meaning to the application of the proposed methods. According to the study by Arroyo F., Cano J., Arroyo, M. (2020) at the National Technological Institute of Mexico (TecNM)/Cancun Technological Institute (ITC), high failure rates in engineering degrees occur at the beginning of higher education studies. This is because new students often fail in basic skills such as those necessary for optimal academic development, and they do not find a real application for the content in the academic programs. Although they have excellent results in the CENEVAL 2022 exam, when they join TecNM.

Although there are various applications and proposals in railways, such as presented by Amrapali, S. & Nandkishor, S. (2022), which provide solutions for bridge construction, or Deepak, S. & Patel, R. (2022) obtain results with the help of finite elements or deformations in catenaries and support posts, tools are required that students at this academic level of the degree program do not yet have. In addition, the use of application programs such as ANSYS$_{\hat{a}}$ or SAP2000$_{\hat{a}}$, among others, is required. These programs require a foundation in subjects that are covered in the final semesters of the degree program in question. Therefore, it is important to provide students with problems that they can solve with the mathematical tools they have acquired halfway through their studies.

MATERIALS

To improve the high dropout and failure rates, activities have been implemented to connect them with real problems in railway engineering in the classroom. Below are some examples from the course.

Example 1: Consider a lattice structure or a Warren-type railway bridge (Griggs Jr. F. 2015) or lattice (Azizi, M., et al. 2022), where you want to calculate the stresses at different points in the set of elements. The forces $\Psi_{(1)}$, $\Psi_{(2)}$, and $\Psi_{(3)}$ are divided into three components: $\Psi_{(x)}$, $\Psi_{(y)}$, **and** $\Psi_{(z)}$, where it can be three-dimensional and its distribution is shown in Figure 1.

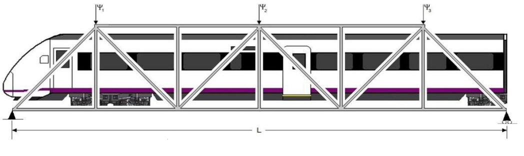

Figure 1. Lattice structure applied to a Warren--type bridge with simple support for calculating the resistance of its supports.

The problem consists of solving the system of linear equations of "m."lines and "n" unknowns as follows:

$$\begin{aligned}
\Psi_{1,1}x + \Psi_{1,2}y + \Psi_{1,3}z + \cdots + \Psi_{1,n}m &= \Psi_1 \\
\Psi_{2,1}x + \Psi_{2,2}y + \Psi_{2,3}z + \cdots + \Psi_{2,n}m &= \Psi_2 \\
\Psi_{3,1}x + \Psi_{3,2}y + \Psi_{3,3}z + \cdots + \Psi_{3,n}m &= \Psi_3 \\
\vdots \quad + \quad \vdots \quad + \quad \vdots \quad + \quad + \quad \vdots \quad &= \vdots \\
\Psi_{m,1}x + \Psi_{m,2}y + \Psi_{m,3}y + \cdots + \Psi_{m,n}m &= \Psi_m
\end{aligned}$$

In n x m (square) matrix notation, it can be written as follows:

$$\begin{vmatrix} \Psi_{1.1} & \Psi_{1.2} & \Psi_{1.3} & \cdots & \Psi_{1,n} \\ \Psi_{2.1} & \Psi_{2.2} & \Psi_{2.3} & \cdots & \Psi_{2.n} \\ \Psi_{3.1} & \Psi_{3.2} & \Psi_{3.3} & \cdots & \Psi_{3,n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \Psi_{m.1} & \Psi_{m,2} & \Psi_{m,3} & \cdots & \Psi_{m,n} \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ \vdots \\ m \end{vmatrix} = \begin{vmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \vdots \\ \Psi_m \end{vmatrix}$$

Or it can also be expressed as:

$$\Psi\mathbf{x} = \psi \qquad \text{(equation 1)}$$

Where Ψ represents the coefficient matrix for the system, the vector to be solved is called x= $[x, y, z, \ldots n]^T$**,** and $\psi = [\Psi_1, \Psi_2, \ldots \Psi_3]^T$corresponds to the other matrix of non-dependent terms. There are several different solution methods, as in (Chapra & Canale, 2015), (Mahesh et al. 2021) & (Reina-Pérez, F. 2022), using the methods of: Gaussian elimination, Gaussian pivoting, Gauss-Jordan, Gauss-Seidel, or the Jacobi procedure.

The Warren bridge example will be solved using the Gauss-Seidel method, starting from the equation $\Psi\mathbf{x} = \psi$ to obtain the solution.

$$\Psi\mathbf{x} - \psi = \mathbf{0} \qquad \text{(equation 2)}$$

To facilitate the solution, a vector equation corresponding to f(x) = 0 is considered, and the matrix **F** is to be found with the vector **C**, as follows:

$$\mathbf{x} = \Psi\,\mathbf{x} + \mathbf{c} \qquad \text{(equation 3)}$$

To arrive at the solution, take a matrix with a value for the first approximation called $x^{(0)}$, which will also be the initial approximation of the vector called **x**. Then calculate the following approximations from the value $\mathbf{x}^{(1)to}$ $\mathbf{x}^{(n)}$ in the equation $\mathbf{x} = \Psi\,\mathbf{x} + \mathbf{c}$, as shown below:

$$\boldsymbol{x}^{(k+1)} = \Psi \boldsymbol{x}^{k} + \boldsymbol{c}\text{, where k = 0, 1, 2, ..., n}$$

$$\text{Thus:}\boldsymbol{x}^{(k)} = [\boldsymbol{x}_1{}^{k}\boldsymbol{x}_2{}^{k}\boldsymbol{x}_3{}^{k} \ldots \boldsymbol{x}_n{}^{k}]^{T}$$

Cycles or iterations starting from 0 to value **n** are represented as: $\mathbf{x}^{(0)}$, $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, … $\mathbf{x}^{(n)}$, when properly set up, will tend toward a convergence of the solution vector **x**, so that the value $x_\alpha{}^m$, $1 \leq a \leq n$ (the components of the vector $x^{(m)}$ should approximate x_α, where $1 \leq a \leq n$ and should correspond to: **x**), so the absolute values:$|x_\alpha{}^m - x_\alpha|$, $1 \leq a \leq n$ must be less than a previously given value, which generally tends to be small and should give meaning to the characteristics of the problem and thus also to the result. Likewise, the values of the iteration vectors are preserved as:

$$\lim_{m\to\infty} x_\alpha{}^m = x_\alpha \qquad 1 \leq \alpha \leq n \quad \text{(equation 4)}$$

For the example in question, the algorithm and numerical value of convergencee are determined; proposed for the systemY x =y , the simplest way is to clear the values from x1 in the first equation, x2 in the second, and so on until the last value is reached. The results obtained for the main diagonal must be different from "0," and the solution is reached more quickly when the quantities are approximately equal to |1|.

The set or system of linear equations is represented as follows:

$$\begin{aligned}
\Psi_{1,1}\,x + \Psi_{1,2}\,y + \Psi_{1,3}\,w + \Psi_{1,4}\,z &= \psi_1\\
\Psi_{2,1}\,x + \Psi_{2,2}\,y + \Psi_{2,3}\,w + \Psi_{2,4}\,z &= \psi_2\\
\Psi_{3,1}\,x + \Psi_{3,2}\,y + \Psi_{3,3}\,w + \Psi_{3,4}\,z &= \psi_3\\
\Psi_{4,1}\,x + \Psi_{4,2}\,y + \Psi_{4,3}\,w + + \Psi_{4,4}\,z &= \psi_4
\end{aligned}$$

The following are proposed:

$$x^{(0)} = \frac{\psi_1}{\Psi_{1,1}}\ ,\quad y^{(0)} = \frac{\psi_2}{\Psi_{2,2}}\ ,$$

$$w^{(0)} = \frac{\psi_3}{\Psi_{3,3}}\ ,\quad z^{(0)} = \frac{\psi_4}{\Psi_{4,4}}$$

To check the proposed value ofe , it must be substituted into the original system of equations as shown below:

$$\begin{aligned}
\Psi_{1,1}\,x^{(0)} + \Psi_{1,2}\,y^{(0)} + \Psi_{1,3}\,w^{(0)} + \Psi_{1,4}\,z^{(0)} &= \psi_1{}^{(0)}\\
\Psi_{2,1}\,x^{(0)} + \Psi_{2,2}\,y^{(0)} + \Psi_{2,3}\,w^{(0)} + \Psi_{2,4}\,z^{(0)} &= \psi_2{}^{(0)}\\
\Psi_{3,1}\,x^{(0)} + \Psi_{3,2}\,y^{(0)} + \Psi_{3,3}\,w^{(0)} + \Psi_{3,4}\,z^{(0)} &= \psi_3{}^{(0)}\\
\Psi_{4,1}\,x^{(0)} + \Psi_{4,2}\,y^{(0)} + \Psi_{4,3}\,w^{(0)} + \Psi_{4,4}\,z^{(0)} &= \psi_4{}^{(0)}
\end{aligned}$$

Thus, the values

$$\begin{aligned}
\Delta\,\psi_1^{(0)} &= \psi_1 - \psi_1{}^{(0)}\\
\Delta\,\psi_2^{(0)} &= \psi_2 - \psi_2{}^{(0)}\\
\Delta\,\psi_3^{(0)} &= \psi_3 - \psi_3{}^{(0)}\\
\Delta\,\psi_4^{(0)} &= \psi_4 - \psi_4{}^{(0)}
\end{aligned}$$

The results obtained are compared againste , yes the values: $\Delta\psi^{(U)}$, $\Delta_1\psi^{(U)}$,$\Delta_2\psi^{(U)}$ $\Delta_3\psi^{(U)}$ and $\Delta_4\psi^{(U)}$ are less than or equal to$\boldsymbol{\varepsilon}$ and the procedure is interrupted.

To calculate the second element of the matrix called $x^{((1))}$,$x^{(0)}$is substituted·In this particular case, we obtain:

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$$x^{(1)} = \frac{\psi_1}{\Psi_{1,1}} - \frac{\Psi_{1,2}}{\Psi_{1,1}} y^{(0)} - \frac{\Psi_{1,3}}{\Psi_{1,1}} w^{(0)} - \frac{\Psi_{1,4}}{\Psi_{1,1}} z^{(0)} =$$

$$y^{(1)} = \frac{\psi_2}{\Psi_{2,2}} - \frac{\Psi_{2,1}}{\Psi_{2,2}} x^{(1)} - \frac{\Psi_{2,3}}{\Psi_{2,2}} w^{(0)} - \frac{\Psi_{2,4}}{\Psi_{2,2}} z^{(0)} =$$

$$w^{(1)} = \frac{\psi_3}{\Psi_{3,3}} - \frac{\Psi_{3,1}}{\Psi_{3,3}} x^{(1)} - \frac{\Psi_{3,2}}{\Psi_{3,3}} y^{(1)} - \frac{\Psi_{3,4}}{\Psi_{3,3}} z^{(0)} =$$

$$z^{(1)} = \frac{\psi_4}{\Psi_{4,4}} - \frac{\Psi_{4,1}}{\Psi_{4,4}} x^{(1)} - \frac{\Psi_{4,2}}{\Psi_{4,4}} y^{(1)} - \frac{\Psi_{4,3}}{\Psi_{4,4}} w^{(1)} =$$

$$\frac{\psi_1 - \Psi_{1,2} y^{(0)} - \Psi_{1,3} w^{(0)} - \Psi_{1,4} z^{(0)}}{\Psi_{1,1}}$$

$$\frac{\psi_2 - \Psi_{2,1} x^{(1)} - \Psi_{2,3} w^{(0)} - \Psi_{2,4} z^{(0)}}{\Psi_{2,2}}$$

$$\frac{\psi_3 - \Psi_{3,1} x^{(1)} - \Psi_{3,2} y^{(1)} - \Psi_{3,4} z^{(0)}}{\Psi_{3,3}}$$

$$\frac{\psi_4 - \Psi_{4,1} x^{(1)} - \Psi_{4,2} y^{(1)} - \Psi_{2,3} w^{(1)}}{\Psi_{4,4}}$$

To verify the maximum error value ofe, the values obtained are replaced in the original equation and presented as follows:

$$\Psi_{1,1}\, x^{(1)} + \Psi_{1,2}\, y^{(1)} + \Psi_{1,3}\, w^{(1)} + \Psi_{1,4}\, z^{(1)} = \psi_1^{(1)}$$
$$\Psi_{2,1}\, x^{(1)} + \Psi_{2,2}\, y^{(1)} + \Psi_{2,3}\, w^{(1)} + \Psi_{2,4}\, z^{(1)} = \psi_2^{(1)}$$
$$\Psi_{3,1}\, x^{(1)} + \Psi_{3,2}\, y^{(1)} + \Psi_{3,3}\, w^{(1)} + \Psi_{3,4}\, z^{(1)} = \psi_3^{(1)}$$
$$\Psi_{4,1}\, x^{(1)} + \Psi_{4,2}\, y^{(1)} + \Psi_{4,3}\, w^{(1)} + \Psi_{4,4}\, z^{(1)} = \psi_4^{(1)}$$

Thus, the values

$$\Delta \psi_1^{(1)} = \psi_1 - \psi_1^{(1)}$$
$$\Delta \psi_2^{(1)} = \psi_2 - \psi_2^{(1)}$$
$$\Delta \psi_3^{(1)} = \psi_3 - \psi_3^{(1)}$$
$$\Delta \psi_4^{(1)} = \psi_4 - \psi_4^{(1)}$$

The aim is that: $\Delta\psi^{(0)}{}_1 \geq \Delta\psi_1{}^{((1)}, \Delta\psi_2{}^{((0)} \geq \Delta\psi_2{}^{((1)}, \Delta\psi_3{}^{((0)} \geq \Delta\psi_3{}^{((1)}, \Delta\psi_4{}^{((0)} \geq \Delta\psi_4{}^{(1)}$.

Once again, the maximum error values are comparede , yes:$\Delta\psi^{(1)}{}_1 \leq \varepsilon$,$\Delta\psi^{(1)} \leq_2 \varepsilon$,$\Delta\psi^{((1))}{}_3 \leq e$ &$\Delta\psi^{(1)} \leq \varepsilon_4$. The process is interrupted.

To calculate the next element of the matrix or vector $y^{(2),}$ substitute $x^{(1)}$and obtain the results for: $y^{(2)}$, $w^{(2)}$, and $z^{((2))}$.

$$x^{(2)} = \frac{\psi_1 - \Psi_{1,2} y^{(1)} - \Psi_{1,3} w^{(1)} - \Psi_{1,4} z^{(1)}}{\Psi_{1.1}}$$

$$y^{(2)} = \frac{\psi_2 - \Psi x^{(2)} - \Psi w^{(1)} - \Psi z^{(1)}}{\Psi_{2.2}}$$

$$w^{(2)} = \frac{\psi_3 - \Psi_{3.1} x^{(2)} - \Psi_{3.2} y^{(2)} - \Psi_{3.4} z^{(1)}}{\Psi_{3.3}}$$

$$z^{(2)} = \frac{\psi_4 - \Psi_{4.1} x^{(2)} - \Psi_{4.2} y^{(2)} - \Psi_{2.3} w^{(2)}}{\Psi_{4.4}}$$

And the process continues.

To present the values of x, y, w, & z in a clearer way, such as those of

Iterations	Values $_{x}$(n)	Dy1 (n)	Values $_{y}$(n)	$\Delta\psi 2$ (n)	Values $_{w}$(n)	$\Delta\psi 3$ (n)	Values $_{z}$(n)	$\Delta\psi 4$ (n)
0	$_{x}{}^{(0)}$	Dy1 (0)	$_{y}{}^{(0)}$	$\Delta\psi 2$ (0)	$_{w}{}^{(0)}$	$\Delta\psi 3$ (0)	$_{z}{}^{(0)}$	$\Delta\psi 4$ (0)
1	$_{x}{}^{(1)}$	Dy1 (1)	$_{y}{}^{(1)}$	$\Delta\psi 2$ (1)	$_{w}{}^{(1)}$	$\Delta\psi 3$ (1)	$_{z}{}^{(1)}$	$\Delta\psi 4$ (1)
2	$_{x}{}^{(2)}$	Dy1 (2)	$_{y}{}^{(2)}$	$\Delta\psi 2$ (2)	$_{w}{}^{(2)}$	$\Delta\psi 3$ (2)	$_{z}{}^{(2)}$	$\Delta\psi 4$ (2)
3	$_{x}{}^{(3)}$	Dy1 (3)	$_{y}{}^{(3)}$	$\Delta\psi 2$ (3)	$_{w}{}^{(3)}$	$\Delta\psi 3$ (3)	$_{z}{}^{(3)}$	$\Delta\psi 4$ (3)
4	$_{x}{}^{(4)}$	Dy1 (4)	$_{y}{}^{(4)}$	$\Delta\psi 2$ (4)	$_{w}{}^{(4)}$	$\Delta\psi 3$ (4)	$_{z}{}^{(4)}$	$\Delta\psi 4$ (4)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1. Summary of each iteration, values, andε obtained using the Gauss-Seidel iterative method.

Gauss-Seidel iterative method.

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$\Delta\psi 1,\Delta\psi\ 2,\Delta\psi\ 3$ &$\Delta\psi\ 4$, a table is created that summarizes this data as follows:

Table 1 shows the reduction of$\Delta\psi\ 1,\Delta\psi\ 2,\Delta\psi\ 3$ &$\Delta\psi\ 4$ in each iteration; if they increase, it is due to a calculation error.

Thus, the number of iterations from “0” to m that are necessary to complete the method will depend on the given value of ε that was defined at the beginning. This numerical procedure has the following advantages:

- The approximate solution can be reached from the first rounds of the process.
- In each iteration, it calculates and identifies the error made.
- For systems of linear equations, it is more efficient than direct methods.
- It is possible to determine the required level of precision from the outset.
- It uses less memory than other methods when scheduling calculations.

It has some disadvantages:

- The element $\mathbf{A}^{-1}$is not obtained, nor is the determinant of **A.**
- It may be that when executing the calculations to converge to the result, it is slower because it can oscillate or if high accuracy is desired.

In modern railways, the poles or supports of the catenary systems are essential as they support the porticos, semi-porticos, and other parts that supply electrical power to the electric motors of the train. In order to comply with the above, certain basic characteristics must be met, such as:

The calculation to support the stresses to which they are subjected, as they contain the catenary, brackets, supports, etc.

Its deformation under various loads is calculated, and its geometry must not be altered, as this would also be reflected in the catenary and could fall outside the limits standardized by ISO 13920:2023. An example in static form is shown below.

Example 2. A support is shown where the railway catenaries are installed as indicated in Figure 2. A horizontal force is applied, caused by the weight of the supports and the cables**,** which will generate the force named F at the top**,** which deforms a distanced each time the “convoy pantographs” pass, which is a function of F or can be written asd 0 = **f** *(F0*) as follows:

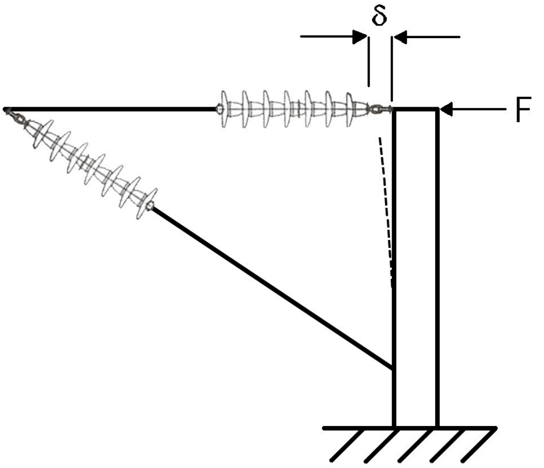

Figure 2. Application of a force to the catenary support and its deformationd .

Subsequently, each of the deformationsd is evaluated each time the force F is increased, and the results can be summarized as shown in Table 2:

Measurements	Forces	Deformations
0	F_0	δ_0
1	F_1	δ_1
2	F_2	δ_2
3	F_3	δ_3
4	F_4	δ_4
5	F_5	δ_5

Table 2. Applied force versus deformationsd .

The force ***F*** applied is perpendicular to the support of the beam that supports the catenary, and it is assumed that it will maintain its shape when the load is applied. The question is to find the deformationd **6** when applying the force ***F6,*** since when exceeding its maximum deformation $\delta_{\mathbf{max}}$ the element could break.

This method uses extrapolation by polynomial approximation to solve linear equations. It is simple with few data points (around 5), but becomes more complicated with a larger number of data points.

The solution begins as follows: the equation is expressed in general terms and then applied to the catenary support (including supports and cables). The first-degree function f(x) is proposed; it is tabulated and expressed as follows:

p(x)=a o(x-x1)+a 1 (x-xo) (equation 5)

The arguments of the function begin with the set of points x0, f(x0), & x1, f(x1), and the valuesa 0, & α 1 to be found; and to determine the coefficienta 0, it is assumed that x = x0 in the previous equation, resulting in:

$$\alpha_0 = \frac{p(x_0)}{x_0 - x_1} = \frac{f(x_0)}{x_0 - x_1} \quad \text{(equation 6)}$$

Next, to find the value of α 1, x is replaced by x1, and the following result is obtained:

$$\alpha_1 = \frac{p(x_1)}{x_1 - x_0} = \frac{f(x_1)}{x_1 - x_0} \quad \text{(equation 7)}$$

Substituting equations (6) and (7) into equation 5, we arrive at the following expression:

$$p(x) = \frac{f(x_0)}{x_0 - x_1}(x - x_1) + \frac{f(x_1)}{x_1 - x_0}(x - x_0) =$$

$$f(x_0)\frac{(x - x_1)}{x_0 - x_1} + f(x_1)\frac{(x - x_0)}{x_1 - x_0}$$

To find the quadratic or parabolic function, we have the following:

$P_2(x) = \alpha_o(x-x_1)\ (x-x_2)\ + a_1(x-x_o)\ (x-x_2)$ $+a_2(x-x_o)\ (x-x_1)$

$$\alpha_2 = \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

In the example, Newton's polynomial approximation is used for the solution, where:

$$p_1(x) = f\,[x_0,x_1](x - x_0) + f(x_0)$$

For this procedure it is proposed that:

$$f[x_0,x_1] = \frac{f(x1) - f(x0)}{x1 - x0}$$

The quadratic equation is expressed as:

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p_2 (x) $=p_1$ (x) $+f\ [x_0,x_1,x_2](x-x_{(0})$ $(x-x_1)$

$=f\ (x_0)\ +f\ \ [x_0,x_1]\ (x\ -x_0)\ +f$ $[x_0,x_1,x_2](x-x_0)(x-x_1)$

The third-degree equation is:

$p_3(x)\ =p_2(x)\ +f\ [x_0,x_1,x_2,x_3](x\ -$ $x_0)(x-x_1)(x-x_2)f\ (x_0)\ +f\ [x_0,x_1]$ $(x-x_0)$

$=f\ [x_0,x_1,x_2](x\ -x_{(0})(x\ -x_1)\ +f$ $[x_0,x_1,x_2,x_3](x-x_{(0})(x-x_1(x-x_2)$

Table 3 is constructed in order to obtain and observe its tabulation for each term in a simpler way.

The 4th order polynomial:

$p_4(x) =f\ (x_0) +f\ [x_0,x_1](x-x_0) +f$ $[x_0,x_1,x_2](x-x_0)(x-x_1)$

$+f\ [x_0\ ,x_1\ ,x_2\ ,x_3\ ](x-x_0\ )(x-x_1\ )$ $(x-x_2\ )$

$+\ (f\ [x_0,x_1,x_2,x_3,x_4]x-x_{(0})(x-x_1(x$ $-x_2(x-x_{(3})$

And to find the equation of the 5th order polynomial, we have:

$p5(x) =f\ (x0) +f\ [x0,x1](x-x0) +f$ $[x0,x1,x2](x-x0)(x-x1)$

$+f\ [x_0\ ,x_1\ ,x_2\ ,x_3\ ](x-x_0\ )(x-x_1\ )(x-$ $x_2\ )$

$+f\ [x_0\ ,x_1\ ,x_2\ ,x_3\ ,x_4\ ](x-x_0\ )(x-x_1\ )$ $(x-x_2\ )(x-x_3\ )$

$+f\ [x_0,x_1,x_2,x_3,x_4,x_5](x\ -x_{(0})(x\ -x_1(x$ $-x_2(x-x_{(3})(x-x_4)$

After substituting the values of F0, F1, … F5, calculations must be performed to find the function f(x0), f(x1), … f(x5), which in this case are the results obtained in the deformations:δ_0,δ_1, … δ_5. The results are intended to be accurate in order to help find the extrapolation when replacing the values of F6, and it is assumed that by correctly applying the method, a value lower than δ max (maximum deformation in the structure) is reached so that there is no failure or fracture in the catenary support.

Example 3. Bacterial growth is shown on the exterior windows of the train, which can obstruct the driver's visibility. The weekly increase is shown in Table 5.

Measurements	Week	Bacteria count
0	*0*	31.6
1	1	100
2	3	1 x10^{15}
3	4	3.1622776 x 10^{33}

Table 5. Bacterial growth in different weeks.

The estimated bacterial count for week 5ªis sought; in the case of very large values, this can be solved using base 10 logarithms, as shown in Table 6.

Measurements	Week	Bacterial count	Log 10x
0	*0*	31.6	1.5
1	1	100	2
2	3	1 x10^{15}	15
3	4	3.1622776 x 10^{33}	33.5
4	5	??	

Table 6. Logarithm values for bacteria counts.

The resulting polynomials are:

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X	f(x)	1st term	2nd term	3rd term
x0	f(x0)			
		$f[x_0,x_1]=\frac{f(x_1)-f(x_0)}{x_1-x_0}$		
x1	f(x1)		$f[x_0,x_1,x_2]=\frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$	
		$f[x_1,x_2]=\frac{f(x_2)-f(x_1)}{x_2-x_1}$		$f[x_0,x_1,x_2,x_3]=\frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$
x2	f(x2)		$f[x_1,x_2,x_3]=\frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$	
		$f[x_2,x_3]=\frac{f(x_3)-f(x_2)}{x_3-x_2}$		$f[x_1,x_2,x_3,x_4]=\frac{f[x_2,x_3,x_4]-f[x_1,x_2,x_3]}{x_4-x_1}$
x3	f(x3)		$f[x_2,x_3,x_4]=\frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$	
		$f[x_3,x_4]=\frac{f(x_4)-f(x_3)}{x_4-x_3}$		$f[x_2,x_{(3)},x_4,x_5]=\frac{f[x_3,x_4,x_5]-f[x_1,x_2,x_3]}{x_5-x_2}$
X4	f(x4)		$f[x_3,x_4,x_5]=\frac{f[x_4,x_5]-f[x_3,x_4]}{x_5-x_3}$	
x5	f(x5)	$f[x_4,x_5]=\frac{f(x_5)-f(x_4)}{x_5-x_4}$		

Table 3. General formulas for polynomial approximations from the 1st to the 3rd term.

X	f(x)	4th term	5th term
x1	f(x1)		
x2	f(x2)	$f[x_0,x_1,x_2,x_3,x_4]=\frac{f[x_1,x_2,x_3,x_4]-f[x_0,x_1,x_2,x_3]}{x_4-x_0}$	
x3	f(x3)	$f[x_1,x_2,x_3,x_4,x_5]=\frac{f[x_2,x_3,x_4,x_5]-f[x_1,x_2,x_3,x_4]}{x_5-x_1}$	$f[x_0,x_{(1)},x_2,x_3,x_4,x_5]=\frac{f[x_1,x_2,x_3,x_4,x_5]-f[x_0,x_1,x_2,x_4,x_5]}{x_5-x_0}$
x4	f(x4)		

Table 4. General formulas for 4th and 5th degree polynomial approximations.

It can be solved by applying

f(x0)

$$f[x_0,x_1]=\frac{2-1.5}{1-0}=0.5$$

f(x1)

$$f[x_1,x_2]=\frac{15-2}{3-1}=6.5$$

f(x2)

$$f[x_2,x_3]=\frac{33.5-15}{4-3}=18.5$$

$$f[x_0,x_1,x_2]=\frac{6.5-0.5}{3-0}=2$$

$$f[x_1,x_2,x_3]=\frac{18.5-6.5}{4-1}=4$$

$$f[x_0,x_1,x_2,x_3]=\frac{4-2}{4-0}=0.5$$

$p_1(x) = f[x_0, x_1](x - x_0) + f(x_0) = 1.5 + 0.5x$

$p_2(x) = p_1(x) + f[x_0, x_1, x_2](x - x_{(0)})(x - x_{(1)}) = 1.5 + 0.5x + x(x - 1)2 = 2x^2 - 1.5x + 1.5$

$p_{(3)}(x) = p_{(2)}(x) + f[x_0, x_1, x_2, x_3](x - x_{(0)})(x - x_{(1)})(x - x_2) f x_0) + f[x_0, x_1](x - x_0) = = 2x^2 - 1.5x + 1.5 + x(x - 1)(x - 3)(0.5) = 0.5x^3 + 1.5$

Evaluating the equation with the initial values gives the values

Week	Bacteria count	$y(x) = 0.5x3 + 1.5$	alog 10^x
0	31.6	$y(x) = 0.5(0)^3 + 1.5 = \mathbf{1.5}$	31.5
1	100	$y(x) = 0.5(1)^3 + 1.5 = \mathbf{2}$	100
3	1×10^{15}	$y(x) = 0.5(3)^3 + 1.5 = \mathbf{15}$	1×10^{15}
4	3.1622776×10^{33}	$y(x) = 0.5(4)^3 + 1.5 = \mathbf{33.5}$	3.1622776×10^{33}
5	¿?	$y(x) = 0.5(5)^3 + 1.5 = \mathbf{64}$	**1×10^{64}**

Table 7. Values before and after obtaining the polynomials.

As can be seen, all the values match the originals, so the equation is correct. To obtain the value in the 5^a week' the base 10 antilogarithm of 64 is applied, resulting in **1×10^{64} bacteria.**

With this method, results or events can be predicted by knowing the previous values.

RESULTS

When explaining the Gauss-Seidel method, start as if working with a vertical beam supported at its base and apply the method to extrapolate using polynomial or Lagrange approximation, and present the post to support the catenary by applying a horizontal force **F** against its deformations.

δ Railway Engineering students not only learn the procedure, but also become more interested in solving problems in this area. Similar efforts were made at TecNM in the area of Civil Engineering, as mentioned by Arroyo F. et al. (2021), with excellent results and a significant reduction in the failure rate in Numerical Methods.

CONCLUSIONS

By applying exercises focused on railway engineering, the pass rate for this subject improved by 50% when using examples specific to its area of application.

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REFERENCES

Amrapali, S. & Nandkishor, S. (2022). Comparative Analysis and design of Pratt Truss Bridge and Warren truss Bridge as per AISC, and ASSHTO LRFD 200 by using Autodesk Structural Analysis Software. International Research Journal of Engineering and Technology (IRJET). Vol. 09 Issue 06, jun. 2022. e-ISSN: 2395-0056. P-ISSN: 2395-0072.

Andrade, H. (2014). Estructuras 1. Universidad de Cuenca ISBN 978-9978-14- 244-8.

Arroyo, F. (2024). Apuntes de asignatura Métodos Numéricos. Tecnológico Nacional de México / Instituto Tecnológico de Cancún. México.

Arroyo, F., Arroyo, M, & Arroyo, M. (2021). Métodos Numéricos Aplicados a la Ingeniería Civil. Atena Editora. https://doi.org/10.22533/at.ed.533210212

Arroyo, F., Cano, J., Arroyo, M. (mayo 2020). Detección de deficiencias académicas en los aspirantes en 2019 a las ingenierías del Instituto Tecnológico de Cancún. Memorias del Congreso Internacional de Investigación Academia Journals.Chetumal. Consulta enero 2021. https://drive.google.com/drive/folders/1nVmDJmY8gBjI7gKB5EywJuNYS42Ph05c

Azizi, M., Baghalzadeh Shishehgarkhaneh, M. & Basiri, M. (2022). Optimum design of truss structures by Material Generation Algorithm with discrete variables. *Decision Analytics Journal*, 3, ID del artículo: 100043. https://doi.org/10.1016/j.dajour.2022.100043

CENEVAL (Centro Nacional de Evaluación para la Educación Superior). (2022). Guía EXANI-II. Examen Nacional de Ingreso a la Educación Superior 23ª. Edición.Consultaenero2022.https://www.ceneval.edu.mx/documents/20182/232201/2_Guia_EXANI-II_210531.pdf/e5a7e4ca-8a72-4c07-ae8d-1bcab232ae48

Chapra, S., Canale, R. (2015). Métodos numéricos para Ingenieros. 7ª edición. Mc. Graw Hill. ISBN 9786071515949.

Deepak, S. & Patel, R. (2022). Comparative analysis of two different steel bridges considering live project data using analysis tool sap2000. Journal of Xi'an Shiyou University, Natural Science Edition. Vol 18 issue 0ct. 2022 pp. 944- 957. ISSN: 1673-064X

Griggs Jr. F. (2015). El Armazón Warren. *Recurso de primera para ingenieros estructurales en ejercicio.* (The Warren Truss. Premiere resource for practicing structuralengineers). StructureMagazine. https://www.structuremag.org/article/the-warren-truss/

ISO 13920:2023 (2023). General tolerances for welded constructions — Dimensions for lengths and angles, shape and position. [Tolerancias generales para construcciones soldadas]. Secretaría Central de ISO OrganizaciónInternacionaldeNormalización. https://www.iso.org/es/contents/data/standard/08/60/86032.html

Mahesh, T., Barbude, P. & Radke, A. (2021) Finding Span to Depth Ratio of Double Warren Steel Truss for Least Weight Subjected To Double Track Railway Loading. https://doi.org/10.9790/9622-1107042935

Migdalovici, M. & Baran, D. (2005). About the Stability of the Pantograph- Catenary Dynamical System. 67.

Nieves, A., Domínguez, F. (2012) Métodos numéricos aplicados a la Ingeniería. 4ª edición. Grupo Editorial Patria. ISBN 9786074383171

Reina-Pérez, F. (2022). Evaluación Comparativa entre los Métodos de Flujos de Potencia Gauss Seidel y Newton Raphson. Pol. Con. (Edición núm. 70) Vol. 7, No 6 Junio 2022, pp. 892-906. https://doi.org/10.23857/pc.v7i6